Resolve of the Aiming Error Based on Euler Angle Transformation

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Abstract
The strap down inertial navigation system (SINS) is an essential part in the closed-loop control system of aiming mechanism; the breakdown of SINS will lead to the failure in aiming. To solve the problem, to calculate the north azimuth angle and trim angle of the navigation with the initial information of azimuth sensor, pitch sensor and SINS initial value, and then compare it with the output of the SINS to make sure if the output of the SINS is correct. If the error is beyond allowed band, stop the aiming. The testing shows that the calculating result is accordance with the real value with a certain error. It can discover the breakdown of the SINS during the aiming and solve the problem of system error. It is of great practical value.
Key words: EULER ANGLE TRANSFORMATION, INITIAL TRANSFORMATION MATRIX, AIMING ERROR

1. Introduction

The rapid development of micro electronic technology and digital technology has brought about qualitative leap in the area of control technology. Multi-sensor information fuse raises the speed and accuracy as well as the reliability. The servo mechanism of aiming system in weapons is a typical multi-sensor control model. The input and output are complicated and there exists coupling. This derives many kinds of control mode according to different ways of decoupling [1].

At the end of the Second World War, the fire control system in foreign countries has only optics gun sight. Aiming relies on adjusting pitch engine and azimuth machine by hands. There is divided scale on the reticle. It can aim well when the distance is within 900m, otherwise the hit rate decreases. Mechanism and optical range finders were invented in the 50s [2]. The USA firstly used these technologies to calculate the distance of the target and modify the sighting angle. The hit rate rises greatly [3,4].

Early, China uses artificial geodetic to measure set of data without auto-control device. It is a kind of manual mechanism aiming system largely affected by landform, weather and human factors that can hardly aim accurately [5]. In 80s, China start to use electricity-driven device to aim, however, it is only restricted to azimuth or pitch auxiliary system and cannot form close-loop system[6]. Afterwards, with the introduction of the new international experiences and technologies, the internal aiming control system develops more rapidly and accurately. It is uses embedded computer to make coordinate transform and decoupling calculation, and it gets the vertical and horizontal angle between car body and horizontal line with the gravity sensors.

Nowadays, most of the internal fire control system has been optimized with the domestic strap down inertial navigation and pitch sensor and azimuth sensor. The strap down inertial navigation has become the kernel of fire control aiming system which decides the accuracy of gun adjustments. In actual operating process it is found that the strap down inertial navigation will come to breakdowns due to long term working or other reasons. In turn, the north-seeking is not accurate, and the output drifts and jumps. During gun adjustment, the strap down inertial navigation cannot reflect accurately the posture of the gun and lead to gun adjustment error. Therefore, there is great significance in studying how to track the error of strap down inertial navigation.

In consideration of confidentiality, a testing apparatus is used to make simulation. The apparatus constitutes an aiming system including the azimuth sensor, pitch sensor and the strap down inertial navigation. An initial transformation matrix based on Euler Angle coordinate transformation theory is introduced to resolve the azimuth angle and trim angle of aiming. Comparing them with the real north azimuth angle and trim angle got by the strap down inertial navigation, we can get the error of the strap down inertial navigation to know if it is working properly.

2. Structure and Constitution

The testing apparatus is made up with platform, azimuth rotating mechanism, pitch rotating mechanism, aiming mechanism, azimuth sensor, pitch sensor and the strap down inertial navigation, as shown in Figure1. The aiming mechanism is fixed on the pitch rotating mechanism. Driven by the azimuth and pitch rotating mechanism, it can point at the given target. The platform is static, and the azimuth rotating mechanism rotates around the axis of the platform. The angle of rotation can be measured by the azimuth sensor. The axis of pitch rotating mechanism is parallel to the surface of the azimuth rotating mechanism and its two trunnions are fixed on the azimuth rotating mechanism. Driven by the pitch engine, the pitch rotating mechanism rotates around its axis. The angle of rotation can be measured by the pitch sensor.

