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Numerical Mathematical Modeling of Stress-Strain State of Metal during Thin Strips Hot Rolling

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The methods of mathematical modeling of stress-strain state of metal during rolling were analyzed. Based on the received bonds mathematical models for determining the local characteristics of the stress-strain state of metal in the zone of plastic forming were established. The data on the size is somewhat smaller than the data obtained on the basis of numerical recurrent solutions of finite-difference form of the equilibrium condition. It was found that the degree of mismatch of the energy and force approaches was less than 3%.

Keywords: ENERGY CONSUMPTION BALANCE, EQUILIBRIUM CONDITION, STRESS-STRAIN STATE OF METAL, MATHEMATICAL MODEL, PLASTIC FLOW OF METAL, THIN SHEETS AND STRIPS

Introduction

Under conditions of up-to-date economic situation it is extremely important to save material resources. Therefore, further development of metallurgy on the basis of experimental studies is impossible. Mathematical modeling of the stress-strain state of metal during rolling can solve this problem.

In works [1, 2] there is a description of mathematical models of the stress-strain state of metal during hot rolling of relatively thin strips. However, these studies take into account the effect of friction on the basis of Coulomb-Amontons law; whereas work [3] states that in relation to hot rolling the use of the law is impractical. In this case, the authors of work [3] recommend the use of plastic friction law (Siebel friction law).

Consequently, the creation of mathematical models that take into account possible factors influencing the stress state of metal during hot rolling on broad-strip mills is up to date.

Results and Discussion

During the numerical mathematical modeling of the stress-strain state of metal during hot rolling of relatively thin sheets and strips one should not forget about the influence of boundary conditions, and consequently the possibility of using one-dimensional approximations of kinematics involving simultaneously the most accurate count of the real nature of the distributions of geometrical parameters, mechanical properties and the conditions of contact friction lengthwise the deformation region.

There are many methods of mathematical modeling of the stress-strain state of metal during rolling. Consider two of them. Both of these techniques are based on the decomposition of the X-axis the entire length of the zone of plastic and elastic form change (**Figure 1**) L_{pl} and L_{el} into a finite set of i elementary volumes (**Figure 2**). The first method is based on the recurrent solutions of finite-difference form of the energy consumption balance, considered within each of the selected elementary volume. The second type of mathematical modeling is performed by numerical recurrent solutions of finite-difference form of the equilibrium condition.

Boundaries of the zones of plastic and elastic form change L_{pl} and L_{el} were defined vertical, and the length of the zone of plastic form change, following the conventional kinematic concepts [1, 2], included the backward creep zones L_{b1}, L_{b2} and zone of slippage L_{s1} and L_{s2}, located on the contact surfaces of driving and driven working rolls with radius R_1 and R_2, respectively (**Figure 1**).

Besides, a number of assumptions for both techniques, similar to the works [4], the main ones are:

- Deformation of the rolled strip is flat and steady over time, the kinematics of plastic flow of metal in the deformation region subjects to Bernoulli hypothesis [1, 2], and the normal axis stress and doubled shearing resistance , varying along the length of the zone of plastic form change, the height of each individual cross-section remain constant;

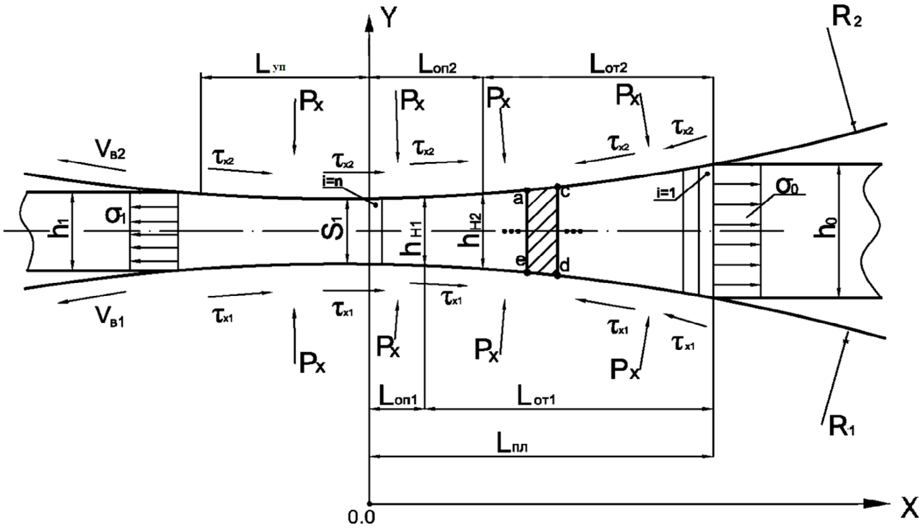

Figure 1. Design pattern of the integral process of the deformation region during hot rolling of relatively thin sheets and strips

