Image Restoration Based on Multi-Wavelet Domain Resolution Analysis

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Abstract
Image restoration is a fundamental means to process the bottom layer of the image for the purpose of improving the visual effect of the original image and the denoising so as to facilitate the subsequent image analysis and pattern recognition. To solve the problem that the degraded image has little information, this paper adopts multi-wavelet resolution to transform the high-frequency and low-frequency information of the decomposed image. After wavelet decomposition is performed in the image, its noises and most details remain in the high-frequency part. On the basis of the extraction of high-frequency detailed information, conduct equant correction of the grayscale range to the histogram equalization to make the processed image more layered and smoother and natural. Besides, the processed image is greatly enhanced in the contrast and definition and no blocking effect ever occurs. Moreover, it can reserve the image detailed information to the utmost extent while weakening blocking effect, reinforces the contrast of the degraded blurry image and realize the purpose of restoration of degraded image.
Key words: IMAGE RESTORATION, MULTI-WAVELET ANALYSIS, DEGRADED IMAGE

1. Introduction

Generally, degradation comes in many forms such as motion blur, noise, image transmission, scanning and camera misfocus. Due to insufficient or excessive lighting conditions in photographing, the image can be too dark or too bright. The image can be blurred by the distortion, relative motion and atmospheric motion of the optical system and various kinds of noises can be introduced in the transmission[1]. To solve the above-mentioned problems, image restoration technology can lend a hand with its ultimate objective to be used in the processing and to achieve the tasks of visual effects of human eyes in accordance with specific requirements. Image restoration is to restore the important information and weaken or even remove the unnecessary information as required of the image. Proper restoration processing performed on the images obtained from different methods can make the blurry original images into the clear images with a lot of useful information and effectively eliminate the noises in the images so as to make it easier to detect and measure the interesting targets in the images [2]. The research on the restoration technology to be used in the images shot under the low-visibility conditions has great application value in target identification and remote-sensing, in fact, currently, it has become a research hotspot in image processing and computer vision. It is possible to come up with an very good estimate of some functions and algorithms to restore the original image [3].

The international research on the degraded visual image mainly includes the following aspects. At present, there are plenty of algorithms to enhance the contrast, among which histogram equalization is the simplest and feasible method, however, this algorithm only adjusts the probability distribution function of the grayscale of the image automatically while according to the theory of atmospheric scattering, fog attenuation increases exponentially with the changes of the scene depth, therefore, its effect on such image is not ideal. The traditional image enhancement methods can basically divided into spatial-domain image enhancement method and frequency-domain image enhancement method. The spatial domain is the collection of the pixels which constitute the image and the spatial-domain image enhancement method directly pre-processes the grayscale value of the image pixels, including gray level transformation, histogram equalization, spatial-domain smoothing and restoration processing of the image and bit color processing[4]. On the other hand, the frequency-domain image enhancement method operates on the spectral components after the image has performed Fourier transform and then reverses Fourier Transform to get the necessary result, including low-pass filtering technology, high-pass filtering technology, band-pass and band-stop filtering and homomorphic filtering. So far, people have been concentrating on the enhancement method based on wavelet changes and that based on fuzzy mathematics [5].

The core of wavelet analysis is the construction of wavelet function and multi-scale analysis. The main characteristics of wavelet function are the rapid attenuation and oscillatory and its sub-functions are mutually orthogonal to each other. In the applications of image processing, orthogonality can preserve energy and symmetry is not only suitable for the visual system of human eyes, but it can also make the signal edge easy to process, therefore, it is of great importance to possess these two features[6]. However, in the field of real number, ortho-symmetric smooth single-wavelet doesn't exist, leaving people no choice but to compromise between orthogonality and symmetry. Extend the multi-resolution analysis space generated by single scaling function to that generated by multi-scaling function to get more freedom. Multi-wavelet can have such features as orthogonality, smoothness, compact support and symmetry which are important to image processing while maintaining the strengths of wavelet. Therefore, multi-wavelet has a bigger potential in image applications[7]. This paper firstly elaborates the image restoration processing. Then it analyzes multi-wavelet transform and its typical characteristic, based on which it proposes the combined image restoration method based on multi-wavelet domain. And finally, it is the experimental simulation and analysis.