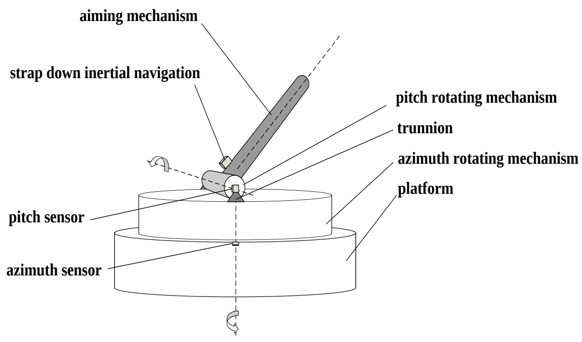

Figure 1. Schematic diagram of the structure and constitution of aiming testing apparatus

3. Principle of Aiming Control

The aiming system is made up with coordinate transform unit, controller unit and azimuth and pitch rotating mechanism unit. The given targets are the north azimuth angle and trim angle in geodetic coordinate system. During aiming, the given targets should be transformed to azimuth and pitch angle in platform coordinate system to adapt to the control mode of azimuth and pitch engine based on platform. The coordinate transform unit makes the coordinate transform. When the target is set, the coordinate transform unit outputs the azimuth and pitch angle according to the information given by the azimuth sensor, pitch sensor and the navigation. The computer compares the outputs and the feedbacks of azimuth and pitch sensor to get the control errors. The errors drive the azimuth and pitch rotating mechanism engine to aim, as shown in Figure 2.

The coordinate transform is based on order Euler angle transform. Three rectangular angle systems are connected through Euler angle and take part in rotation transformation. The definitions of the three rectangular angle systems are as follow.

Geodetic coordinate system: O-XeYeZe, right handed system. Xe axis, Ye axis and Ze axis directs separately to east, north and up, as shown in Figure 3.

Trunnion coordinate system: O-XtYtZt, right handed system, linked with the trunnion of the aiming mechanism. The coordinate origin is on the centre of the rotation. The Xt axis is coincided with the axis of trunnion of the aiming mechanism. The Yt is coincided with the axis of the aiming mechanism. Zt axis is decided with right-handed system, as shown in Figure 4.