- Changes of the current values of thickness $h_{xi} = h_{xi1}...h_{xi2}$, as well as normal $p_{xi} = p_{xi1}...p_{xi2}$ and tangential $\tau_{xi} = \tau_{x1(2)i1}...\tau_{x1(2)i2}$ contact voltages along the length of each of the selected *i*-th elementary volume are of linear character (**Figure 2**)

$$\mu_{\tilde{o}b_{1(2)}} = \mu_{0b_{1(2)}} \left(\frac{\tilde{o} - L_{s_{1(2)}}}{L - L_{s_{1(2)}}} \right)^{\grave{a}_{f1(2)}};$$

$$\mu_{xb_{1(2)}} = -\mu_{0s_{1(2)}} \left(\frac{L_{s_{1(2)}} - \tilde{o}}{L_{s_{1(2)}}} \right)^{\grave{a}_{f1(2)}}. \qquad (1)$$

Taking to the account the assumptions and the finite-difference form of presenting the main stress-strain state active components to the left side, and reactive, including iteration, components to the right side, the condition of energy consumption balance, considered within each of the selected *i*-th elementary volume (**Figure 2**) can be written as:

$$N_{\sigma i1} + N_{\tau 1i} + N_{\tau 2i} = N_{\sigma i2} + N_{\tau c1i} + N_{Fi} + N_{ui} \qquad (2)$$

where $N_{\sigma i1}, N_{\sigma i2}$ - capacities of normal axial stresses acting respectively in the initial and final boundary sections of the selected *i*-th elementary volume; $N_{\tau 1i}, N_{\tau 2i}$ - capacities supplied to the backward creep zones and backed away in slippage zones by skin-friction force on contact surfaces of the upper and lower working rolls; $N_{\tau c1i}, N_{\tau c2i}$ - current capacities of relative slip, spent on overcoming skin-friction force on contact surfaces of the upper and lower working rolls; N_{Fi}, N_{ui} - capacities spent directly on the plastic forming and overcoming the inertial forces of the *i*-th selected elementary volume.

After describing each component of expression (2) in accordance with works [1, 2], we obtain

$$\sigma_{xi1}h_{xi1}V_{xi1}+\frac{\tau_{x1i1}+\tau_{x1i2}}{2\cos\alpha_{x1i}}\Delta x_iV_{b1}+\frac{\tau_{x2i1}+\tau_{x2i2}}{2\cos\alpha_{x2i}}\Delta x_iV_{b2}=$$

$$=\sigma_{xi2}h_{xi2}+\frac{1}{2}\left[\tau_{x1i1}\left(V_{b1}-\frac{V_{xi1}}{\cos\alpha_{x1i}}\right)+\tau_{x1i2}\left(V_{b1}-\frac{V_{xi2}}{\cos\alpha_{x1i}}\right)\right]\times$$

$$\times\frac{\Delta x_i}{\cos\alpha_{x1i}}+\frac{1}{2}\left[\tau_{x2i1}\left(V_{b2}-\frac{V_{xi1}}{\cos\alpha_{x2i}}\right)+\tau_{x2i2}\left(V_{b2}-\frac{V_{xi2}}{\cos\alpha_{x2i}}\right)\right]\times$$

$$\times\frac{\Delta x_i}{\cos\alpha_{x2i}}+\frac{1}{2}K_\Lambda\left(2K_{xi1}+2K_{xi2}\right)h_{xi2}\ln\left(\frac{h_{xi1}}{h_{xi2}}\right)V_{xi2}+\rho_M\frac{h_{xi1}+h_{xi2}}{2}\Delta x_ia_{xi}V_{xi2},\qquad(3)$$

