Transfer matrix algorithm of the longitudinal seismic response of multi-span simply supported girder bridge

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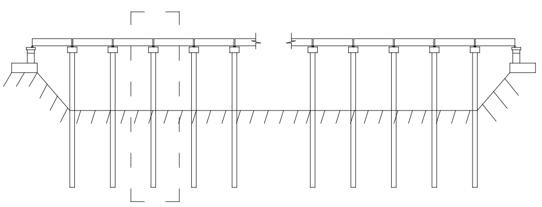
Abstract
Most of railway and highway bridges employ multi-span simply supported girder system. Investigations about seismic damage to bridge both at domestic and abroad show that for multi-span bridges, longitudinal seismic damage of the bridge is generally more serious, moreover, the girder falling is one of the most serious destructions of the upper structure. As for the multi-span simply supported beam bridge, computerization of specification is recommended. For the number of bridge span is small, finite element calculation is feasible. However, when the number of bridge span is large (even reaches hundreds of thousands), modeling will be very complicated. In order to calculate efficiently the seismic response of simply supported Girder Bridge with long multi-span, this paper proposes a transfer matrix algorithm, in which, using half sides of both pier and structure to form a unit, to establish the relationship of the dynamic state response between the two sides, then calculate the dynamic response of any parts via response state transfer rule, at the same time, take the seismic input non-uniformity of each pier into consideration. Taking the six-span simply supported girder bridge as an example, it proves that this method is reliable, when compared with the finite element numerical calculation results.
Key words: MULTI-SPAN, SIMPLY SUPPORTED GIRDER BRIDGE, LONGITUDINAL, SEISMIC RESPONSE, TRANSFER MATRIX ALGORITHM

Introduction

Simply supported girder bridge is widely applied in Chinese railways and highways. Investigations about seismic damage to bridge both at domestic and abroad show that for multiple span bridges, longitudinal seismic damage of the bridge is generally more serious. For simply supported system bridges, structural seismic damages generally do not affect main girder in the earthquake, but the displacement of main beam is a multiple seismic disaster, once the girder bridge falling happens, it will lose its efficacy [1, 2]. Because of no lateral force for the bearing in simply supported girder bridge, the horizontal rigid body displacement of girder is limited by the bearing under earthquake. The girder will bear large inertia force horizontal under horizontal component of seismic momentum, this horizontal inertial force will transfer to the pier through the connecting component, if the horizontal inertial force is larger than the shear strength of the bearing and the connecting components, the bearing will be damaged, the horizontal rigid body displacement produced by girder will lose restriction, producing relatively larger displacement between beam and pier. If there are no other constraints at this time, the relative longitudinal displacement between the beam and the bridge pier will increase further. When the displacement exceeds the pier width range, longitudinal girder falling will occur. At present, the calculation of the longitudinal seismic response of the long multi-span bridge is still mainly using finite element numerical simulation. In manual algorithm, besides the specifications of elemental point can simplify computation, there are algorithms derived by many experts and scholars through the Lagrange equation and the pseudo excitation method [3-10] can also simplify. However, these algorithms are mainly aimed at solving problems of the bridge structure with large span, which is not suitable for long multi-span bridge structures. When the overall bridge structure is relatively simple with small number of spans, it can use the finite element software to establish the whole bridge model to resolve numerical solution. However, when the number of bridge span is large (even reaches hundreds of thousands), modeling will be very complicated. In order to calculate efficiently the seismic response of simply supported girder bridge with long multi-span, this paper established a reasonable control equation about the multi-span simply supported girder bridge by model simplification, using transfer matrix algorithm, and derived the longitudinal bridge seismic response expressions of multi-span simply supported girder bridge with arbitrary span number.

2. Model Simplification

The actual engineering structure has an infinite number of degrees of freedom, which need further simplification to obtain a simplified mechanical model including the concepts of particle and rigid body. This will in turn to simplify the practical engineering structure with infinite number of degree of freedom to a discrete model with finite degree of freedom. A simplified model of simply supported beam bridge with n span is shown in Figure 1. Selecting bridge pier, capping beam and the half of both left and right connecting bridge in upper part as the computing unit, namely the dotted line part in figure(1).