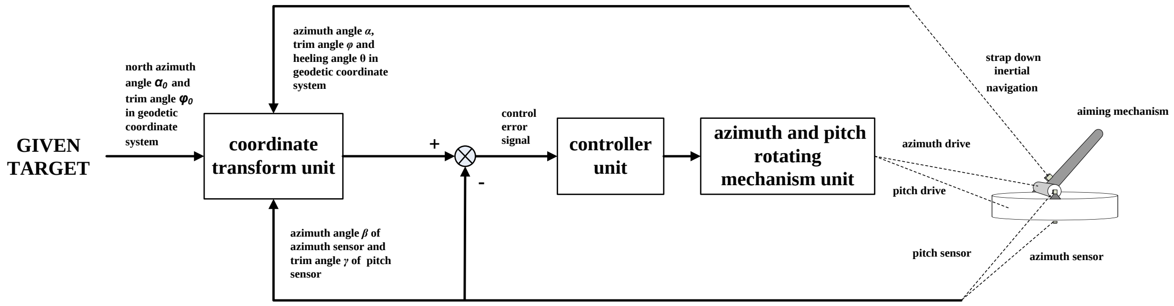

Figure 2. The principle of aiming control system

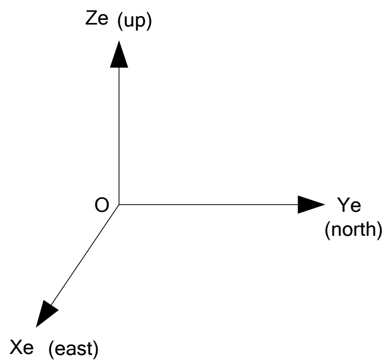

Figure 3. Geodetic coordinate system

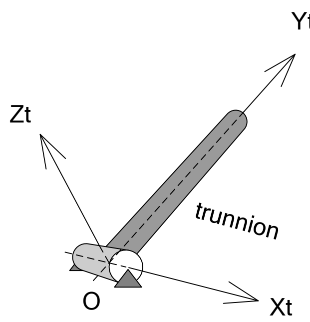

Figure 4. Trunnion coordinates system

Platform coordinate system: O-XpYpZp, right handed system, linked with the center of the platform. The coordinate origin is in the center of the platform. Yp is coincided with the base line of the platform, and the Xp axis and Zp axis are decided with right-handed system, as shown in Figure 5.

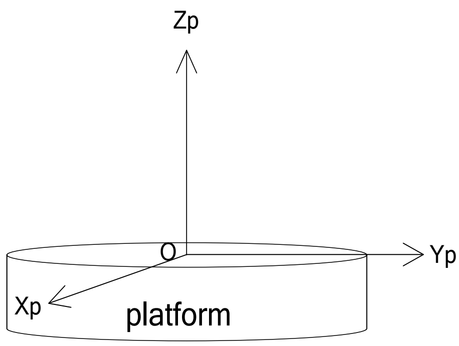

Figure 5. Platform coordinates system

The Euler angle between the trunnion coordinates system and the geodetic coordinates system is decided by azimuth angleα, trim angleφand heeling angle θ output by strap down inertial navigation. According to order Euler theory, the trunnion coordinate system's rotation towards platform coordinates system equals to three order rotations. the trunnion coordinates system rotates around OZt axis forα. and then it rotates around OXt axis anti-clockwise for φ. In the end, it rotates around OYt axis anti-clockwise for θ. The Euler angle between trunnion coordinate and platform coordinate is β output by azimuth sensor and γ output by pitch sensor. The rotation of trunnion around platform equals to two order rotation. It firstly rotates around axis OZt clockwise for β, then rotates around axis OXt anti-clockwise for γ. For the trunnion mechanism is fixed on azimuth rotation mechanism, the trunnion coordinate, relative to platform coordinates system, does not rotate around axis OYt, so it does not have the third Euler angle.

When there is certain Euler angle between two coordinates, there will be three Euler angle transform matrixes. The rotation matrix of two coordinates is the product of the three Euler transform matrixes. for example, the north orientation angle between trunnion axis and geodetic coordinates is α, the trim angle is φ, and the heeling angle is θ.

azimuth transform matrix

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

trim transform matrix

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\varphi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \quad (2)$$

heeling transform matrix

$$C = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \quad (3)$$

The transform matrix from geometric coordinate to trunnion coordinates is

$$D = C \cdot B \cdot A \quad (4)$$

The trunnion coordinate has azimuth angle β and pitch angle γ towards platform coordinate, then azimuth transform matrix

$$F = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

trim transform matrix

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix} \quad (6)$$

The transform matrix from trunnion coordinate to geometric coordinate is

$$H = (E \cdot F)^{-1} \quad (7)$$

Then the matrix M from geometric coordinate to platform coordinate is

$$M = H \cdot D \quad (8)$$

When the transform matrix M is established, the coordinate of the target in platform coordinate can be got. For example, the target's vector in geometric coordinate is R, and then R vector in platform coordinate is T.

$$T = M \cdot R \quad (9)$$

Therefore, we have the target in platform coordinate. Compared with the output of azimuth and pitch sensor, the error can be got and the azimuth and pitch engine drives to aim at the target.

4. The Analysis of the Errors

In control system, the strap down inertial navigation is an angle sensor. It feeds back the north azimuth and trim angles to coordinates transform unit. Therefore, it is essential to the accuracy of aiming. The accuracy and construction of navigation is much higher than that of sensors, and so is the breakdown rate. the normal breakdowns includes: the prohibit in north-seeking, drifting, jumping in output and sluggish, which leads to the change of parameter after coordinate transform and cause the overshoot, shake and out of control.