2. Image Restoration Processing

The basic objective of image restoration is to process the image to make it more suitable for the specific application fields than the original image. Image restoration processing highlights the interesting information of the image, weakens or removes the unnecessary information and enhances the useful information according to certain requirements. Besides, considering the influence of additive noise $n(x,y)$, the degradation process can be indicated by Formula (1) and Fig.1.

$$g(x,y) = H * f(x,y) + n(x,y) \quad (1)$$

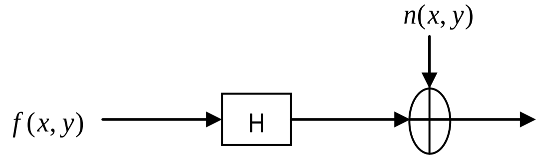

Figure 1. Image degradation model

The commonly-used image restoration methods include:

Histogram Equalization

Histogram equalization emphasizes the global or local features of the give image in purpose, makes the originally unclear image become clear or stresses certain interesting features, corrects the histogram of the original image into the histogram with uniformly-distributed grayscale through grayscale transformation function and then corrects the original image according to the equalized histogram. Its transformation function depends on the cumulative distribution function of grayscale histogram of the image. Some images have bigger frequency in the low-value grayscale regions, making the image details in the dark regions unclear. At this time, histogram equalization can separate the grayscale range of the image, increase the grayscale with low frequency and automatically increases the contrast of the entire image by adjusting the dynamic range of the image grayscale value to make the image have bigger contrast and clear details[8].

Contrast Enhancement

Contrast enhancement is an important part in image processing technology and its purpose is to increase the grayscale change range and enrich the grayscale level of the image by adjusting the distribution of the image grayscale value so as to significantly improve the visual effects of the image and provide the image which is more suitable to be analyzed. Some images have lower contrast, making the entire image fuzzy and blurry. The grayscale of every pixel of the original image can be modified according to certain rules to change the dynamic range of the image grayscale[9].

Noise Smoothing

Noise smoothing is the operation to be made on image noises, which is aimed to eliminate noises. Noise smoothing is a spatial-domain filtering technique with enhanced low frequency and its purposes include: fuzziness and denoising. Mean filtering and median filtering are frequently used in image smoothing and the smoothing filtering in the spatial domain usually adopts the simple average method, namely to seek the average brightness value of the surrounding pixel points. The size of the neighborhood is directly related to the smoothing effect. The bigger the neighborhood, the better the smoothing effect, however, if the neighborhood is too big, the smoothing will cause more loss of the edge information, making the output image fuzzy; therefore, reasonable choice shall be made in selecting the size of the neighborhood[10].

Restoration

Noise smoothing usually leads to fuzzy image edge. Since average and integral operation can blur the image, perform inverse operation on it and adopt differential operator to use template and statistics difference to sharpen the image enhancement. The commonly-used restoration template is Laplacian template, as indicated in Formula (2).

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad (2)$$

The constitution of Laplacian template can be seen easily: subtract it and the surrounding 8 pixels, suggesting the differences and take the sum of this difference and itself as the grayscale of a new image. It is obvious that if a bright spot occurs in a dark region, the result of the restoration processing makes it brighter and enhances the image noises[11].

3 Multi-wavelet Transform and Its Basic Characteristics

3.1 Single-wavelet Transform

Wavelet refers to a waveform with limited length and a mean value of 0. Wavelet function is exactly defined as:

$$WT_f(a,b) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t)\overline{\psi\left(\frac{t-b}{a}\right)}dt, \quad a \neq 0 \quad (3)$$

Or in the form of inner product:

$WT_f(a,b) = \langle f, \psi_{a,b} \rangle$. In this formula,

$$\psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right) \quad (4)$$

The existence of inverse transform requires $\psi(t)$ to meet the following admissible condition:

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \quad (5)$$

In this formula, $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$.

