Spectral mesh smoothing for virtual simulation

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Abstract
Mesh surface smoothing has the important effect for improving the reality and immension of virtual simulation system. In this paper, we propose a novel triangular mesh surface smoothing method based on differential coordinates. We use a spectral processing method on the differential coordinates to smooth the mesh using convolution. We then reconstruct the new Cartesian coordinates using the smoothed differential coordinates with face barycenter constraints. The algorithm can preserve and enhance certain user-specified geometric features and smooth the mesh. The presented approach is computationally simple, robust, and can effectively remove substantial noise. Our experimental results demonstrated that the proposed method can overcome surface shrinkage and shape distortion during the smoothing processing, and can partly preserve geometric details of the mesh surface.

Key words: MESH SMOOTHING, VIRTUAL SIMULATION, SPECTRAL METHOD

Introduction

Virtual simulation technology is a combination of computer technology, computer graphics, computer vision and other disciplines Technology. The fidelity and real-time interaction is the most important feature of a virtual simulation system. The sense of reality in virtual simulation is based mainly on realistic rendering of 3D models. We only pay attention to the overall geometry of a 3D model in order to meet the real-time requirement, and geometric details of a 3D model can be omitted. For the high resolution 3D model, mesh surface smoothing can greatly improve the render processing. Hence, robust and efficient 3D mesh smoothing techniques must be applied before realistic rendering of 3D models.

Taubin [1] used a single processing approach for mesh surface smoothing. They proposed a low-pass filter with positive and negative damping factors, which is known as the $\lambda | \mu$ algorithm. In this method, the filter is $f(\mathrm{x}) = (1-\lambda k)(1-\mu k)$ where $\lambda > 0$, $\mu < -\lambda$. The first step is to filter using λ to smooth the mesh surface, and the second step is to filter with μ to expand the surface and compensate for shrinkage. Taubin’s smoothing can stop mesh surface shrinkage, and performs well if λ and μ

are appropriately chosen. The algorithm can be numerically unstable if the filter's parameters are not carefully chosen. Additionally, Taubin's smoothing method requires more iteration steps to reach a particular level of smoothing, when compared with other methods. Zhang used a spectral analysis of the mesh geometry based on a graph Laplacian for mesh smoothing. Projecting a 3D mesh surface (a 3D signal) onto the basis formed by the eigenvectors of the Laplace operator is equivalent to the Fourier transformation. We can remove the mesh surface noise by removing the high frequency spectral coefficients and then recover the mesh surface. A very smooth model that has few details may be reconstructed using a small number of spectral coefficients. The details can be refined by increasing the number of coefficients used in the reconstruction. This method distorts geometric details and protruding features.

In this paper, we consider a computer graphics object represented by dense triangular meshes. We propose using the spectrum of the Laplace-Beltrami operator defined on differential coordinates to smooth the mesh surface. In contrast to traditional global Cartesian coordinates, differential coordinates do not only represent the spatial location of each vertex of the mesh surface; they also contain information about the local shape of the mesh surface, and the size and orientation of local details. We regard the differential coordinates as a signal defined on the mesh surface. Thus, we can use the spectral processing method on differential coordinates to smooth the mesh. We can then reconstruct the new Cartesian coordinates to fit the smoothed differential coordinates using barycenter constraints on the mesh faces. The new method is simple, stable, and can effectively preserve geometric details of the mesh surface.

Previous Works

A wide variety of mesh smoothing algorithms has been proposed. In general, these mesh smoothing algorithms can be grouped into three classes: isotropic, anisotropic, and hybrid categories. Most early mesh smoothing methods are isotropic, which means that the filter is independent of the surface geometry. Laplace smoothing is a very simple and effective technique for mesh smoothing. It is the most frequently used mesh smoothing method. It repeatedly and simultaneously adjusts the location of each mesh vertex to the geometric center of the neighborhood vertices. However, mesh surface shrinkage and shape distortion may occur during the smoothing process. Taubin[1] introduced signal processing techniques to mesh surface smoothing to overcome the shrinkage problem. Taubin's method is a two-step filtering algorithm. The first step is to smooth the mesh surface, and the second step expands the surface to compensate for shrinkage. However, Taubin smoothing requires more iteration steps to achieve a required level of smoothing, when compared with Laplace smoothing. Some other smoothing methods [2–4] are also considered isotropic. Isotropic methods do not preserve surface features.