where $\tau_{x1i1},\tau_{x1i2},\tau_{x2i1},\tau_{x2i2}$ - contact stresses on the upper and lower working rolls (**Figure 1**); V_{b1},V_{b2} – peripheral speeds of rotation of the lower and upper working rolls; V_{xi1},V_{xi2} - velocity of the rolled material in the initial and final sections of the boundary sections, respectively; $\alpha_{x1i},\alpha_{x2i}$ - the current values of the contact angles on the upper and lower working rolls; K_Λ - non-smoothness coefficient of plastic deformation, the physical meaning and methods of determining of which are described in detail in works [2, 4]; $2K_{xi1}=1{,}155\sigma_{sxi1}$, $2K_{xi2}=1{,}155\sigma_{sxi2}$ - current values along the length of the zone of plastic form change of doubled shearing resistance of rolled metal in the initial and final boundary sections; ρ, a_{xi} - metal density of the rolled workpiece and the value of its acceleration, which takes place within the *i*-th volume element, can be defined as

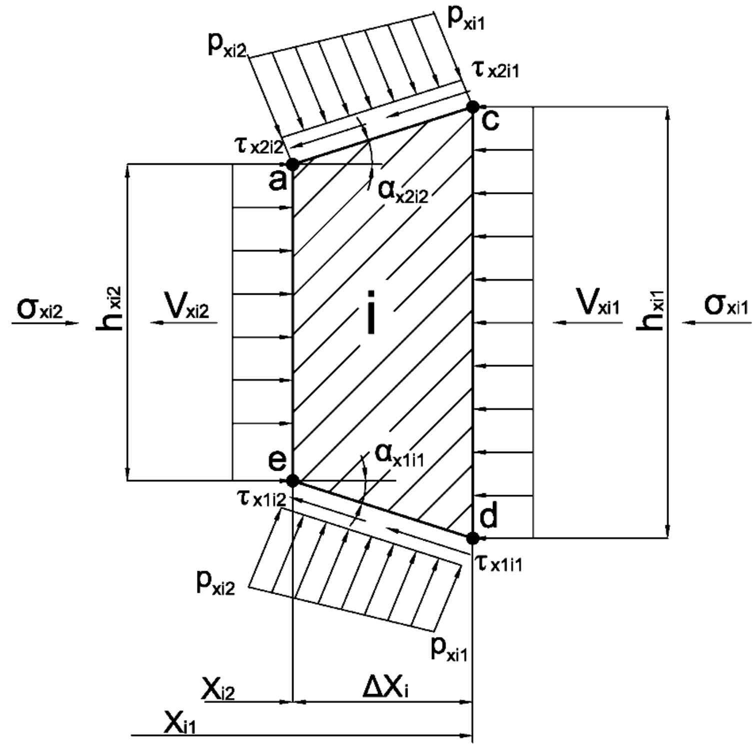

Figure 2. Design pattern of the selected elementary volume of plastic zone of form change in relation to the numerical mathematical modeling of hot rolling of relatively thin sheets and strips

$$a_{xi} = (V_{xi2} - V_{xi1})(V_{xi1} + V_{xi2})/(2\Delta x) = (V_{xi2}^2 - V_{xi1}^2)/(2\Delta x)$$

By analogy with the previous equation, we can write the equation of equilibrium for the selected volume element:

$$\sigma_{xi2} \cdot h_{xi2} - \sigma_{xi1} \cdot h_{xi1} - \frac{\tau_{x1i1} + \tau_{x1i2}}{2} \cdot \frac{\Delta x \cos\alpha_{x1i2}}{\cos\alpha_{x1i1}} -$$

$$- \frac{\tau_{x2i1} + \tau_{x2i2}}{2} \cdot \frac{\Delta x \cos\alpha_{x2i2}}{\cos\alpha_{x2i2}} + \frac{p_{xi1} + p_{xi2}}{2} \cdot \frac{\Delta x \sin\alpha_{x1i1}}{\cos\alpha_{x1i1}} +$$

$$+ \frac{p_{xi1} + p_{xi2}}{2} \cdot \frac{\Delta x \sin\alpha_{x2i2}}{\cos\alpha_{x2i2}} + \rho\left(\frac{h_{xi1} + h_{xi2}}{2}\right)\Delta x \cdot a_{xi} = 0 \qquad (4)$$

