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Boundary element method in problems of plate elements bending of engineering structures

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Abstract
The problem of increase of accuracy of Kantorovich-Vlasov's variational method is considered. It is proposed to simplify the method of applying a number of members of the method under various boundary conditions. The solutions of isotropic bending problems of thin rectangular plates with hinged support and rigid fixing, wherein there is a significant increase in accuracy as compared with the case of using one member of number, are presented. The calculation results are obtained by numerically-analytical variant of the method of boundary elements in the environment of MATLAB.
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Solution of the problems of bending elements of engineering structures in the form of rectangular plates and plate systems (in such fields as aerospace, aircraft industry and instrumentation) analytically have considerable mathematical difficulties. The Kontorovich-Vlasov variational method is one of these analytical methods. This method, developed of the Fourier method of variables separation, allows us to obtain an approximate analytical solution of boundary value problems for partial differential equations. For this reason, the scope of its application is very extensive and it is a powerful and effective tool for solving complicated tasks of modern science. In particular, deformable solid mechanics, structural mechanics, dynamics and strength of machines, etc. this method helped to obtain solutions of many problems of statics, dynamics and stability of disc and shell structures. This method has been criticized for the fact, that it is very difficult to take into account two or more members of the desired solution. The only exception is hinged support facility. We propose to improve the accuracy of the method by using multiple terms in the series at different (not hinged) support conditions. We introduce this proposal on the examples of solving the bending of isotropic thin rectangular plates, where the method of Kantorovich-Vlasov was used for the first time.

The differential equation of bending of thin discs is reduced to the form (equation of Jermain-Lagrange) [1]

$$\frac{\partial^4 w(x,y)}{\partial y^4} + 2\frac{\partial^4 w(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y)}{\partial x^4} = \overline{q}(x,y)/D, \quad (1)$$

where $w(x,y)$ – the deflection of the middle plane of the plate;

$\overline{q}(x,y)$ – lateral load on the plate;

$D = Eh^3/12(1-\mu^2)$ – cylindrical rigidity;

E – modulus of elasticity of the 1st type;

h – the thickness of the plate;

μ – Poisson's ratio.

Kinematic and static parameters of equation (1) are represented as functional series. For example, deflection and bending moment take the following form

$$w(x,y) = W_1(y)X_1(x) + W_2(y)X_2(x) + ... = \sum_{i=1}^{\infty} w_i(x,y), \quad (2)$$

$$M_y(x,y) = -D\left[W_1^{II}(y)X_1(x) + \mu W_1(y)X_1^{II}(x)\right] - -D\left[W_2^{II}(y)X_2(x) + \mu W_2(y)X_2^{II}(x)\right] - ... = \sum_{i=1}^{\infty} M_{yi}(x,y), \quad (3)$$

where $X_i(x)$, $i=\overline{1,\infty}$ – the given functions system of the variable x;

$W_i(y)$, $i=\overline{1,\infty}$ – the required functions system of the variable y.

This view distinguishes between two areas of the disc - cross coincide with the direction of the axis Ox, and longitudinal, coinciding with the direction of the axis Oy, (Figure 1). As a given system of functions $X_i(x)$ it is convenient to take the form of natural oscillations of the housing strut with supports similar to the support conditions of the longitudinal edges of the disc [1,2]. The essence of the mathematical transformation of the equation of bending of the disc in the Kantorovich-Vlasov method is the substitution of the range (2) into the equation (1), multiplying both parts by the selected functions system $X_i(x)$ and integration within the width of the disc from 0 to l_1. A system of linear differential equations for the unknown functions is obtained $W_i(y)$ [1].

