Modeling and analysis of automatic sorting system of logistics with finite capacity

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Abstract
The order processing strategy of automatic sorting system is studied for the finite capacity, which is not covered in present literatures. First the lengths of order working area are discredited and grouped according to the lengths; then the Markov chain model of the system is built to solve the problem about order sorting in the decision-making process. The decision-making variables are the next order groups need to be processed, and the decision-making variables are restricted by the frequencies of different order groups. After the parameters of automatic sorting system are determined, the analytical solution of the optimal ordering strategy can be calculated and then the utilization rate of sorting system can be found out. Besides, the error upper bound is analyzed because of discretization of the order working area lengths. The processing capacity of automatic sorting system can be analyzed based on the above model, and the optimal system parameters can be selected according to the cost of the sorting system.
Key words: AUTOMATIC SORTING SYSTEM, ORDER SEQUENCING, LINEAR PROGRAMMING

Introduction

In recent years, due to the rapid changes of the commodity markets and demands of manufacturing companies for "Just in Time", orders have been characterized by multiple species and small batch. Job of distribution center also has

been characterized by frequent ordering, shorter delivery period, higher precision, which put forward higher requirements for sorting system. At present, widely used sorting technique is RFID (Radio Frequency Identification) sorting by hand. The sorting technique can greatly reduce error rate, calculate optimum route by the information system, decrease labor intensity by indication signs in storehouse and improve work efficiency. Manual sorting can process multiple orders once time and walking routes can increase the work efficiency after studying on the batch of orders and optimized of picking members. The study focuses on storehouse layout, orders' characteristics and the location of goods. However, with the development of the circulation of commodities and commodity economy, the weaknesses of manual sorting are become visible such as inefficiency, high labor intensity and insoluble error. At the same time, due to the continuous improvement of logistics technology and management level, especially supply chain management, many people use the automatic sorting system to sort the goods which are characterized by multiple species, small batch, continual orders and regular pack, such as tobacco, CD, medicines, books and so on. Logistics automatic sorting system can be divided into two kinds of logistics automatic sorting system according to the ways of the warehouse entry, which are manual loading and automatic loading. The order and spacing between goods are not determined in manual warehouse entry sorting, but determined in automatic warehouse entry sorting.

The theoretical research on manual warehouse entry sorting system shows as follows. Johnson [1] studied the separation after picking (Accumulation-Sortation System), and pointed out that according to goods' arriving time order to choose the next receipt channel and concluded that this way improves the efficiency than last way by which each order enters its channel by manual. Meller [2] studied that according to goods' arriving time order to distribute exit of orders in real-time when receiving channel space is finite. The moving loops of goods on the conveyors will be reduced and the storing efficiency will be increased after adopting this way. Park, Jeong-Hyun [3] described the method for the carrier sorting plan and sorting rate improvement at the situation where the number of pocket of LSM is small. The sorting rate improvement effect more than 20% was showed by applying the proposed carrier sorting plan to two areas. There is a lot of research about automatic sorting system of automatic loading. Caputo [4] took the example of pharmaceutical industry and by using the simulation method studied automatic sorting system of goods selection, under the condition of multiple species, small batch and frequent ordering. He also studied the distribution of warehouse, replenishment points of warehouse and the number of the upper bound value of automatic sorting system. And then, according to the proposed decision principle he established the expert system which can automatically take corresponding control measures under the condition of order law has been changed. Kim [5] studied the automatic warehouse which is loaded by Gantry Robot. He brought forward a simple heuristic algorithm and partial algorithm method in dealing with the order process. Yani, I., Scavino, E. [6] described the prototype implementation of a real-time automatic identification and sorting system for recyclable beverage cans using an intelligent computer vision technique.

The optimization of channel, goods, disposition and other aspects to increase of efficiency of automatic sorting system are studied in the above research, but not considering the capacity of the receiving area of automatic sorting system which doesn't match the actual finite logistics automatic sorting system. The paper argues that the capacity and receiving abilities of the receiving areas automatic sorting system are finite: Because different orders of work zones have different lengths, it is necessary to enlarge receiving area capacity in order to make receiving area not cause sorting delay, but too big capacity of the receiving area can result in excessive capacity and low utilization efficiency of the receiving area. Therefore, the capacity of cago-receicing areas of the automatic sorting system is fully considered in the paper, and objective function is constructed aiming at the minimum delay time of goods in the receiving area. In the paper, a finite capacity automatic sorting system model has been constructed, and the best order processing strategy has been discussed. This research will be mainly used in the design stage of automatic sorting system, to study the effect of system parameters on system efficiency in the long-time running.

