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Calculation of Heating Duration for Complex Shaped Objects by Equivalent Size Method

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During the heating calculation of products in the heating furnaces a simplified method for determining the heating duration gained currency. According to it a heated complex shape product is replaced by a simple shaped equivalent. The article describes the theoretically based method of determining the choice of the size of an equivalent object, at which the permissible error of calculation is provided.

Keywords: COMPLEX SHAPED OBJECTS, HEATING DURATION, EQUIVALENT SIZE, SIMPLY SHAPED OBJECT

Introduction

Heating calculation of products in industrial furnaces is in most cases reduced to determination of the heating duration to the desired temperature at the surface and on the axis (in the center) of the product.

In practice of such calculations a simplified engineering approach, according to which products in the shape of prism, brick, cylinder of finite length are substituted for calculating with the equivalent simply shaped objects: unlimited cylinder, unbounded plate, sphere, 2D and 3D temperature fields of thermally massive objects become one-dimensional.

The validity of a simplified approach proves allowable discrepancy in the results of calculations of heating complex and simply shaped objects.

Results and Discussion

The validity of a simplified approach depends on the correct choice of the equivalent simply shaped object: radius of a cylinder/ a sphere R_{eq}, plate thickness S_{eq}.

For example, in [1] when calculating the heating of ingots in heating wells an ingot with square cross section 2S•2S was replaced by an equivalent cylinder the radius of which is found by equating the cross-sectional areas.

$$(2S)^2 = \pi \cdot R_{eq}^2 \qquad (1)$$

The discrepancy in the results of the calculation of heating duration of ingots was 6.4%.

In the reference book the set of billets heated in a furnace and located on the hearth parallel to each other with gaps of uniform thickness is recommended to consider a continuous plate, the thickness of which S_{eq} depends on the ratio of gap and billet width, as well as the emissivity coefficient of the furnace system. Accuracy of calculations is not assessed; the method of choice S_{eq} is not explained.

The purpose of this study is to offer a theoretically based method for choosing the size R_{eq} (S_{eq}) and check its adequacy by comparing the results of calculations of heating of the prism and the equivalent cylinder of infinite length.

The heat balance of the solid heating is considered as

$$\bar{q} \cdot F_{surf} \cdot \tau = M \cdot \Delta i \qquad (2)$$

where $\bar{q}$ - the average density of the resulting surface heat flow of the object, kW/m2; F_{surf} - heat-absorbing surface area, m^2; τ - the duration of the heating process from the initial to the final temperature, sec; M - mass, kg. $\Delta i = (\bar{c}_0^{\bar{t}f} \cdot \bar{t}_f - \bar{c}_0^{\bar{t}i} \cdot \bar{t}_i)$ - increase in the specific enthalpy of the object during the heating from the initial bulk temperature $\bar{t}_f$ to the initial one $\bar{t}_i$, kJ/kg; $\bar{n}_0^{\bar{t}i}$, $\bar{n}_0^{\bar{t}f}$ - mean specific heat of the object in the range from 0°C to $\bar{t}_f$ and $\bar{t}_i$ respectively. From the balance equation (2) for the object of any shape it follows that

$$\tau = \frac{M \cdot \Delta i}{F_{surf} \cdot \bar{q}} . \quad (3)$$

For the equivalent cylinder or sphere in accordance with [3] it can be written

$$\frac{M}{F_{surf}} = \frac{R_{eq} \cdot \rho}{\hat{E}_1} ,$$

from which

$$\tau_{eq} = \frac{R_{eq} \cdot \rho \cdot \Delta i_{eq}}{\bar{q}_{eq} \cdot \hat{E}_1} , \quad (4)$$

where R_{eq} – radius or a cylinder or a sphere; $\hat{E}_1$ - mass index; for a plate $\hat{E}_1$ = 1; a cylinder $\hat{E}_1$ = 2; or a sphere $\hat{E}_1$ = 3; ρ – density, kg/m^3.

The heating duration of the complex shaped object (3) and the equivalent simple shaped object (4) must coincide, i.e. $\tau = \tau_{eq}$. Besides the problem situation requires that $\Delta i = \Delta i_{eq}$. If during the transition to an equivalent simple shaped object, we keep the same M / F_{surf} as for the complex shaped object, then we come to the necessity of equality $\bar{q} = \bar{q}_{eq}$, which is not always fulfilled. This fact is a source of short-cut calculation error.

Thus, it is suggested to calculate the size of a simple shaped object, which is equivalent to the complex shaped object on the basis of equality of the following quantities: heating duration, enthalpy expansion, the ratio of the mass heat-absorbing surface area and of the mean density of resulting surface heat flow.