At the time, the inverse transform is

$$f(t) = C_\psi^{-1} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \psi_{a,b}(t) WT_f(a,b) db \frac{da}{|a|^2}$$

(6)

C_ψ has restricted the category of function ψ which belongs to $L^2(R)$ and which can serve as “basic wavelet (or mother wavelet)”. Especially when ψ is required to be a window function, ψ has to belong to $L^1(R)$, namely $\int_{-\infty}^{\infty} |\psi(t)| dt < \infty$, therefore, $\hat{\psi}(\omega)$ is a continuous function of R, $\hat{\psi}$ must be 0 in the starting point, namely

$$\hat{\psi}(0) = \int_{-\infty}^{\infty} \psi(t) dt = 0 \quad (7)$$

It can be seen from Formula (7) that wavelet function absolutely has oscillatory. Fig. 2 is the time-domain and frequency-domain graph of Morlet wavelet[12,13].

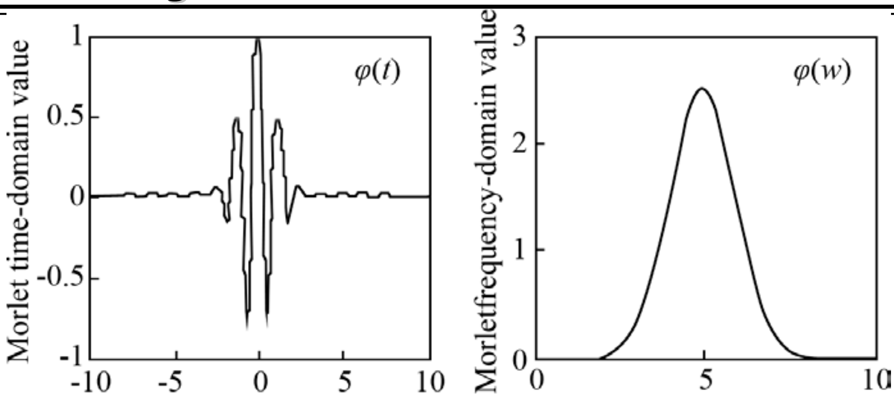

Figure 2. Time-domain and frequency-domain graph of Morlet

3.2 Multi-wavelet Resolution Analysis

Multi-wavelet is the wavelet generated from $r(r \geq 2, r \in N)$ functions as scale components. Multi-wavelet can perfectly integrate symmetry, compact support, orthogonality and high-order vanishing moment together, therefore, it has advantages over single-wavelet in signal processing. The formation of multi-wavelet is as follows:

(1) There are N scaling functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$, which can be presented by $\Phi(t) = [\phi_1(t), \phi_2(t), \ldots, \phi_N(t)]^T$. Translate the scaling function to get $\phi_1(t-k), \phi_2(t-k), \ldots, \phi_N(t-k)$ and generate a space V_0. If $\Phi(t)$ meets the conditions of $\left\langle \phi_i(\cdot - k), \phi_j(\cdot - l) \right\rangle = \delta_{i,j}\delta_{k,l}, 0 \leq i, j \leq N-1$, then $\Phi(t)$ is orthogonal multi-scaling function[14].

(2) Assume that S is the real matrix of 2×2 and $S^2 = I_2$ and $\det(S) = -1$, to a certain integer N, if a matrix filter H satisfies the following conditions, then it means that H is symmetric around the symcenter $\frac{N}{2}$ under S.

$$SH_{N-k}S = H_k \qquad (k \in Z, cen(H) = \frac{N}{2}) \qquad (8)$$

If the filter is symmetric, then it has linear phase and it can ensure that the signal to be processed by the filter won't have distortion, therefore, when designing a filter, it is usually hoped that it is symmetric[15].

(3)Mark $\Phi_{j,k}(t) = 2^{j/2}\Phi(2^j t - k) = [2^{j/2}\phi_1(2^j t - k), 2^{j/2}\phi_2(2^j t - k)]^T$, then V_j can be presented as

$$V_j = span\{2^{j/2}\phi_l(2^j t - k), l = 1, 2, j \in Z, k \in Z\} \qquad (9)$$

In $L^2(R)^2$, the orthogonal complementary space of V_j is W_j and then W_j is presented as

$$W_j = span\{2^{j/2}\psi_l(2^j t - k), l = 1, 2, j \in Z, k \in Z\} \qquad (10)$$

The space series $\{V_n\}$ and $\{W_n\}$ as well as their relationship can be visually presented in Fig.3.