Various anisotropic methods have been proposed to effectively prevent shape distortion during smoothing [5–8]. These methods modify the diffusion equation so that it is nonlinear or anisotropic, to preserve the sharp geometric features of 3D mesh surface. Some anisotropic methods [1,9–15] were based on anisotropic diffusion methods from image processing. Anisotropic methods can effectively preserve the geometric details of a mesh surface. However, most of these methods are computationally expensive and have many parameters.

Some hybrid methods [16–18] have been proposed to make smoothing methods more robust. In general, these methods combine multiple approaches to make the smoothing algorithms more robust. However, they are complicated to implement.

The Spectral Method and Differential Coordinates

Spectral Method

There has been an increasing amount of interest in signal processing approaches to 3D mesh manipulation in recent years. The mesh vertex coordinates are represented as a 3D signal defined over the underlying mesh graph. Taubin[1] first used mesh Laplacian operators for mesh surface processing. The spectral analysis of a mesh Laplacian is generalization of a Fourier analysis on a 3D mesh.

Let A be the adjacency matrix defined by the n-vertex mesh topology, i.e.

$$A_{ij} = \begin{cases} 1 & i \; and \; j \; are \, 1-ring \; neighbors \\ 0 & otherwise \end{cases} \tag{1}$$

Let D be the diagonal matrix such that $D_{ii} = 1/d_i$, where d_i is the degree (valence) of vertex i. Then, $L = I - DA$ is the mesh Laplacian, i.e.,

$$L_{ij} = \begin{cases} 1 & i = j \\ -1/d_i & i \; and \; j \; are \, 1-ring \; neighbors \\ 0 & otherwise \end{cases} \tag{2}$$

The eigenvectors of L form an orthogonal basis of the R^n analogy basis functions used in the discrete Fourier transform. The associated eigenvalues can be considered as “frequencies”, and the projection of the coordinate vectors of a 3D mesh on the eigenfunctions are the spectrum of the mesh surface. We can derive the smooth mesh surface by reconstructing the low-frequency components in the spectral domain.

Differential Coordinates

Different coordinates were first proposed by Marc Alexa [19]. Let $\mathrm{M}=(V,E,F)$ be a given mesh surface with n-vertices. V is the set of vertices, E is the set of edges, and F is the set of faces. Each vertex $i\in M$ is represented as $v_i=(x_i,y_i,z_i)$ using absolute Cartesian coordinates. We can write the differential coordinate vector at vertex v_i as

$$\delta_i=\frac{1}{d_i}\sum_{j\in N(i)}\left(v_i-v_j\right) \tag{3}$$

Where $N(i)=\{j\,|\,(i,j)\in E\}$, and $d_i=|N(i)|$ is the degree of v_i.

From a differential geometry view, the $\delta-coordinates$ is a discretization form of the continuous Laplace-Beltrami operator [20]. The $\delta-coordinates$ of v_i are a vector in R^3. The direction of the $\delta-coordinates$ approximates the local normal direction, and the magnitude approximates the local mean curvature [21]. Therefore, differential coordinates can successfully preserve local detail properties of the mesh surface. Differential coordinates can display the size and orientation of local geometric details of the mesh surface.

Mesh Smoothing Process

In this section, we discuss the implementation of spectral processing on differential coordinates for mesh smoothing. We first calculate the differential coordinates of every vertex of the global Cartesian coordinates according to Equation 1. Second, we apply spectral processing to these differential coordinates to obtain the smoothed differential coordinates. Finally, we reconstruct a smoothed mesh by solving a least squares problem, with barycenter constraints for each face of the mesh. The specific steps are as follows.