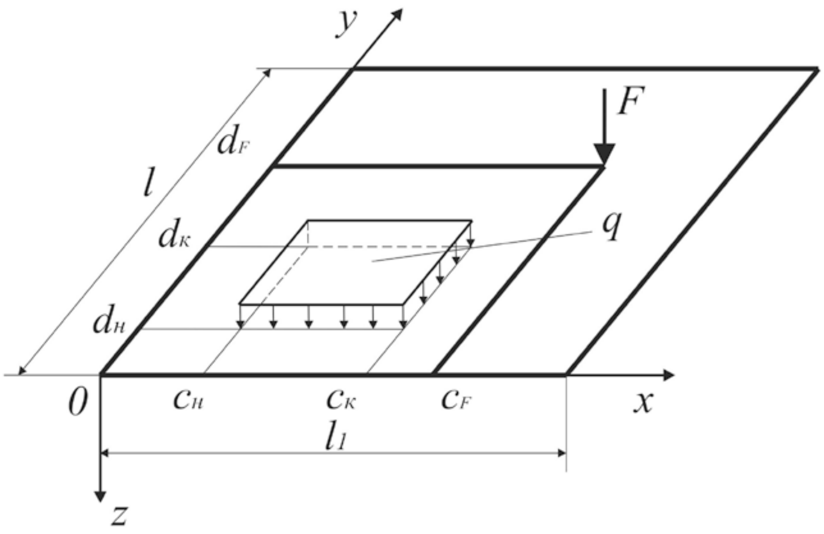

Figure 1. Computational scheme of the square block

$$\sum_{k=1}^{\infty} a_{ik} W_k^{IV}(y) - 2\sum_{k=1}^{\infty} b_{ik} W_k^{II}(y) + \sum_{k=1}^{\infty} c_{ik} W_k(y) = q_i(y)/D, \quad (4)$$

$(i = 1, 2, ...)$

where coefficients are evaluated according to formulas

$$a_{ik} = \int_0^{l_1} X_k X_i dx; \; b_{ik}\int_0^{l_1} X_k^I X_i^I dx - \frac{\mu}{2}\left[X_k X_i^I + X_k^I X_i\right]\Big|_0^{l_1}; \quad (5)$$

$$c_{ik} = \int_0^{l_1} X_k^{II} X_i^{II} dx; \; q_i(y) = \int_0^{l_1} \overline{q}(x,y) X_i(x) dx.$$

By simply supported longitudinal edges of the plates

$$X_i(x) = \sin\frac{i\pi x}{l_1} \quad (6)$$

and a common system differential linear equations(4) disintegrates into separate equations (owing to the function features(6))

$$\begin{aligned} &W_1^{IV}(y) - 2r_1^2 W_1^{II}(y) + s_1^4 W_1(y) = q_1(y)/D;\\ &W_2^{IV}(y) - 2r_2^2 W_2^{II}(y) + s_2^4 W_2(y) = q_2(y)/D;\\ &W_3^{IV}(y) - 2r_3^2 W_3^{II}(y) + s_3^4 W_3(y) = q_3(y)/D;\\ &- - - - - - - - - - - - - - - - - -, \end{aligned} \quad (7)$$

where

$$r_i^2 = -B_i/A_i;\ s_i^4 = C_i/A_i;\ q_i(y) = \int_0^{l_1} \bar{q}(x,y) X_i(x)dx / A_i;$$

$$A_i = \int_0^{l_1} X_i^2(x)dx;\ B_i = \int_0^{l_1} X_i^{II}(x) X_i(x)dx;\ C_i = \int_0^{l_1} X_i^{IV}(x) X_i(x)dx. \quad (8)$$

Functions system $W_1(y)$, $W_2(y)$, $W_3(y)$,… is found as a solution of the corresponding equations (7) with the support conditions on the transverse edges of the plate. The essence of suggestions of this work is to determine the system functions $W_i(y)$ of the individual equations (7) under the condition of leaning against longitudinal edges different from joint ones.

In this case supplementary coefficients of the joint system of equalizations (4) are not taken into account and functions $W_i(y)$ will be defined with some error.

Thus, in the method of Kantorovich-Vlasov a rectangular plate is designed by two beams. In the direction of axis of Ox a beam allows to choose functions $X_i(x)$, in the direction of axis of Oy - to find functions $W_i(y)$ with the help of Cauchy solution bend of beam. Cauchy solution for equations (7) is possible to present in a matrix form (here and below the indexes of range members are suppressed) [2, 3].