Equating (3) and (4), we obtain

$$\frac{M \cdot \Delta i}{F_{surf} \cdot \bar{q}} = \frac{R_{eq} \cdot \rho \cdot \Delta i_{eq}}{\bar{q}_{eq} \cdot \hat{E}_1} ,$$

from which after the shortcut we have

$$R_{eq} = \frac{M \cdot K_1}{F_{surf} \cdot \rho} \text{, m} \quad (5)$$

If it is convenient for an equivalent object to take the form of an infinite plate, the calculated thickness of Seq is also found with the help of formula (5).

In order to test the adequacy of the suggested method of determining the equivalent size of the simple shaped object, two examples of calculating the heating of a prism with square cross section 2S•2S and the equivalent cylinder, the radius of which is determined from expressions (1) and (5).

Example 1. A prism shaped bloom with square cross section *2S•2S* of infinite length is heated at a constant temperature of the furnace from all sides.

Assumptions:

Thermophysical properties of the metal in the heating process do not change. The heat-transfer coefficient from the furnace to the metal surface is assumed to be constant.

Source data:

Furnace temperature t_{fur} = 1000 °C;
Initial temperature of the product t_i = 20 °C;
General heating duration τ = 3600 с;
Heat-transfer coefficient α = 200 W/(m2·K);
Heat-conduction coefficient λ = 35 W /(m·K);
Specific heat capacity of the metal c_m = 700 J/(kg·K);
Metal density ρ = 7800 kg/m3;
Prism square section side 2*S* = 0,15 м.

Equivalent bloom is accepted to be in the form of a cylinder of infinite length.

From the expression (1) we obtain

$$R_{eq(1)} = \sqrt{\frac{(2S)^2}{\pi}} = 0.0846 \text{ m}$$

From the expression (5) it follows that

$$R_{eq(5)} = \frac{M \cdot K_1}{F_{surf} \cdot \rho} = \frac{(2S)^2 \cdot 2}{4 \cdot 2S} = 0.075 \text{ m.}$$

Calculation of heating of the prism of infinite length with the given parameters was performed by the analytical solution of the heat conduction equation, presented in [4].

Temperature at different moments was calculated at characteristic points of heated prism, which are shown in **Figure 1**. The values of simplexes x/S, y/S at these points are equal:

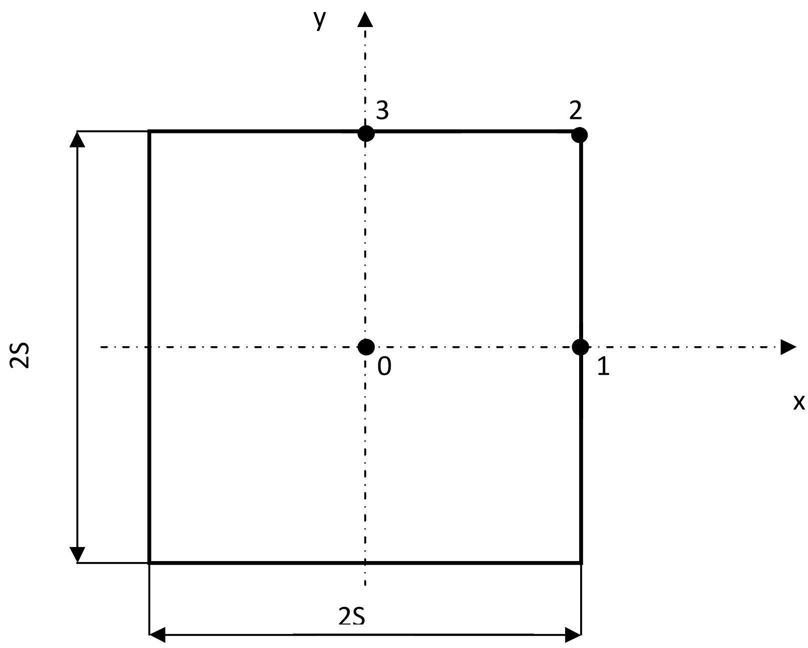

Figure 1. Scheme of the prism cross section with characteristic points of the temperature calculation (**Example 1**)