5. Errors Solution and its Principle

The testing apparatus should be parked on the ground with incline smaller than 5°. When its position is set, the geometric coordinate is at rest to the platform coordinate. The transform matrix from platform coordinate to geometric coordinate is a constant matrix which does not change with time. The matrix can be got with initial parameter of navigation and sensor, and it is called initial transform matrix. It is a constant matrix but it is not unique. It is decided by the initial posture of the platform and the initial position of the aiming mechanism on the platform.

The initial transform matrix reflects the constrained relationship between the parameters of the navigation and the sensor. The north azimuth and trim angle can be got by the azimuth, the pitch and the initial transform matrix. When the difference between the real value and the calculating result is within a certain amount, the output of the navigation is considered correct, otherwise it is not right.

The initial transform matrix J0 can be got by inverting matrix M0 which is the transform matrix from geometric coordinate to platform coordinate under initial condition.

$$J_0 = M_0^{-1} \quad (10)$$

Consuming the azimuth angle of the aiming mechanism isβ, the pitch angle is γ, the coordinate value can be got by transforming it from the polar coordinate to the rectangular coordinate:

$X=\cos\gamma.\sin\beta$;

$Y=\cos\gamma.\cos\beta$;

$Z=\sin\gamma$

The rectangular coordinate value of the aiming mechanism in geometric coordinate is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = J_0 \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (11)$$

With the rectangular coordinate, the north azimuth angle and trim angle of the navigation of aiming mechanism in geometric coordinate are

$$\alpha = tg^{-1}(\frac{x}{y}) \quad (12)$$

$$\phi = tg^{-1}(\frac{z}{\sqrt{x^2 + y^2}}) \quad (13)$$

For example, if the perigon is 6400mil, under the initial condition, the north azimuth angle of the navigation is 340.70mil, the trim angle is 210.70mil, the heeling angle is 84.50mil, the output of the azimuth sensor is 12.34mil and the output of the pitch sensor is 67.89mil. The initial transform matrix can be got according to the formula 10.

$$J_0 = \begin{bmatrix} 1.0313 & 0.3345 & 0.0352 \\ -0.325 & 1.0206 & -0.172 \\ -0.0861 & 0.1529 & 1.0705 \end{bmatrix}$$

During the aiming, when the azimuth and pitch sensor outputs (β, γ) are (200mil, 350mil), there are

X= cos(γ. π/3200) .sin(β. π/3200) = 0.1837

Y= cos(γ. π/3200) .cos (β. π/3200) = 0.9235

Z= sin(γ. π/3200) = 0.3369

And the formula

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = J_0 \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.0313 & 0.3345 & 0.0352 \\ -0.325 & 1.0206 & -0.172 \\ -0.0861 & 0.1529 & 1.0705 \end{bmatrix} \cdot \begin{bmatrix} 0.1837 \\ 0.9235 \\ 0.3369 \end{bmatrix} = \begin{bmatrix} 0.5102 \\ 0.8248 \\ 0.4860 \end{bmatrix}$$

The north azimuth angle of the navigation is

$$\alpha = tg^{-1}(\frac{x}{y}) .3200/\pi=564.25 \text{（mil）}$$

The resolving result of trim angle is

$$\phi = tg^{-1}(\frac{z}{\sqrt{x^2+y^2}}) .3200/\pi=473.19 \text{（mil）}$$

6. Analysis of test results

Park the testing apparatus on the slope of 3°,set the aiming mechanism to zero. After the north-seeking of the navigation, calculate initial transform matrix J0 with the target platform coordinate（-27.2mil, -10.8mil）on first row (targets on platform) in the following table and the output of the navigation（-2999.60mil, -0.2 mil, 64.1 mil）. The value of J0 (initial transform matrix) is shown in table 1.