$$\begin{Bmatrix} DW(y) \\ D\theta(y) \\ M(y) \\ Q(y) \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & -A_{13} & -A_{14} \\ A_{21} & A_{22} & -A_{23} & -A_{13} \\ -A_{31} & -A_{32} & A_{22} & A_{12} \\ -A_{41} & -A_{31} & A_{21} & A_{11} \end{bmatrix} \begin{Bmatrix} DW(0) \\ D\theta(0) \\ M(0) \\ DW(0) \end{Bmatrix} + \int_0^y \begin{Bmatrix} A_{14}(y-\xi) \\ A_{13}(y-\xi) \\ -A_{12}(y-\xi) \\ -A_{11}(y-\xi) \end{Bmatrix} q(\xi)d\xi \quad , \quad (9)$$

where $W(y), \theta(y), M(y), Q(y)$ – bending, turning angle, flexion moment and transversal force of conditional beam, replacing a plate in the direction of axis O*y*.

Let us estimate an error of the offered approach on concrete examples.

Square plate with the joint leaning on a perimeter, loaded with evenly distributed load $\bar{q}(x,y) = q = const$ and concentrated force of F in a center. In this case $r_i = s_i = i\pi$. Fundamental orthonormal functions and elements look like from loading [3].

$$\begin{aligned} &\Phi_1(y) = y\,chry;\ \Phi_2(y) = chry;\ \Phi_3(y) = shry;\ \Phi_4(y) = y\,shry;\ \omega_i = i\pi;\\ &X_i(x) = \sin(i\pi x/l_1);\ A_{11} = \Phi_2(y) - (1-\mu)r\,\Phi_4(y)/2;\\ &A_{12} = (1-\mu)\Phi_1(y)/2 + (1+\mu)\Phi_3(y)/(2r);\ \ A_{13} = \Phi_4(y)/(2rA);\\ &A_{14} = (r\Phi_1(y) - \Phi_3(y))/(2r^3A);\ A_{21} = r\,(1+\mu)\Phi_3(y)/2 - (1-\mu)r^2\Phi_1(y)/2;\\ &A_{22} = \Phi_2(y) + (1-\mu)r\,\Phi_4(y)/2;\ \ A_{23} = \Phi_1(y)/(2A) + \Phi_3(y)/(2rA);\\ &A_{31} = -(1-\mu)^2 r^3 A\Phi_4(y)/2;\ \ A_{32} = A\left[(1-\mu)^2 r^2\Phi_1(y) + (1-\mu)(3+\mu)r\Phi_3(y)\right]/2;\\ &A_{41} = A\left[(1-\mu)^2 r^4\Phi_1(y) - (1-\mu)(3+\mu)r^3\Phi_3(y)\right]/2; \end{aligned} \quad , \quad (10)$$

$$B_{11}=q\frac{\left[r\Phi_4(y-d_{н})_+ + 2H(y-d_{н}) - 2\Phi_2(y-d_{н})_+\right]\gamma_q(\omega)\Big|_{c_{н}}^{l_1}}{2r^4A}-$$

$$-q\frac{\left[r\Phi_4(y-d_{к})_+ + 2H(y-d_{к}) - 2\Phi_2(y-d_{к})_+\right]\gamma_q(\omega)\Big|_{c_{к}}^{l_1}}{2r^4A}+$$

$$+F\gamma_F(\omega)\frac{r\Phi_1(y-d_F)_+ - \Phi_3(y-d_F)_+}{2r^3A};$$

$$B_{21}=\frac{q\left[r\Phi_1(y-d_{н})_+ - \Phi_3(y-d_{н})_+\right]\gamma_q(\omega)\Big|_{c_{н}}^{l_1}}{2r^3A}-$$

$$-q\frac{\left[r\Phi_1(y-d_{к})_+ - \Phi_3(y-d_{к})_+\right]\gamma_q(\omega)\Big|_{c_{к}}^{l_1}}{2r^3A}+F\gamma_F(\omega)\frac{\Phi_4(y-d_F)_+}{2rA};$$

$$B_{31}=q\frac{\left[(1-\mu)r\Phi_4(y-d_{н})_+ + 2\mu\Phi_2(y-d_{н})_+ - 2\mu H(y-d_{н})\right]\gamma_q(\omega)\Big|_{c_{н}}^{l_1}}{2r^2}- \quad , \qquad (10)$$