Step 1. Compute the differential coordinates δ_i for each vertex v_i, that is,

$$\delta_i=\left(\delta_{ix},\delta_{iy},\delta_{iz}\right)=v_i-\sum_{j\in N(i)}\frac{1}{|N(i)|}v_j \tag{4}$$

where $N(i)$ is the set of $1-ring$ neighbourhood of vertex v_i, and $|N(i)|$ is the degree of v_i.

Step 2. The smoothed differential coordinates $\delta_i^{'}$ are calculated using spectral processing.

We compute the eigen-decomposition of the Laplacian operator (L) on the mesh surface, transform the differential coordinate δ_i to the spectrum domain, and apply a low-pass filter. We then reconstruct the differential coordinate $\delta_i^{'}$ using the inverse transformation.

Step 3. The Cartesian coordinates of the mesh are then reconstructed from the smoothed differential coordinates, with barycenter constraints for every face to prevent skinking [1][21]. That is,

$$AV_i'=\begin{pmatrix}L\\F\end{pmatrix}V_i'=\begin{pmatrix}\delta_i\\b_i\end{pmatrix}=B_i \tag{5}$$

Where V_i' represents the Cartesian coordinates of the reconstructed mesh. F is a matrix in which each row contains a constraint on the position of the barycenter of the corresponding face. Its elements are

$$F_{kj}=\begin{cases}1/3 & j\in\{vertex\ of\ one\ face\}\\ 0 & otherwise\end{cases} \tag{6}$$

Where $1\le k\le m$, and m is the index of the face. B_i is an $m-vector$ with elements $b_{ik}=\frac{1}{3}\left(v_{ri}+v_{si}+v_{ti}\right)$, where $\{v_r,v_s,v_t\}$ is the vertex of a face.

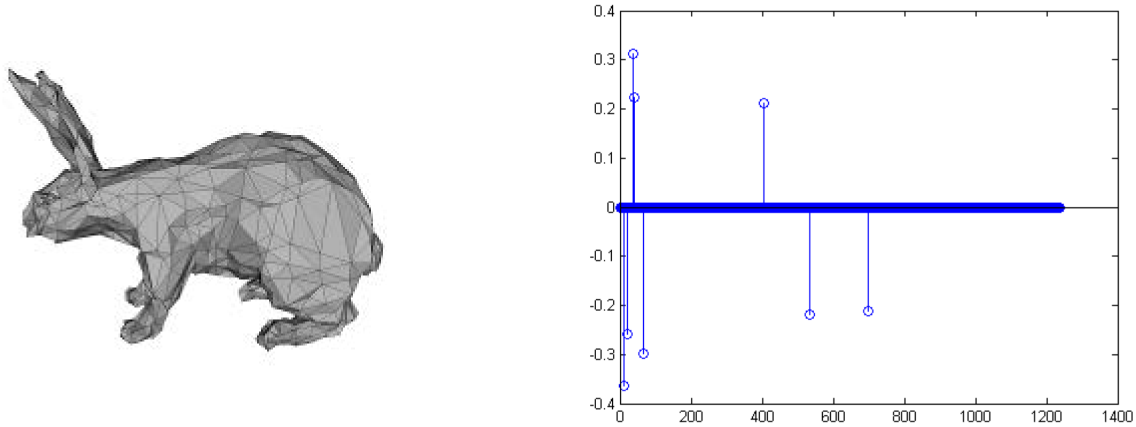
A complete smoothing process is composed of the application of step1+step2+step3. In practice, the choice of filter in step2 will affect the smoothing results.

Experiments and Results

The method proposed in this paper is simple, stable and able to effectively remove noise, and does not induce surface shrinkage and shape distortion during the smoothing process. And unlike some of the previous methods with many parameters, our method has only one parameter, the cut-off frequency, needs to set up.