Set 13 groups of targets, when the aiming mechanism is in place, take record of the output of the navigation and calculate the errors of azimuth angle and trim angle.

Table 1. The calculated result and the output of navigation

Sn	targets on platform		initial transform matrix	calculating result of the navigation		output of the navigation			errors	
	azimuth	pitch		azimuth angle	trim angle	azimuth angle	trim angle	heeling angle	azimuth angle	trim angle
1	-27.2	-10.8	-1.0229，-0.2335，-0.0627 0.2325，-1.0248，0.0236 -0.0664，0.0091，1.0490	-2999.60	-0.20	-2999.6	-0.20	64.1	0	0
2	52.5	294.4		-2900.11	299.33	-2900.1	299.8	67.3	-0.01	-0.47
3	29.1	591.9		-2900.17	597.57	-2900.1	600.0	77.3	-0.07	-2.43
4	-6.6	889.8		-2900.94	896.64	-2900.3	899.7	101.3	-0.64	-3.06
5	252.2	306.5		-2700.30	298.71	-2700.0	299.8	67.0	-0.3	-1.09
6	228.4	604.8		-2700.36	597.76	-2700.0	599.8	77.2	-0.36	-2.04
7	192.8	902.5		-2700.44	896.59	-2699.9	899.8	101.8	-0.54	-3.21
8	452.9	317.0		-2499.97	296.74	-2499.2	300.2	64.3	-0.77	-3.46

9	429.1	617.0		-2500.89	597.48	-2500.0	600.2	74.7	-0.89	-2.72
10	393.8	914.5		-2501.48	896.07	-2500.1	899.9	99.4	-1.38	-3.83
11	-145.0	280.8		-3098.76	298.16	-3099.8	299.9	65.2	1.04	-1.74
12	-167.7	579.3		-3099.06	597.37	-3099.8	599.9	74.8	0.74	-2.53
13	-202.6	875.5		-3101.20	894.71	-3099.7	899.7	97.4	-1.5	-4.99

The data shows that the north azimuth angle and trim angle got by the calculation of the initial transform matrix can follow the north azimuth angle and trim angle of the navigation well with a certain error. However with the difference of the pitch gravitational torque of the aiming mechanism, the platform has elastic deformation during aiming, which changes the initial transform matrix from the platform coordinate to geometric coordinate. The analyze shows that the greater the elastic deformation is, the bigger the error between the calculated parameters of navigation with initial transform matrix J0 and the real result is. However, multi testing results calculate out the biggest tracking error due to elastic deformation. if the result is bigger than the biggest tracking error, the navigation output is considered wrong.

7. Conclusion

The essay studies the principle of how the aiming mechanism aims at a given target and analysis the possibility of using initial transform matrix to calculate the parameters of the navigation. When the strap down inertial navigation is breakdown, it uses the initial transform matrix and the real-time output of azimuth and pitch sensor to track the output of the navigation to correct the error in order to control the aiming process. It is of great practical value.

Acknowledgements

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References

1. Xiaotin Rui, Guoping Wang, Yuqi Lu (2012) Advances in Rocket Launch Dynamics. 20th International Symposium on Ballistics. *Pennsylvania: Destech Publications*, p.p.408-415.
2. Andrew S.(2011) *Computer Networks, 3rd ED*. Tsinghua University Press: BeiJing.
3. Morris Driels (2011) Linear Control Systems Engineering. Tsinghua University Press: BeiJing.
4. G.Y.Hu.(2013) Improvement Design Of X-type Rocket Gun. *Fire Control & Command Control*, 28(6), p.p.23-45.
5. Wai R.(2013)Development of intelligent position control system using optimal design technique. *IEEE Transactions on Industrial Electronics,* 50(1), p.p.218-231.
6. Chen X K.(2006)Adaptive Sliding Mode Control for Discrete-time Multi-input Multi-output Systems. *Automatica*, 42(3), p.p.427-435.