$$-q\frac{\left[(1-\mu)r\Phi_4(y-d_{к})_+ + 2\mu\Phi_2(y-d_{к})_+ - 2\mu H(y-d_{к})\right]\gamma_q(\omega)\Big|_{c_{к}}^{l_1}}{}+$$

$$+F\gamma_F(\omega)\frac{(1-\mu)r\Phi_1(y-d_F)_+ + (1+\mu)\Phi_3(y-d_F)_+}{2r};$$

$$B_{41}=q\frac{\left[(3-\mu)\Phi_3(y-d_{н})_+ - (1-\mu)r\Phi_1(y-d_{н})_+\right]\gamma_q(\omega)\Big|_{c_{н}}^{l_1}}{2r}-$$

$$-q\frac{\left[(3-\mu)\Phi_3(y-d_{к})_+ - (1-\mu)r\Phi_1(y-d_{к})_+\right]\gamma_q(\omega)\Big|_{c_{к}}^{l_1}}{2r}+$$

$$+F\gamma_F\left[\Phi_2(y-d_F)_+ - (1-\mu)r\Phi_4(y-d_F)_+/2\right];$$

$$\gamma_{qi}(\omega)=\int_0^{l_1}X_i(x)dx;\quad \gamma_{Fi}(\omega)=\sin(i\pi c_F/l_1);$$

$H(y-d_{н})$ – Heavyside's unit function displaced at the point $d_{н}$;

$\Phi_4(y-d_{н})_+$ – spline function of sight

$$\Phi_4(y-d_{н})_+=\begin{cases}0, (y-d_{н})<0\\ \Phi_4(y-d_{н}), (y-d_{н})\geq 0\end{cases}.$$

The required function $W(y)$ from the equation (9) is determined in such a way:

$$DW(y)=DW(0)\cdot A_{11}(y)+D\theta(0)\cdot A_{12}(y)-M(0)\cdot A_{13}(y)-Q(0)\cdot A_{14}(y)+B_{11}(y), \qquad (11)$$

where initial parameters can be defined at the decision of boundary problem for a beam on the numeral-analytical method of border elements [2, 3] ($c_{н}=0$; $c_{к}=l_1$; $d_{н}=0$; $d_{к}=l$; $\mu=0,3$; $q=1$; $F=1$; $d_F=l/2$; $c_F=l_1/2$; $l_1=l=a=1$).

	1	2	3	4				
1		A_{12}		$-A_{14}$	$D\theta(l)$		$-B_{11}(l)$	
2	-1	A_{22}		$-A_{13}$	$D\theta(0)$	=	$-B_{21}(l)$	. (12)
3		$-A_{32}$		A_{12}	$Q(l)$		$B_{31}(l)$	

4		$-A_{31}$	-1	A_{11}	$Q(0)$	$B_{41}(l)$

In the table 1 the results of calculations (2), (3) are presented, where even terms are absent, because they are practically equal to zero.

Table 1. The values of bendings and moments in the joint leaning plate

Number of the row terms range membrs	Terms of leaning	Loading	Bending in the center of plate $w(l_1/2, l/2)$	Flexion moments in the center of plate $M_y(l_1/2, l/2)$
1	The joint on a perimeter	$\overline{q}(x,y)=q$	$41,093\cdot 10^{-4}\ qa^4/D$	$4,920\cdot 10^{-2}\ qa^2$
3			$-0,505\cdot 10^{-4}$	$-0,155\cdot 10^{-2}$
5			$0,047\cdot 10^{-4}$	$0,0312\cdot 10^{-2}$
7			$-0,00778\cdot 10^{-4}$	$-0,0113\cdot 10^{-2}$
9			$0,00221\cdot 10^{-4}$	$0,00531\cdot 10^{-2}$
Σ			$40,624\cdot 10^{-4}$	$4,790\cdot 10^{-2}$
1	The joint leaning on a perimeter	$\overline{q}(x,y)=F\times \times\delta(x-l_1/2)\times \times\delta(y-l/2)$	$107,665\cdot 10^{-4}\ Fa^2/D$	$21,756\cdot 10^{-2} F$
3			$5,962\cdot 10^{-4}$	$6,901\cdot 10^{-2}$
5			$1,290\cdot 10^{-4}$	$4,138\cdot 10^{-2}$
7			$0,470\cdot 10^{-4}$	$2,956\cdot 10^{-2}$
9			$0,221\cdot 10^{-4}$	$2,299\cdot 10^{-2}$
Σ			$115,609\cdot 10^{-4}$	$38,049\cdot 10^{-2}$