Taking into account the analytical description of the conditions of the external contact friction and the engineering version of plasticity (1) [1, 2]

$$\sigma_{xi2} = p_{xi2} - 2K_{xi2}, \qquad (5)$$

as well as by assuming as a result of the insignificance of the current values of the contact angles α_{x1i} and α_{x2i}, value of their cosines of the equal $\cos\alpha_{x1i} \approx \cos\alpha_{x2i} \approx 1{,}0$ (3) and (4) can be transformed into the following equations, respectively:

$$\sigma_{xi1} h_{xi1} V_{xi1} + \frac{1}{2}\Delta x V_{b1}(2K_{xi1}\mu_{x1i1} + 2K_{xi1}\mu_{x1i2}) + \frac{1}{2}\Delta x V_{b2}(2K_{xi2}\mu_{x2i1} +$$

$$+ 2K_{xi2}\mu_{x2i2}) = p_{xi2} h_{xi2} - 2K_{xi2} h_{xi2} + \frac{1}{2}\Delta x\left[2K_{xi1}\mu_{x1i1}(V_{b1} - V_{xi1}) +\right.$$

$$\left.+ 2K_{xi1}\mu_{x1i2}(V_{b1} - V_{xi2})\right] + \frac{1}{2}\Delta x\left[2K_{xi2}\mu_{x1i2}(V_{b2} - V_{xi1}) + 2K_{xi2}\mu_{x1i2} \times\right.$$

$$\left.\times (V_{b2} - V_{xi2})\right] + \frac{1}{2}K_{\Lambda} h_{xi2} V_{xi2}(2K_{xi1} + 2K_{xi2})\ln\left(\frac{h_{xi1}}{h_{xi2}}\right) + \frac{1}{2}\rho\Delta x a_{xi} V_{xi2}(h_{xi1} + h_{xi2}); \qquad (6)$$

$$p_{xi2} h_{xi2} - 2K_{xi2} h_{xi2} - \sigma_{xi1} h_{xi1} - \frac{1}{2}\Delta x(2K_{xi1}\mu_{x1i1} + 2K_{xi1}\mu_{x1i2}) -$$

$$- \frac{1}{2}\Delta x(2K_{xi2}\mu_{x2i1} + 2K_{xi2}\mu_{x2i2}) + \frac{1}{2}\Delta x(p_{xi1} + p_{xi2}) + \frac{1}{2}\rho\Delta x a_{xi}(h_{xi1} + h_{xi2}) = 0 \qquad (7)$$

Taking into account the Bernoulli hypothesis

$$V_{xi1} = V_1 h_1 / h_{xi1}; \quad V_{xi2} = V_1 h_1 / h_{xi2}, \qquad (8)$$

as well as, carrying out the appropriate mathematical transformations and further reductions we obtain for the energy approach

$$p_{xi2} = \sigma_{xi1} + 2K_{xi2} + \frac{2K_{xi1}\Delta x}{2}\left(\frac{\mu_{x1i1}}{h_{xi1}} - \frac{\mu_{x2i1}}{h_{xi1}}\right) + \frac{2K_{xi2}\Delta x}{2}\left(\frac{\mu_{x1i2}}{h_{xi2}} - \frac{\mu_{x2i2}}{h_{xi2}}\right) -$$

$$-0{,}5K_{\Lambda}(2K_{xi1} + 2K_{xi2})ln\left(\frac{h_{xi1}}{h_{xi2}}\right) - \frac{\rho_M}{4h_{xi2}}(h_{xi1} + h_{xi2})(V_{xi2}^2 - V_{xi1}^2) \qquad (9)$$

and force approach

$$p_{xi2} = \frac{2(\sigma_{xi1}h_{xi1} + 2K_{xi2}h_{xi2}) - p_{xi1}(h_{xi1} - h_{xi2}) + 2K_{xi1}(\mu_{x1i1} + \mu_{x2i1})}{h_{xi1} + h_{xi2}} +$$

$$+ \frac{2K_{xi2}(\mu_{x1i2} + \mu_{x2i2}) - \Delta x \cdot \rho \cdot a_i(h_{xi1} + h_{xi2})}{h_{xi1} + h_{xi2}} \qquad (10)$$

where, based on the condition of plasticity (5) it can be determined the corresponding value of the normal axial stresses acting in the final boundary section of the selected *i*-th elementary volume of the plastic form change (**Figure 2**).