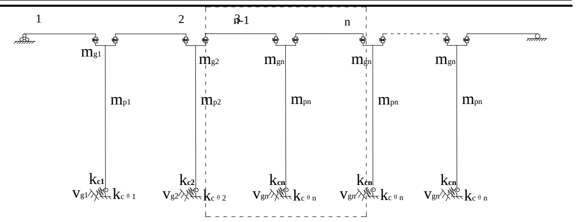


Figure 1. The simplified calculation model

3. The Equation Deduction

3.1. The control equation

When long multi-span bridge structure is discretized into unit, control equation of isolated unit body is obtained based on the differential equations of motion. When motion problem in time domain is Fourier transformed, it can be converted into frequency domain [11]. In this case, by taking the influence of the incentive effect of support on the structure and Fourier transforming motion equations of the structure into consideration, problems in time domain will be transformed into frequency domain:

$$-\omega^2 MU + i\omega CU + KU = F - MU_g \text{ ,} \qquad (1)$$

in which, M, C, K are the structure mass, damping and stiffness matrices, F is the external force system received, U is the displacement of the structure, U_g is longitudinal bridge displacement along the seismic wave.

The damping matrix is using proportional damping, i.e.:

$$C = \alpha M + \beta K \text{ ,} \qquad (2)$$

in which, α is the coefficient proportional to mass, and β is the coefficient proportional to the stiffness.

To make

$$S = \left(-\omega^2 + i\omega\alpha\right)M + \left(i\omega\beta + 1\right)K, \qquad (3)$$

Then the damping matrix is substituted into the Equation(1), and it can be solved as Equation(3): Equation (1) can be re-written as

$$SU = F - MU_g \text{ ,} \qquad (4)$$

3.2. The transfer matrix equation

Further discrete the unit with the unit number and node number as shown in figure 2. Supposing a total of j nodes, assuming the number on left pier beam end is 1, and the number on right side of the beam end node is j. Where u1 is the initial displacement of the beam end of the left unit with an assumed known quantity, initial displacement unit on the right side of the beam end, as the unknown quantity ultimately need to be solved; l was half length of the beam, h represents the height of Pier 1, 2... j are the code number for node, ○,1、○,2…○,n are the discrete unit number.

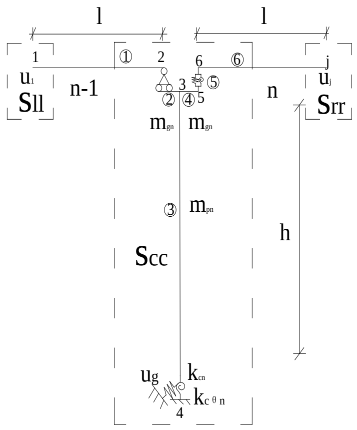


Figure 2. The calculation element model

Arrange equation (4) and write it in the form of a matrix block, i.e.:

$$\begin{bmatrix} S_{ll} & S_{lc} & S_{lr} \\ S_{cl} & S_{cc} & S_{cr} \\ S_{rl} & S_{rc} & S_{rr} \end{bmatrix} \begin{Bmatrix} U_l \\ U_c \\ U_r \end{Bmatrix} = \begin{Bmatrix} F_l \\ F_c \\ F_r \end{Bmatrix} - \begin{Bmatrix} M_l \\ M_c \\ M_r \end{Bmatrix} U_g , \quad (5)$$

In which, $S_{ll} = s_{11}$, $S_{rr} = s_{jj}$, $U_l = u_1$, $U_r = u_j$,

$$[S_{lc}] = \begin{bmatrix} s_{12} & \cdots & s_{1n} & \cdots & s_{1,(j-1)} \end{bmatrix},$$

$$[S_{cl}] = \begin{bmatrix} s_{21} & \cdots & s_{n,1} & \cdots & s_{(j-1),1} \end{bmatrix}^T,$$

$$[S_{cr}] = \begin{bmatrix} s_{2,j} & \cdots & s_{n,j} & \cdots & s_{(j-1),j} \end{bmatrix}^T,$$

$$[S_{rc}] = \begin{bmatrix} s_{j,2} & \cdots & s_{j,n} & \cdots & s_{j,(j-1)} \end{bmatrix},$$

$$U_c = \begin{Bmatrix} u_2 & \cdots & u_n & \cdots & u_{(j-1)} \end{Bmatrix}^T,$$

$$[S_{cc}] = \begin{bmatrix} s_{2,2} & \cdots & s_{2,n} & \cdots & s_{2,(j-1)} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ s_{n,2} & \cdots & s_{n,n} & \cdots & s_{n,(j-1)} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ s_{(j-1),2} & \cdots & s_{(j-1),n} & \cdots & s_{(j-1),(j-1)} \end{bmatrix},$$