Model Description

The automatic sorting system studied in this paper focuses on the automatic sorting system with the funtion of automatically loading goods. In practice, the logistics automatic sorting systems are all different because of the difference of their service level and service objects, but the main

work processes of various sorting systems are the same. Therefore, this paper assumes that the quantity of the boxing channels of the sorting system is N, orders are sorted on the main conveyor belt and orders with the order of the first, the second, the third, …the Nth, the ($N+1$)th, the ($N+2$)th…, enters successively the boxing channels with the order of the first, the second, the third, …the Nth, the ($N+1$)th, the ($N+2$)th…. After all packing goods orders are completed, the next order should wait for the time T and then can enter its boxing channel. What can be done in time T is to seal, label, equip boxes and so on.

The optimal length of the respective pre-operation area (the length of the working area is the shortest) can be calculated in the automatic sorting system of automatic loading, so the length of the work area each order can be gotten. Time interval between adjacent work areas can be calculated in this system according to time T, i.e., the waitng time of the next order (W_i in Fig. 1). It is assumed that the speed of the main conveyor is 1, the length of the interval of adjacent work area is equal to the waiting time of the next order and the length of order value work area is equal to the length of the order processing time.

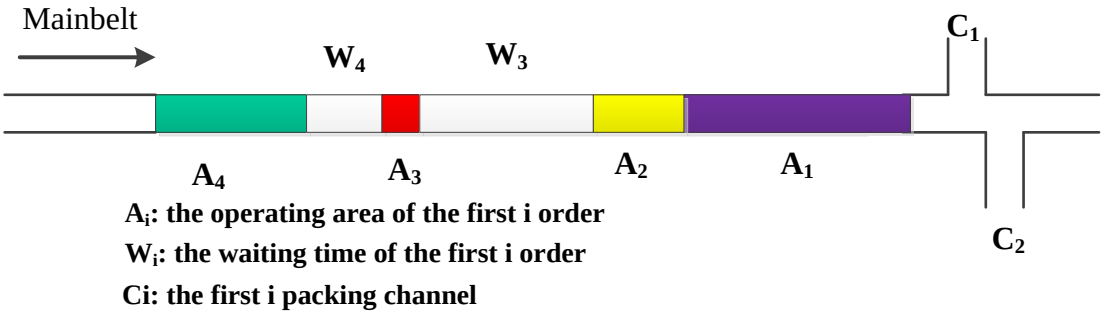

Figure 1. The automatic sorting system of automatic loading and finite capacity

The order processing time distribution can be gotten by the historical data. The order processing time is continuous, so it is difficult to be analyzed. All orders can be grouped by the type of order processing time, and it is assumed that the span of each group is D, the processing time t is in the range of 0 to T_m, and the proportion of the i th group order is P_{r_i}, then:

$$\Pr_i = \Pr(t \in [(i-0.5)D, (i+0.5)D]) \quad (1)$$

$$0 \le i \le N_o$$

$$N_o = \min\{n \mid (n+0.5)D \ge T_m, n = 1,2,3...\} 。$$

P_{r_i} is the proportion of the i th group order processing time in the total processing time.

N_0 is the processing time of the order.

In the above logistics automatic sorting system, processing time of orders is related to the logistics capacity of the system and related to the processing time of each order. The time needed of every order from entering the system to complement of its treatment includes: the processing time of this group and the previous one (i.e. the waiting time of this order). Thus, through the analysis of order processing time, a reasonable order processing strategy can be found to reduce waiting time and processing time of the orders.

Model Analysis and Solution

1. Model Building

1. Constraints Condition

The capacity of logistics storing system is finite. If it is assumed that the capacity of logistics automatic sorting system is $Q_{\text{gross capacity}}$, so the total capacity of the arriving orders will be no more than $Q_{\text{gross capacity}}$.

2. Influencing Factor

The total time of each order in the automatic sorting system includes processing time and waiting time of sorting and processing, and the waiting time is determined by the processing state of the precious order.