We compared the smoothing results of our method with the two other approaches. The algorithms described in this paper have been implemented in Matlab 2012b, on a PC with a 3.2GHz Intel Xeon CPU and 4.0 GB of RAM. We applied the methods to the rabbit model shown in Figure1.

Vertices of rabbit model can be expressed as $V=(x_i,y_i,z_i)\ (i=1,2,\cdots,n)$. The 3D model has three components (X、Y、Z). FIGURE 1(b) shows X-component spectral coefficients of the rabbit model's Cartesian coordinates. If we want to smooth the 3D model, we must smooth the three components independently. In this paper, local details can be defined in a more implicit way with a single-resolution mesh, by using differential coordinates. It's handy when we smooth the model by using spectral coefficients of Laplace coordinates only. As the same time, spectral coefficients of Laplace coordinates have the higher discrimination in the high frequency part of signal than Cartesian Coordinates' spectral coefficients.



(a)

(b)

Figure 1. Spectral decomposition. (a) The Rabbit model with noise, (b) the spectral coefficients of Laplace coordinates.

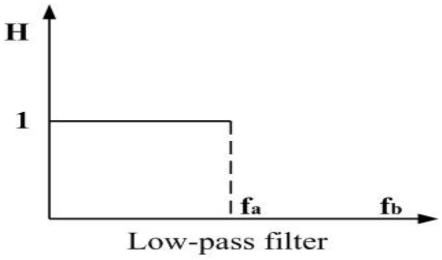
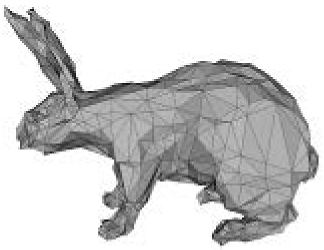
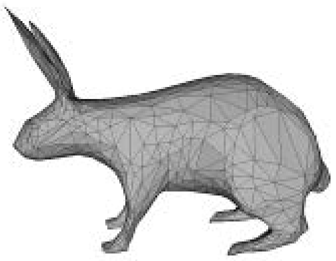
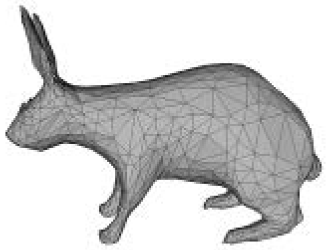
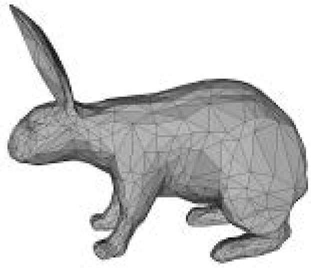


Figure 2. Low-pass filter

The low-pass filter is a simple damping operator in signal processing for decaying high frequency features. In this paper, the low-pass filter, shows in FIGURE 2, is generalized for decaying the Laplacian coordinates. Different f_a will produce different smoothing results. We make $f_a = 200$ for Rabbit model smoothing in this section. Table 1 shows that the Laplace smoothing method can make the model surface collapse, and spectral processing can lose some geometric details.

Table 1. Origin model and the results of the different smoothing methods

	
Model with noise	Laplace smoothing

	
Spectral processing	Our method

Conclusions

In this paper, we developed and successfully applied a novel denoising technique based on the differential coordinates of a triangular mesh surface. Differential coordinates can describe local details and can be treated as a signal defined on the mesh. Therefore, we can apply spectral processing to the signal to smooth the differential coordinates and remove noise. The noisy mesh is smoothed by solving a sparse linear system. We added barycenter constraints to the linear system to avoid shrinkage and distortion during the smoothing process. The proposed method is non-iterative, fast, and effective.

Our approach can be improved and extended. We have used barycenter constraints when reconstructing the mesh, but a variety of constraints for specific purposes can be included by adding more appropriate equations to the linear system. And different filter can get diverse smoothing results.

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