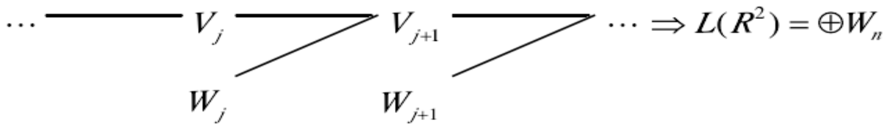

Figure 3. Spaces W_n and V_n

(4) Extend the analysis and composition algorithm of orthogonal single-wavelet to orthogonal multi-wavelet and we can get:

In this way, Discrete Multi-Wavelet Transform (DMWT) is

$$C_{j-1,k} = \sqrt{2}/2\sum_n H_{n-2k}C_{j,n} \qquad (11)$$

$$D_{j-1,k} = \sqrt{2}/2(\sum_n G_{n-2k}C_{j,n}) \qquad (12)$$

And Inverse Discrete Multi-Wavelet Transform (IDMWT) is

$$C_{j,k} = \sqrt{2}/2(\sum_n H^T_{k-2n}C_{j-1,n} + \sum_n G^T_{k-2n}D_{j-1,n}) \qquad (13)$$

Formulas (12) and(13) are referred to as Mallat algorithm, which plays the same role in DMWT and DWT[16,17].

4. Image Restoration Combination Method Based on Multi-wavelet Domain

Since the noises and most details of the image which has conducted wavelet decomposition, remain in the high-frequency part while the overall visual sensation of the image is determined by the low-frequency information, the following has provided how the combination method proposed in this paper enhances the image restoration.

(1) Input the image to be processed, perform wavelet scale decomposition on the noisy image andget the first-order high-frequency and low-frequency subband images.

(2) Perform soft-threshold denoising on the extracted high-frequency information according to Formula (14).

Conduct wavelet threshold denoising on the first-order high-frequency subband image after wavelet transform and enhance the image. Perform secondary scale transform on the low-frequency subband image after the wavelet transform, get the second-order high-frequency and low-frequency subband images, conduct wavelet threshold denoising on the second-order low-frequency subband image and enhance it, perform wavelet self-adaptive enhancement on the second-order high-frequency subband image and get the enhanced high-frequency subband image.

$$\tilde{w}_{j,k} = \begin{cases} \operatorname{sgn}(w_{j,k})(|w_{j,k}| - \mu), & |w_{j,k}| \geq \mu \\ 0, & |w_{j,k}| < \mu \end{cases} \quad (14)$$

In here, $\operatorname{sgn}(\ )$ is sign function.

(3) Process the low-frequency image extracted with histogram equalization according to Formula (15).

The histogram equalization transformation function F_k, namely the grayscale cumulative distribution function of the image is:

$$F_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \quad (15)$$

F_k is the normalized grayscale.

Therefore, the grayscale intervals of the image after histogram equalization is pulled away or the gray scale is uniformly distributed so as to increase contrast, make the image details clear and achieve the purpose of enhancement.

(4) Conduct pixel-level fusion on the information processed in (2) and (3).

(5) Reconstruct the image with inverse wavelet transform by the wavelet coefficient of enhancement and noise suppression, get the recovery image of the original one, as indicated by Formula (16):

$$V_{out} = \begin{cases} V_{in} & , \quad V_{in} > \mu_2 \\ W*(V_{in} - \mu_1) + \mu_2 & , \quad \mu_2 \geq V_{in} > \mu_1 \\ W*V_{in} & , \quad \mu_1 \geq V_{in} \geq \mu_1 \\ W*(V_{in} + \mu_1) - \mu_2 & , \quad -\mu_1 > V_{in} \geq \mu_2 \\ V_{in} & , \quad V_{in} < -\mu_2 \end{cases} \quad (16)$$

μ_1 and μ_2 are the low and high thresholds of the low-frequency part after wavelet scale decomposition respectively. $\mu_1 = \alpha\sqrt{\log n} = \sqrt{n}$ (Here, n is the image size and α is the noise variance) while $\mu_2 = \alpha V_{\max j} (0 < \alpha < 1, V_{\max j})$ is the maximum wavelet coefficient of the low-frequency part of the decomposed image. W is the gain factor, which is related to the significant noise index w. When the noise is small, the gain on the wavelet coefficient increases and vice versa, thus realizing the objective of adaptive enhancement.