Therefore, in order to optimize the order processing strategy, the aim in the model is to analysis objective function which is to get the minimal value of the total time in the sorting system. Factors that affect the objective function are: automatic sorting order allocation situation (the target is to get the minimal waiting time for all orders in the system) and sorting efficiency.

3. Model Description

As an example to analyze the logistics operation of automatic sorting system all day long, there are M （$M \le Q_{\text{gross capacity}}$） orders to be sorted, then there are $M!$ kinds of arrangement, and heuristic algorithms is generally used. The optimal operating strategy of long-term operation of the system is studied in this paper to confirm capabilities and parameters of the system. In this

case[7], since the waiting time of this order only depends on the state of its previous $N-1$ orders, the system has history–independent. Orders can be considered to be added one by one. The system is observed when one order is going to add to the queue, the condition of the system can be denoted by S_i, and then:

$$S_i = (n_{i1}, n_{i2} ... n_{i,N-1})$$

n_{ij} is the sum of processing time and waiting time of the last $N-j$ orders.

The state space of the system is noted by $S = \{S_i\}$, in the worst case, the order processing time is T_m and the waiting time is T.

$$N_w = \min\{n \mid (n+0.5)D \geq T, n = 1,2,3...\}$$

Then, there are $(N_o + N_w)^{N-1}$ kinds of state in total.

In summary, the state of the automatic sorting system can be seen as a Markov chain. The system is studied by Markov decision process, and the decision variable is the next order when the system is on the state S_i. The next order will be processed in time a, and the order choose probability is $f(i,a)$. Then:

$$a \in A, A = \{0,1,2...N_o\}$$

When the system is in state S_i, the cost of the next order which will be processed in time a is $c(i,a)$, which means waiting time of the next one. Then:

$$c(i,a) = \begin{cases} 0, \sum_{k=1}^{N-1} n_{i,k} < T, a \geq n_{i,1} \\ n_{i,1} - a, \sum_{k=1}^{N-1} n_{i,k} < T, a < n_{i,1} \\ 0, \sum_{k=1}^{N-1} n_{i,k} \geq T, \sum_{k=2}^{N-1} n_{i,k} + a \geq T \\ T - (\sum_{k=2}^{N-1} n_{i,k} + a), \sum_{k=1}^{N-1} n_{i,k} \geq T, \sum_{k=2}^{N-1} n_{i,k} + a < T \end{cases} \quad (2)$$

When the system is in state S_i and the next state is j ,the probability of the next order which will be processed in time a is $p_{i,j}(a)$, then $p_{i,j}(a) = 1$ and S_j will satisfy:

$$\begin{aligned} & n_{j,1} = n_{i,2} \\ & n_{j,2} = n_{i,3} \\ & \ldots\ldots \\ & n_{j,N-2} = n_{i,N-1} \\ & n_{j,N-1} = \begin{cases} T - \sum_{k=1}^{N-1} n_{i,k} + a, \sum_{k=1}^{N-1} n_{i,k} < T \\ a, \sum_{k=1}^{N-1} n_{i,k} \geq T \end{cases} \end{aligned} \quad (3)$$

In other cases: $p_{i,j}(a) = 0$.

Smooth distribution of state i is π_i, note $x_{i,a} = \pi_i f(i,a)$.

Then the sytem can be descripted by the following linear programming model [8]:

$$\begin{aligned} & \min C = \sum_{i \in S} \sum_{a \in A} x_{i,a} c(i,a) \\ & s.t. \\ & \sum_{a \in A} x_{j,a} = \sum_{i \in S} \sum_{a \in A} x_{i,a} p_{i,j}(a), j \in S, \\ & \sum_{i \in S} \sum_{a \in A} x_{i,a} = 1, \\ & \sum_{i \in S} x_{i,a} = \Pr_a, \\ & x_{i,a} \geq 0, i \in S, a \in A \end{aligned} \quad (4)$$

The cost of system is C. Decision variable of the system is $x_{i,a}$, the quantity of decision variable is $N_o(N_o + N_w)^{N-1}$.

Linear programming software such as Lindo can be used to solve the problem in this linear system.