Inaccuracies at the action of the evenly distributed load at bendings (exact data are taken from [4, 5])

$$\Delta_1 = \frac{40,624-40,6}{40,6}\cdot 100\% = 0,06\% \quad (13)$$

at flexion moments

$$\Delta_2 = \frac{4,79-4,79}{4,79}\cdot 100\% = 0,0\% \quad (14)$$

At the action of the concentrated force in the center of plate at bendings

$$\Delta_3 = \frac{115,609-116,0}{116,0}\cdot 100\% = 0,34\% \quad (15)$$

In reference data [4, 5] the values of flexion moments in the center of the plate (point of application of the concentrated force F) are absent. Evidently, that the results of method are practically exact.

Square plate with rigid fixing along the perimeter, loaded with evenly distributed load and concentrated force in a center. In these terms $|s|>|r|$, frequency of rigidly fixed beam should be found searched from equation $\cos(\omega)\cdot ch(\omega)=1$. The elements of equation (9) become [3].

$$\Phi_1(y)=ch(\alpha y)\sin(\beta y);\ \Phi_2(y)=ch(\alpha y)\cos(\beta y);\ \Phi_3(y)=sh(\alpha y)\cos(\beta y);$$
$$\Phi_4(y)=ch(\alpha y)\sin(\beta y);\ \alpha=\sqrt{(s^2+r^2)/2};\ \beta=\sqrt{(s^2-r^2)/2};$$
$$A_{11}=\Phi_2(y)-(1-\mu)r^2\Phi_4(y)/(2\alpha\beta);\ A_{12}=(s^2-\mu r^2)\Phi_1(y)/(2\beta s^2)+$$
$$+(s^2+\mu r^2)\Phi_3(y)/(2\alpha s^2);\ A_{13}=\Phi_4(y)/(2\alpha\beta A);\ A_{14}=(\alpha\Phi_1(y)-\beta\Phi_3(y))/(2\alpha\beta s^2 A);$$
$$A_{21}=(s^2+\mu r^2)\Phi_3(y)/(2\alpha)-(s^2-\mu r^2)\Phi_1(y)/(2\beta);$$
$$A_{22}=\Phi_2(y)+(1-\mu)r^2\Phi_4(y)/(2\alpha\beta);\ A_{23}=(\alpha\Phi_1(y)+\beta\Phi_3(y))/(2\alpha\beta A);$$
$$A_{31}=A\left[\mu r^4(2-\mu)-s^4\right]\Phi_4(y)/(2\alpha\beta);$$
$$A_{32}=A\left[-s^4+2(1-\mu)s^2r^2+\mu^2r^4\right]\Phi_1(y)/(2\beta s^2)+A\left[s^4+2(1-\mu)s^2r^2-\mu^2r^4\right]\Phi_3(y)/(2\alpha s^2);$$
$$A_{41}=A\left[-s^4+2(1-\mu)s^2r^2+\mu^2r^4\right]\Phi_1(y)/(2\beta)-A\left[s^4+2(1-\mu)s^2r^2-\mu^2r^4\right]\Phi_3(y)/(2\alpha);$$
$$B_{11}=q\frac{\left\{r^2\Phi_4(y-d_н)_+ +2\alpha\beta\left[H(y-d_н)-\Phi_2(y-d_н)_+\right]\right\}\gamma_q(\omega)\Big|_{c_н}^{l_1}}{2\alpha\beta s^4 A}-$$
$$-q\frac{\left\{r^2\Phi_4(y-d_к)_+ +2\alpha\beta\left[H(y-d_к)-\Phi_2(y-d_к)_+\right]\right\}\gamma_q(\omega)\Big|_{c_к}^{l_1}}{2\alpha\beta s^4 A}+$$