5. Experimental Result and Analysis

This paper takes degraded image as example in the research of the restoration processing of degraded image which are collected under dissatisfactory circumstances. This paper has conducted enhancement processing on the degraded image through traditional method and the algorithm of this paper and it has also made visual evaluation and objective analysis of the experimental result. The concrete experimental result is indicated by Fig.4-Fig.7.

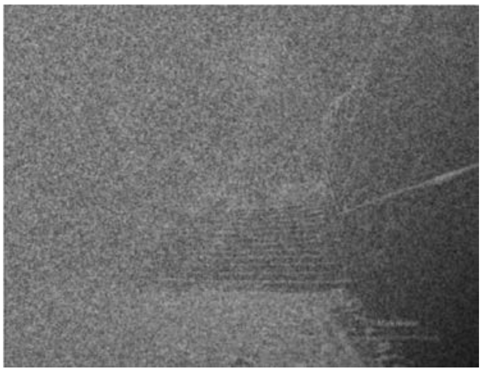

Figure 4. Original degraded image

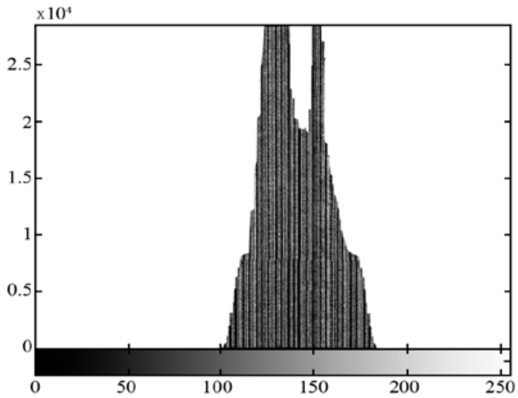

Figure 5. Histogram of original degraded image

Figure 6. Traditional histogram equalization

Figure 7. This paper algorithm

It is not difficult to see from the experimental result of the degraded image that the histogram of the degraded image (Fig.4) is concentratedly distributed with the grayscale value within the range of [100, 200], that the image has no layered sense and it is highly fuzzy, that the scenery in the image is difficult to tell and the entire image quality is not good. The image after being processed by traditional histogram equalization can show part of the scenery of the image, however, due to the flaws of the traditional histogram equalization, the grayscale merger excessively in the equalization with no remedy measures for the reduced grayscale, leading to the loss of a great deal of detail information of the original image. There are few grayscales in the original image and the grayscales reduce after being processed and the edge lines of the scenery in the image are not clear at all. However, the algorithm of this paper can not only improve the layered sense and clearness of the image and make the color, contrast and brightness of the image more suitable for human eyes, but it can also enhance the entropy of the image and highlight the image details.

6. Conclusions

Based on the traditional single-wavelet transform, this paper has proposed a combination method of image restoration method based on multi-wavelet domain on the degraded image. The experimental result has shown that the adoption of the multi-wavelet domain algorithm of this paper can achieve better processing result with profound detailed information, strong image layer, high contrast and excellent image restoration in conducting restoration processing on degraded image.