The average processing time of each order is $E(i) = \sum_{i=0}^{N_o} i \Pr_i$,

The average waiting time of each order is C, the number of orders which can be processed in the system is:

$$NUM = \frac{T_{wd} / D}{E(i) + C} \quad (5)$$

2. Deviation Bounds Analysis

There are definite deviations because discrete state is used to simulate continuous state. Therefore, deviation blinds need to analyzed. The time concerned in the deviation bounds analysis is continuous.

1. If the order processing sequence is not changed and the processing time of the i th order will increase $\Delta t_{p,i}$, then the processing time of order number $i+j$ $(j \geq 0)$ will change $\Delta t_{s,i+j}$.

$$0 \leq \Delta t_{s,i+j} \leq \Delta t_{p,i} \text{。} \quad (6)$$

If the processing time of the i th order increase $\Delta t_{p,i}$ the wait time of the i th order increases $\Delta t_{p,i}$ for the order after it, then $\Delta t_{w,i} = \Delta t_{p,i}$.

(1) When $j = 0$, $\Delta t_{s,i+j} = \Delta t_{w,i} = \Delta t_{p,i}$.

(2) When $j = 1$, the state of the j th order that will arrive at the storing system is $S_i^{'} = (l_{i-(N-2)}, l_{i-(N-1)} ... l_i)$, and all the variables are continuous. When $LS_i = \sum_{k=0}^{N-2} l_{i-k}$, the waiting time of the j th order $t_{w,i+1}$ contents with

$t_{w,i+1} = \begin{cases} 0, LS_i \geq T \\ T - LS_i, LS_i < T \end{cases}$. If the waiting time of the i th order increase $\Delta t_{p,i}$, then LS_i will increase $\Delta t_{w,i}$ and $\Delta LS_{i+j} = \Delta t_{w,i}$ ($j = 1$).Then $-\Delta LS_i \leq \Delta t_{w,i+1} \leq 0$ can be easily proved. Because $\Delta t_{s,i+1} = \Delta t_{w,i} + \Delta t_{w,i+1}$, then $0 \leq \Delta t_{s,i+1} \leq \Delta t_{p,i}$.

(3) When $1 \leq j \leq N-1$, $j = n(1 \leq n \leq N-2)$, $j = n+1$, $\Delta LS_{i+n+1} = \Delta t_{s,i+n}$, so $0 \leq \Delta LS_{i+n+1} \leq \Delta t_{p,i}$ can be gotten.

(4) When $j \geq N-1$ and $j \leq n(n \geq N-1)$, $j = n+1$, $\Delta LS_{i+n+1} = \Delta t_{s,i+n} - \Delta t_{s,i+n-(N-1)}$, $\Delta t_{s,i+n+1} = \Delta t_{s,i+n-(N-1)} + \Delta LS_{i+n+1} + \Delta t_{w,i+n+1}$. If $\Delta LS_{i+n+1} \geq 0$, then $-\Delta LS_{i+n+1} \leq \Delta t_{w,i+n+1} \leq 0$. Likewise, if $\Delta LS_{i+n+1} < 0$, then $\Delta LS_{i+n+1} \leq \Delta t_{w,i+n+1} \leq 0$.

2. If there are M orders need to be processed and the processing sequence is not changed, the i th order ($1 \leq i \leq M$) processing time variation is $\Delta t_{p,i}$, then all the processing time of the all orders changed by $\Delta t_{s,M}$.

$$\sum_{i=1}^{M} \min(0, \Delta t_{p,i}) \leq \Delta t_{s,M} \leq \sum_{i=1}^{M} \max(0, \Delta t_{p,i}) \quad (7)$$

If all the processing time changed like this, when the processing time changes form the first to the i th order, then the last one changed $\Delta t_{s,N_d}^{i}$, and $\min(0, \Delta t_{p,1}) \leq \Delta t_{s,M}^{1} \leq \max(0, \Delta t_{p,1})$. The processing time of the j th order will not influence the previous orders.

$$\min(0, \Delta t_{p,2}) \leq \Delta t_{s,M}^{2} - \Delta t_{s,M}^{1} \leq \max(0, \Delta t_{p,2})$$

$$\min(0, \Delta t_{p,3}) \leq \Delta t_{s,M}^{3} - \Delta t_{s,M}^{2} \leq \max(0, \Delta t_{p,3})$$

……

$$\min(0, \Delta t_{p,M}) \leq \Delta t_{s,M}^{M} - \Delta t_{s,M}^{M-1} \leq \max(0, \Delta t_{p,M})$$

Conclusion can be obtained by adding all the equation above.