Expending the Equation(5), it can be solved as:

$$S_{ll}U_l + S_{lc}U_c + S_{lr}U_r = F_l - M_l U_g \quad (6)$$

$$S_{cl}U_l + S_{cc}U_c + S_{cr}U_r = F_c - M_c U_g \quad (7)$$

$$S_{rl}U_l + S_{rc}U_c + S_{rr}U_r = F_r - M_r U_g \quad (8)$$

Through sorting, state vector of arbitrary pier support form on both left and right sides of the beam end can be obtained.

$$Q_r^{(n)} = R_n Q_l^{(n)} + T_n F_c^{(n)} + L_n U_g^{(n)} \quad (9)$$

$$R_n = \begin{bmatrix} \frac{S_{lc}}{S_{cc}} S_{rr} - S_{lr} & 0 \\ \frac{S_{rc}}{S_{cc}} S_{rr} - S_{rr} & 1 \end{bmatrix}^{-1} \begin{bmatrix} S_{ll} - \frac{S_{lc}}{S_{cc}} S_{cl} & -1 \\ S_{rl} - \frac{S_{rc}}{S_{cc}} S_{cl} & 0 \end{bmatrix} \quad (10)$$

$$T_n = \begin{bmatrix} \frac{S_{lc}}{S_{cc}} S_{rr} - S_{lr} & 0 \\ \frac{S_{rc}}{S_{cc}} S_{rr} - S_{rr} & 1 \end{bmatrix}^{-1} \begin{Bmatrix} \frac{S_{lc}}{S_{cc}} \\ \frac{S_{rc}}{S_{cc}} \end{Bmatrix} \quad (11)$$

$$L_n = \begin{bmatrix} \frac{S_{lc}}{S_{cc}} S_{rr} - S_{lr} & 0 \\ \frac{S_{rc}}{S_{cc}} S_{rr} - S_{rr} & 1 \end{bmatrix}^{-1} \begin{Bmatrix} M_l - \frac{S_{lc}}{S_{cc}} M_c \\ M_r - \frac{S_{rc}}{S_{cc}} M_c \end{Bmatrix} \quad (12)$$

$$Q^{(n)} = \begin{Bmatrix} U \\ F \end{Bmatrix} \quad (13)$$

In which, n represents the isolated body of the number of the across. The relationship with the calculation unit

$$Q_l^{(n)} = Q_r^{(n-1)} \quad (14)$$

For the displacement vector of nth cross beam

$$Q_r^{(n)} = \prod_{n}^{i=1} R_i \cdot Q_l^{(1)} + \sum_{j=1}^{n-1} \prod_{n}^{i=j+1} R_i \left(T_j F_c^{(j)} \cdot + L_j U_g^{(j)} \right) + T_n F_c^{(n)} + L_n U_g^{(n)}$$

(15)

Through the adjacent boundary conditions of the bridge abutment, the displacement and internal force of the first cross can be solved from simultaneous equations, each node displacement of other across can be solved with equation (15), and binding force of other nodes can be calculated with the element stiffness matrix under the local coordinates.

Argument of the Calculation Method

A four-span simply supported girder bridge was shown in Figure 3 with parameters shown in Table 1. Each span length of the bridge is 8 m. It calculated by Rayleigh damping with damping ratio is 0.05. The bridge employs lead rubber as its bearing with parameters selected according to the literature [12], namely: the lateral initial stiffness is 2×10^4 KN/m, damping ratio is 0.05, the yield load is 250 KN, and post yield stiffness is 1.92×10^4 KN/m. Structural parameters are shown in table 1. In consideration of the role of the pile foundation at the bottom of the pier, "m" method is employed to calculate the horizontal, vertical and rotational stiffness of the pile foundation above the soil surface, and then connect the structure to soil with spring elements. Assuming the basic conditions for each pier are the same, by calculation, the lateral stiffness of pile foundation at the bottom of the pier is 2.78×10^5 KN/m, the rotation stiffness is 10.4×10^5 KN/m, vertical and rotational stiffness is 11.0×10^5 KN/m. Assuming the le ft abutment is a fixed hinge support and the right is a sliding one, the displacement and internal force of the first and last cross are solved by the abutment boundary condition. According to the equation (15), the displacements and internal forces of other spans can be solved with Matlab, which are compared with ANSYS numerical simulation results.