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References

1. Ehsan Ollah Sheybani. (2012) An Algorithm for Real Time Blind Image Quality Comparison and Assessment. *International Journal of Electrical and Computer Engineering (IJECE)*, 2(1), p.p.120-129.
2. Wei Feng, Wenxing Bao. (2012) An Improved Technology of Remote Sensing Image Fusion Based Waveled Packet and Pulse Coupled Neural Net. *TELKOMNIKA Indonesian Journal of Electrical Engineering*, 10(3), p.p.551-556.
3. Alan D. Fleming, Sam Philip, Keith A. Goatman, Peter F. Sharp, John A. Olson. (2012) Automated Restoration Assessment of Retinal Images Using Regionally Based Structural and Statistical Measures. *Medical Engineering & Physics*, 34(7), p.p.849-859.
4. Lenny R. Vartanian, Shanta Dey. (2013) Self-concept Restoration, Thin-Ideal Internalization, and Appearance-Related Social Comparison as Predictors of Body Dissatisfaction. *Body Image*, 10(4), p.p.495-500.
5. Xuexin G. Gao, Dobromir Filip, Alaa Rostom, etc. (2012) Colono scopy Withdrawal Velocity and Image Restoration Measurement as A Novel Patient-Centric Real-Time Quality Indicator for Screening Colonoscopy. *Gastro Intestinal Endoscopy*, 75(4), p.p.AB300-AB301.
6. B. Andò, S. Baglio, A. Pistorio. (2014) A Low Cost Multi-sensor Strategy for Early Warning in Structural Monitoring Exploiting a Wavelet Multiresolution Paradigm. *Procedia Engineering*, 87, p.p.1282-1285.

7. Fatemeh Safara, Shyamala Doraisamy, Azreen Azman, etc. (2013) Multi-level Basis Selection of Wavelet Packet Decomposition Tree for Heart Sound Classification. *Computers in Biology and Medicine*, 43(10), p.p.1407-1414.
8. João Miguel Pires Dias, Carlos Manta Oliveira, Luís A. da Silva Cruz. (2014) Retinal Image Quality Assessment Using Generic Image Quality Indicators. *Information Fusion*, 19(9), p.p. 73-90.
9. João Miguel Pires Dias, Carlos Manta Oliveira, Luís A. da Silva Cru. (2012) Evaluation of Retinal Image Gradability by Image Features Classification. *Procedia Technology*, 5(1), p.p. 865-875.
10. Jie Wu. (2014) The Impact on The X-ray Phase Contrast Imaging Restoration of The Light Source Area. *Optik – International Journal for Light and Electron Optics*, 125(1), p.p.345-348.
11. Marcus W. Beck, Bruce Vondracek, Lorin K. Hatch, Jason Vinje. (2013) Semi-automated Analysis of High-resolution Aerial Images to Quantify Docks in Glacial Lakes. *ISPRS Journal of Photogrammetry and Remote Sensing*, 81(7), p.p.60-69.
12. Maya Kallas, Paul Honeine, Cédric Richard, etc. (2013) Non-negativity Constraints on The Pre-Image for Pattern Recognition with Kernel Machines. *Pattern Recognition*, 46(11), p.p. 3066-3080.
13. Mohammad Reza Yousefi, Reza Jafari, Hamid Abrishami Moghaddam. (2014) Imposing Boundary and Interface Conditions in Multi-Resolution Wavelet Galerkin Method for Numerical Solution of Helmholtz Problems. *Computer Methods in Applied Mechanics and Engineering*, 276(1), p.p.67-94.
14. Ali Akbari, Meisam Khalil Arjmandi. (2014) An Efficient Voice Pathology Classification Scheme Based on Applying Multi-Layer Linear Discriminant Analysis to Wavelet Packet-Based Features. *Biomedical Signal Processing and Control*, 10(3), p.p.209-223.
15. Maheswaran Rathinasamy, Jan Adamowski, Rakesh Khosa. (2013) Multiscale Streamflow Forecasting Using A New Bayesian Model Average Based Ensemble Multi-Wavelet Volterra Nonlinear Method. *Journal of Hydrology*, 507(12), p.p.186-200.
16. Ratikanta Behera, Mani Mehra. (2013) Integration of Barotropic Vorticity Equation over Spherical Geodesic Grid Using Multilevel Adaptive Wavelet Collocation Method. *Applied Mathematical Modelling*, 37(7), p.p.5215-5226.
17. Snehamoy Chatterjee, Roussos Dimitrakopoulos. (2012) Multi-scale Stochastic Simulation with A Wavelet-Based Approach. *Computers & Geosciences*, 45(8), p.p.177-189.