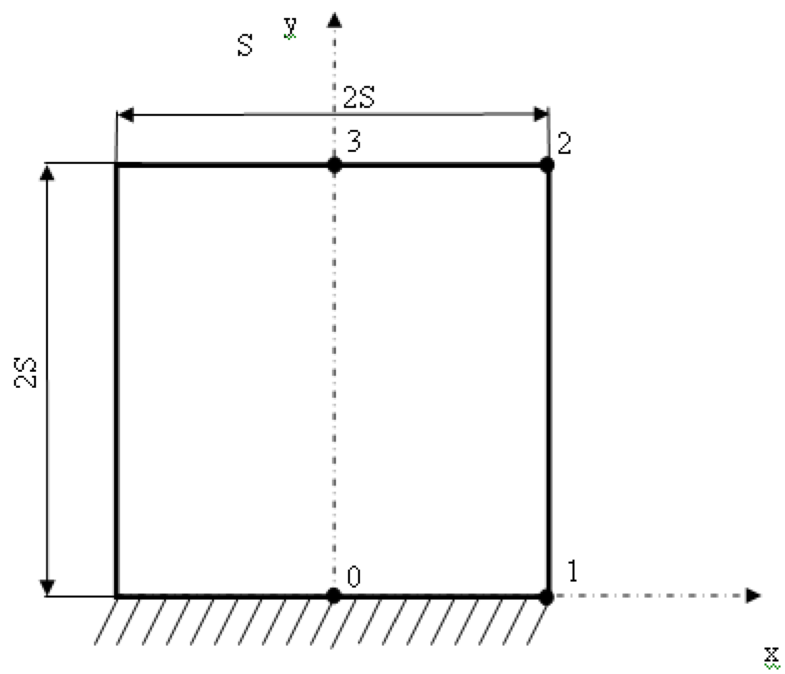

Figure 2. Scheme of the heated on the adiabatic bottom prism cross section with characteristic points of the temperature calculation (**Example 2**)

The average temperature of the prism surface was defined as

$$t_{surf} = (t_1 + 2 \cdot t_2 + t_3)/4$$

The temperature at the axis of the prism is

Table 1. Comparative calculation results of the prism and the equivalent cylinders heating in the **Example 1**

№	Shape	Method of choosing equivalent size	Size, m	Temperature in the end of heating, °C		Heating duration τ, c	Divergence τ, %
				t_{surf}	t_{ax}		
1.	Prism		2S·2S = 0,15·0,15	962	949	3600	
2.	Cylinder	expression (5)	R_{eq} = 0,075	959	949	3500	2,8
3.	Cylinder	expression (1)	R_{eq} = 0,0846	959	949	4000	11

$t_{ax}=t_0$.

Here t_0, t_1, t_2, t_3 – temperatures at the points 0,1,2,3.

Calculation of heating duration of the equivalent cylinder to the temperatures obtained at the end of the prism heating was made with the help of numerical and analytical method [5].

Comparative calculation results are given in **Table 1.**

Example 2. A prism shaped bloom with square cross section 2S•2S is located on the adiabatic bottom and heated on three sides (**Figure 2**) during the time τ at a constant furnace temperature. Assumptions and source data for calculation are the same as in Example 1. According to the equality of cross-section areas (1) the radius the equivalent cylinder will still be equal R_{eq} = 0.0846 m. In accordance with the

Table 2. Comparative calculation results of the prism and the equivalent cylinders heating in the **Example 2**

№	Shape	Method of choosing equivalent size	Size, m	Temperature in the end of heating, °C		Heating duration τ, c	Divergence τ, %
				t_{surf}	t_{ax}		
1.	Prism		2S·2S = 0,15·0,15	917	874	3600	
2.	Cylinder	expression (5)	R_{eq} = 0,1	906	878	3500	2,8
3.	Cylinder	expression (1)	R_{eq} = 0,0846	905	880	2900	19, 4

suggested method the radius of the equivalent cylinder (5) will be equal

$$R_{eq} = \frac{M \cdot K_1}{F_{surf} \cdot \rho} = \frac{(2S)^2 \cdot 2}{3 \cdot (2S)} = 0.1$$

The prism and the equivalent cylinder heating are calculated as well as in **Example 1**. Temperature at different moments was calculated at characteristic points of heated prism, which are shown in **Figure 2.** Calculation results are given in **Table 2**.

According to **Tables 1,2** the divergence between the results of calculation of heating duration of the prism and the equivalent cylinder with the choice of Req from expression (5) doesn't exceed 3 %.

Conclusions

Calculation of heating of the complex shaped objects with their replacement by equivalent simple shaped objects in the form of an unlimited cylinder, an infinite plate or a sphere provides a permissible error and can be used in engineering calculation practice upon condition of determining the size of the equivalent object using the method suggested in this paper

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Расчет продолжительности нагрева тел сложной формы с исполь-зованием метода эквивалентного размера

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При расчете нагрева изделий в нагревательных печах получил распространение упрощенный метод определения продолжительности нагрева, при котором нагреваемое изделие сложной формы заменяют эквивалентным телом простой формы. В статье рассмотрен теоретически обоснованный метод выбора определяющего размера эквивалентного тела, при котором обеспечивается допустимая погрешность расчета.