$$+F\gamma_F(\omega)\frac{\alpha\Phi_1(y-d_F)_+ -\beta\Phi_3(y-d_F)_+}{2\alpha\beta s^2 A};$$
$$B_{21}=q\frac{\left[\alpha\Phi_1(y-d_н)_+ -\beta\Phi_3(y-d_н)_+\right]\gamma_q(\omega)\Big|_{c_н}^{l_1}}{2\alpha\beta s^2 A}-q\frac{\left[\alpha\Phi_1(y-d_к)_+ -\beta\Phi_3(y-d_к)_+\right]\gamma_q(\omega)\Big|_{c_к}^{l_1}}{2\alpha\beta s^2 A}+ \qquad (16)$$
$$+F\gamma_F(\omega)\frac{\Phi_4(y-d_F)_+}{2\alpha\beta A};$$
$$B_{31}=q\frac{\left\{2\mu\alpha\beta r^2\left[\Phi_2(y-d_н)_+ -H(y-d_н)\right]+(s^4-\mu r^4)\Phi_4(y-d_н)_+\right\}\gamma_q(\omega)\Big|_{c_н}^{l_1}}{2\alpha\beta s^4}-$$
$$-q\frac{\left\{2\mu\alpha\beta r^2\left[\Phi_2(y-d_к)_+ -H(y-d_к)\right]+(s^4-\mu r^4)\Phi_4(y-d_к)_+\right\}\gamma_q(\omega)\Big|_{c_к}^{l_1}}{2\alpha\beta s^4}+$$
$$+F\gamma_F(\omega)\frac{(s^2-\mu r^2)\alpha\Phi_1(y-d_F)_+ +(s^2+\mu r^2)\beta\Phi_3(y-d_F)_+}{2\alpha\beta s^2};$$
$$B_{41}=q\frac{\left\{\alpha\left[2\beta^2-(1-\mu)r^2\right]\Phi_1(y-d_н)_+ +\beta\left[2\alpha^2+(1-\mu)r^2\right]\Phi_3(y-d_н)_+\right\}\gamma_q(\omega)\Big|_{c_н}^{l_1}}{2\alpha\beta s^2}-$$
$$-q\frac{\left\{\alpha\left[2\beta^2-(1-\mu)r^2\right]\Phi_1(y-d_к)_+ +\beta\left[2\alpha^2+(1-\mu)r^2\right]\Phi_3(y-d_к)_+\right\}\gamma_q(\omega)\Big|_{c_к}^{l_1}}{2\alpha\beta s^2}+$$
$$+F\gamma_F(\omega)\left[\Phi_2(y-d_F)_+ -(1-\mu)r^2\Phi_4(y-d_F)_+/(2\alpha\beta)\right];$$
$$X(x)=\sin(\omega x/l_1)-sh(\omega x/l_1)-a_z\left[\cos(\omega x/l_1)-ch(\omega x/l_1)\right];$$
$$a_z=(\sin\omega-sh\omega)/(\cos\omega-ch\omega);\ \gamma_q(\omega)=\int_0^{l_1}X(x)dx;$$
$$\gamma_F(\omega)=\sin(\omega c_F/l_1)-sh(\omega c_F/l_1)-a_z\left[\cos(\omega c_F/l_1)-ch(\omega c_F/l_1)\right].$$

A boundary problem of the rigidly fixed beam becomes

$(d_н = 0\,; d_к = l\,; c_н = 0\,; c_к = l_1; d_F = l/2\,; c_F = l_1/2\,; \mu = 0{,}3;\ q = 1;\ F = 1;\ l_1 = l = a = 1)$

	1	2	3	4
1			$-A_{13}$	$-A_{14}$
2			$-A_{23}$	$-A_{13}$
3	*-1*		A_{22}	A_{12}
4		*-1*	A_{21}	A_{11}

$M(l)$
$Q(l)$
$M(0)$
$Q(0)$

=

$-B_{11}(l)$
$-B_{21}(l)$
$B_{31}(l)$
$B_{41}(l)$

. (17)

In a table number 2 the results of calculations of row terms (2) , (3) for the rigidly fixed plate are presented. As well as in a joint leaning plate, here also the even terms of range are equal to zero.