If the real length (no discretization) of all the orders are known and the processing sequence is not changed, then based on the formula (1), formula (5) and formula (7), the bound of the processing capacity should be:

$$upper(NUM) = \frac{T_{wd} / D}{E(i) - 1/2 + C}$$

$$lower(NUM) = \frac{T_{wd} / D}{E(i) + 1/2 + C}。$$

Because C is smaller than $E(i)$, the error of the discrete length mainly depends on the span D of each group. If D is 1% of the average length of the order, then the error of system capacity calculation after the order discretization is only 0.5%, and the error is meet to the error requirement.

3. Conclusion of the Model

On the basis of the analysis of the model, the following conclusion can be gotten: the average processing time is $E(i) = \sum_{i=0}^{N_o} i \Pr_i$ in the finite capacity of logistics storing system. The average waiting time of each order is C. The average quantity of the order which can be handed in the system is $NUM = \frac{T_{wd} / D}{E(i) + C}$.

The decision model studied in this paper is more advanced than the existing ones mainly manifests for the third constraint condition. In all states, the two chances are equal. One is the chance of choosing next order whose processing time is a, another is the chance of producing order whose processing time is a. By error analysis, the results can be gotten: if the span of each group order is 1% of the average length of all the orders, the error of system capacity is 0.5% after discretization. So the model meets the requirements of error.

Experimental Analysis

The example selected is a logistics sorting system of a supermarket chain in the stowage center of one city, which is mainly used for sorting fruits and vegetables. At first, different fruits and vegetables are sorted to different freight cars, and then transported to each supermarket node. The sorting system has two boxing channels, orders line up on the main conveyor belt, the first, the second, the third…order enters into the first, the second, the first, the second …boxing channel respectively. The ratio of the first group orders is 10% and the second is 0.9. The waiting time of the order is two units time.

$N = 2$

$N_w = 2$

$\Pr_1 = 0.1$

$\Pr_2 = 0.9$

Then $S = \{(1), (2), (3)\}$,

The optimal strategy can be shown by the following matrix $f(i, a)$:

$$\begin{bmatrix} 1 & 0 \\ 0.053 & 0.947 \\ 0.053 & 0.947 \end{bmatrix}。$$

If the objective function value is 0.05, then the receiving capacity is shortage in 5% time. Because of the finite capacity of the logistics system, the goods line up and wait in the system.

The above model analysis results can be applicated as follows:

(1) Find the appropriate sorting model. By using the model, the processing capacity of the

logistics system can be analyzed and the appropriate automatic sorting system can be choosen early at the design stage;

(2) Study the optimal configuration of sorting system. For example, in the case of 3 channels with the objective function is 0, by thinking about increasing the cost of adding one more channel and taking advantage over the output of the system with 5% processing capacity, and then we can make a scientific decision.

In addition, in the orders automatic sorting system which runs continuously 24 hours every day and whose orders have a approximate uniform distribution, the optimal decisions in the long run only needs small revisal before application.

Compared with the practical application based on present theoretical research:

(1) Present automatic sorting system study requires a great deal of calculation and the analysis on replenishment model, while in this paper, the application effect of sorting system can be easily obtained with a simple software , and also achieve the goal of optimization of channels according to the aim of optimization.

(2) In present automatic sorting model study, there are many evaluations on the logistics automatic sorting model. But there is not analysis on how to design the system according to the calculation of the amount of orders.

(3) In the optimization of present automatic sorting system, by optimizing the configuration of logistics sorting system in most cases in order to improve the efficiency of system, and the generated optimization results are mostly the results of quantitative analysis. However, this paper makes quantitative analysis on the sorting efficiency according to the working capacity of the channel system.

Conclusion

The model of automatic sorting system with finite capacity, the optimal treatment strategy and the optimal processing ability are studied in this paper. Since the size of the involved linear programming problem of the model is $N_o(N_o+N_w)^{N-1}$. Therefore, under the condition of ensuring the accuracy, using the model under the condition of a small number of boxing channels in receiving areas can be suggested. And this model has outstanding theoretical and practical value. The model and the calculation process can be improved in future to achieve a great number of the receiving area boxing channel sorting system, and the applications can gradually be expanded.

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