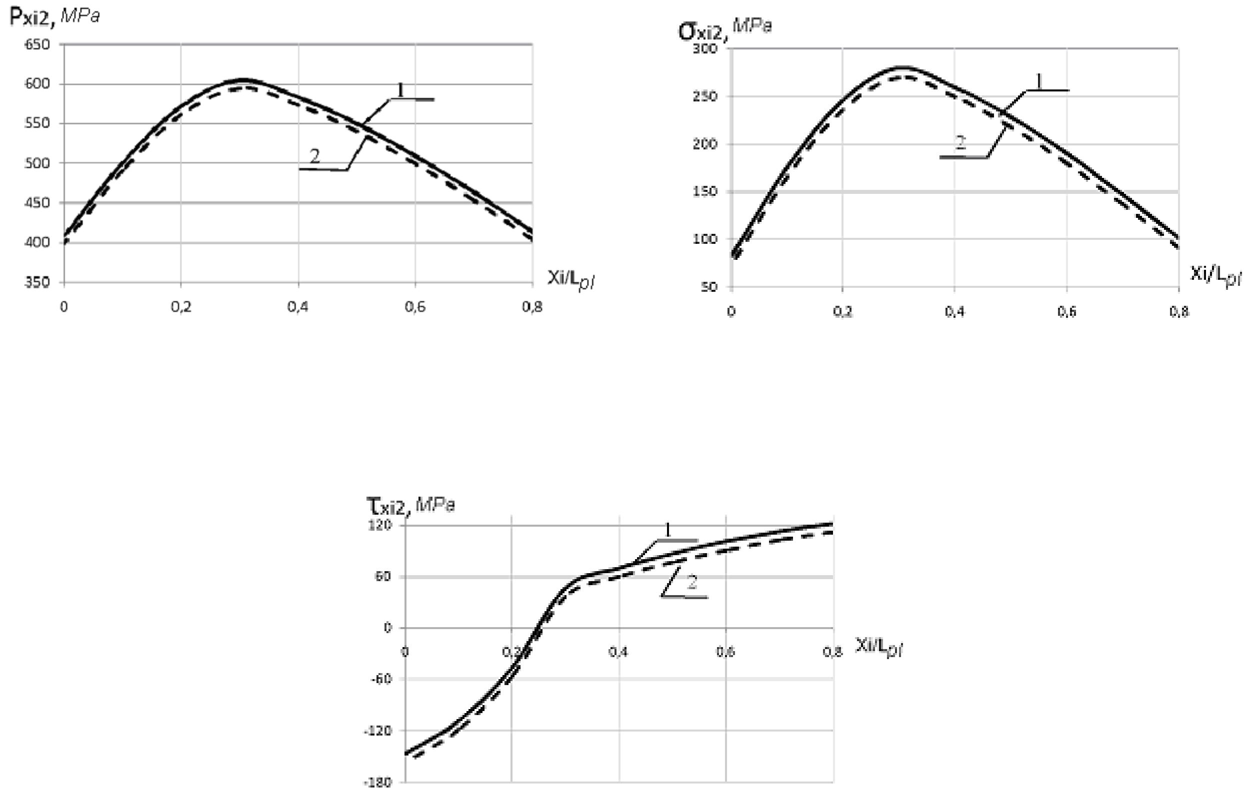

Figure 3. Local characteristics of stress-strain state of metal in the zone of plastic form change during hot rolling of relatively thin strips obtained by mathematical modeling based on a power 1 and energy 2 approaches

Conclusions

Based on the received bonds mathematical models for determining the local characteristics of the stress-strain state of metal in the zone of plastic forming were established. Data obtained on the basis of numerical recurrent solutions of finite-difference form of the energy consumption balance is somewhat smaller than the data obtained on the basis of numerical solution of recurrent finite- difference form of the condition of equilibrium (Figure 3). Besides, it was found that the degree of mismatch of the energy and force approaches was less than 3%.

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Численное математическое моделирование напряженно-деформированного состояния металла при горячей прокатке тонких полос

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Проведен анализ методик математического моделирования напряженно-деформированного состояния металла при прокатке. На основе полученных зависимостей были созданы математические модели для определения локальных характеристик напряженно-деформированного состояния металла в зоне пластического формоизменения. Полученные данные по величине несколько меньше, чем данные, полученные на основе численного рекуррентного решения конечно-разностной формы условия равновесия. При этом было установлено, что степень несоответствия энергетического и силового подходов составила менее 3 %.