Table 1. Parameter structure data

Structure	Density/(kg/m^3)	Length/m	Width/m	Height/m	Modulus of elasticity/MPa
Beam	2500	6.00	1.50	1.5	32500
Pier	2500	1.08	1.13	6.0	28000

By establishing the finite element model with ANSYS, six degrees of freedom is employed with three-dimensional beam unit beam4 for bridge pier and beam, in bridge bearings, combin40 unit model horizontal support is employed; and the combin14 unit is used to simulate the vertical support. Seismic response is input with the high quality method [13, 14] via the mass21 unit. Since total span of this example is relatively small, so input seismic motion with uniform excitation. For the convenience of calculation, assuming the input excitation is EL wave with maximum acceleration 2.15m/s2. ANSYS modeling diagram is shown in Figure 3.

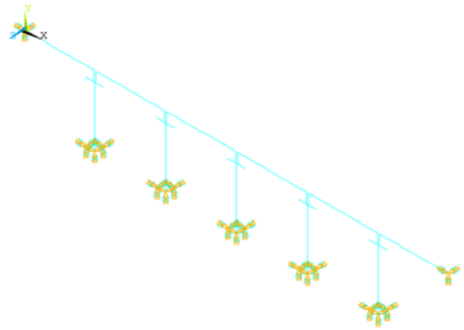


Figure 3. The finite element model

After the discrete of overall structure, the 7 node elements shown in Figure 1 are integrated, that is, in which, j=7 and n=6. When calculate with the deduced formula, it needs to Fourier transform the equations and converts them to the calculation of frequency domain state, and finally converts the calculated results back to the time domain state. the actual bridge body is discretized into finite element body, the lumped mass matrix is employed. The final comparison results between the numerical simulation and formula calculation of longitudinal bridge are shown in table 2, table3,and table 4.

Table 2. Displacement on the top of the pier

displacement(mm)	1st pier top	2nd pier top	3rd pier top	4th pier top	5th pier top
equation	74.5	72.6	72.7	71.3	75.3
simulation	74.2	72.1	72.8	71.2	74.4
deviation	0.30%	0.73%	-0.11%	0.15%	1.21%

Table 3. Shear comparison result of pier bottom

shear(MN)	1st pier	2nd pier	3rd pier	4th pier	5th pier	6thpier
equation	1.800	2.849	3.003	3.004	2.790	1.789
simulation	1.788	2.827	2.978	2.980	2.766	1.769
deviation	0.65%	0.77%	0.83%	0.81%	0.89%	1.09%

Table 4. Moment comparison result 0f pier bottom

Moment (×10	1st pier	2nd pier	3rd pier	4th pier	5th pier	6thpier

MN·m)						
equation	2.250	2.851	3.175	3.168	2.844	2.320
simulation	1.704	2.399	2.492	2.491	2.475	1.766
deviation	0.64%	0.74%	0.72%	0.78%	0.83%	0.98%

By comparing the results of numerical calculation and theoretical calculations, it was found that the visible maximum error displacement appears in the last pie with the value of 1.21%. The error between transfer numerical matrix algorithm and ANSYS numerical calculation is mainly attributed to meshing situations and the different spring simulation at the bottom. But view from the whole aspect, the error is acceptable in engineering range. The transfer matrix method can not only avoid the complex process of modeling long multi span bridges, but also can save more time. It has more advantages over numerical calculation.

Summary

This paper mainly deduces the longitudinal seismic response of long multi-span simply supported girder bridge with transfer matrix algorithm. Through the comparison results with the calculation of numerical simulation, it not only shows that the calculation method proposed in this paper matches well with the results of numerical calculation, but also satisfies the actual engineering precision requirement. The principle of calculation method in this paper is simple and clear, moreover, it can avoid the tedious process of building models for the long multi-span bridge seismic force with the numerical software, at the same time, in post processing, it has great advantage in computing time, can also provide theoretical calculation basis for numerical results.

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