Table 2. The values of bendings and moments in the rigidly fixed plate

Number of the range members	Terms of leaning	Loading	Bendings in the center of plate $w(l_1/2, l/2)$	Flexion moments	
				In the reference cut set $M_y(l_1/2, 0)$	In the center of plate $M_y(l_1/2, l/2)$
1	Tough embedding on a perimeter	$\overline{q}(x,y) = q$	$129{,}917\cdot 10^{-5}\ qa^4/D$	$-538{,}852\cdot 10^{-4}\ qa^2$	$245{,}606\cdot 10^{-4}\ qa^2$
3			$-3{,}386\cdot 10^{-5}$	$42{,}306\cdot 10^{-4}$	$-13{,}872\cdot 10^{-4}$
5			$0{,}367\cdot 10^{-5}$	$-10{,}968\cdot 10^{-4}$	$3{,}304\cdot 10^{-4}$
7			$-0{,}078\cdot 10^{-5}$	$4{,}325\cdot 10^{-4}$	$-1{,}298\cdot 10^{-4}$
9			$0{,}024\cdot 10^{-5}$	$-2{,}128\cdot 10^{-4}$	$0{,}638\cdot 10^{-4}$
Σ			$126{,}844\cdot 10^{-5}$	$-505{,}318\cdot 10^{-4}$	$234{,}379\cdot 10^{-4}$
1	Tough embedding on a perimeter	$\overline{q}(x,y) = F\times \times\delta(x-l_1/2)\times \times\delta(y-l/2)$	$51{,}520\cdot 10^{-4}\ qa^2/D$	$-11{,}897\cdot 10^{-2}\ F$	$19{,}212\cdot 10^{-2} F$
3			$3{,}895\cdot 10^{-4}$	$-0{,}314\cdot 10^{-2}$	$6{,}148\cdot 10^{-2}$
5			$0{,}999\cdot 10^{-4}$	$-0{,}0096\cdot 10^{-2}$	$3{,}877\cdot 10^{-2}$
7			$0{,}391\cdot 10^{-4}$	$-0{,}000266\cdot 10^{-2}$	$2{,}819\cdot 10^{-2}$
9			$0{,}191\cdot 10^{-4}$	$-6{,}208\cdot 10^{-8}$	$2{,}215\cdot 10^{-2}$
Σ			$56{,}966\cdot 10^{-4}$	$-12{,}221\cdot 10^{-2}$	$34{,}272\cdot 10^{-2}$

Unaccuracies of results at the action of evenly distributed load

at bendings

$$\Delta_4 = \frac{126{,}844 - 126{,}0}{126{,}0}\cdot 100\% = 0{,}67\%\,, \tag{18}$$

at bending moments

$$\Delta_5 = \frac{505{,}318 - 513{,}0}{513{,}0}\cdot 100\% = 1{,}50\%\,,$$

$$\Delta_6 = \frac{234{,}379 - 231{,}0}{231{,}0}\cdot 100\% = 1{,}46\%\,. \tag{19}$$

At the action of the concentrated force in the center of plate

at bendings

$$\Delta_7 = \frac{56{,}996 - 56{,}0}{56{,}0}\cdot 100\% = 1{,}78\% \tag{20}$$

at bending moments

$$\Delta_8 = \frac{12,221 - 12,57}{12,57} \cdot 100\% = 2,78\% \qquad (21)$$

As it was mentioned before, values of moments in the center of plate in reference data [4, 5] are absent.

It ensues from presented, that exactness of variation method of Kantorovich-Vlasov at the use of a few row terms (2) substantially increases and the results of calculations practically coincide with the exact meaning. Taking into account that the analytical decision of the Cauchy of differential equalization problem in partials (for example, for a rectangular plate this matrix equalization (9)) within the framework of algorithm of numeral-analytical variant of method of border elements has a considerably more wide use, than simply plates, then it is necessary to conclude that the different thin-walled engineering constructions (uncut plates, cylindrical plicate shells, platebemed systems, polygonal reservoirs etc.) and boarder tasks for linear and in the partials of differential equalizations with variable coefficients [7] can be evaluated in the more exact situation. There are no complications with determination of second and the next terms of row (2), because all calculation correlations depend only on frequency of eigentones of corresponding beams [3, 6].

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