

Hybrid Control Strategy of Vehicle Semi-active Suspension Based on the Genetic Algorithm Optimization

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Abstract

For unfavorable control effect due to difficult precise modeling of system and complex road conditions in the vehicle semi-active suspension control, this paper proposes a fuzzy PID hybrid control strategy based on generic algorithm optimization. This method codes the fuzzy control rule by using the floating number coding, uses the reciprocal of suspension performance evaluation function as the fitness function of genetic algorithm, and optimizes the fuzzy control rules by using the genetic operations such as probability selection, multi-point crossover and single-point mutation. The numerical computing and analysis indicates that the proposed control strategy can effectively improve performance indexes of body vertical acceleration, suspension dynamic deflection and tyre dynamic load. Compared to the passive suspension, PID-controlled and fuzzy PID-controlled semi-active suspension, the fuzzy PID controlled optimized by the generic algorithm can effectively improve vehicle ride comfort, handling stability and security.

Key words: VEHICLE SUSPENSION, SEMI-ACTIVE CONTROL, DYNAMIC FEATURE, FUZZY RULE

1. Introduction

The vehicle suspension is the general name of the force transmission device between the chassis and wheel axle, includes multiple forces, and determines the ride comfort, road handling stability and security of a vehicle [1]. The rigidity and dampness of a traditional passive suspension is not adjustable, so it can only ensure that the system control effect can be optimal under a specific operation condition. It is difficult to meet requirements of ride comfort and handling stability under different operation conditions. The active suspension features excellent performance, but its application in vehicles is restricted due to high energy consumption, complex structure and high cost. The semi-active suspension system features an actuator with lower power consumption, simple structure and low price, and can reach control effect like it of the active suspension [2], so it is extensively focused by the scholars. Many control strategies are proposed for it, e.g. sky-hook control, ground-hook control, optimal control, adaptive control, sliding mode control, PID control, fuzzy control and neural networks control [3-9]. For the complex non-linear vibration system such as vehicle suspension, generally expected control effect cannot be reached by using single control method due to difficult precise system modeling and complex road conditions, so the adaptive hybrid control strategy independent of precise modeling will be mainly studied in future.

The traditional PID control does not require understanding on the mathematic model of the controlled objects. The regulator parameters can be adjusted according to the experiences to get satisfactory results. This method features simple algorithm, better practicability and higher reliability. The weakness is that the set control parameters cannot be adjusted, so this is sensitive to parameter change of the controlled object and the system is not very robust. The fuzzy control does not depend on precise mathematic model, features fuzzy and uncertain processing, and is not sensitive to change of system parameters and environment, so the fuzzy PID hybrid control strategy of both control methods features flexible control, high precise and stronger self-adaptation, overcomes weakness of the PID control methods, and has better control effect [10], but the fuzzy logic control rule depends on knowledge and experiences of experts and is not easy to set. For the weaknesses of the above control methods, this paper proposes a fuzzy PID hybrid control strategy based on the generic algorithm (GA).

This paper designs a fuzzy PID controller for 1/4 vehicle of a sedan with 2 degrees of freedom, optimizes this fuzzy PID control rule by using the generic algorithm, and compares its control effect with it of a passive suspension, PID control and fuzzy PID-controlled semi-active suspension system, and validates control effect of the fuzzy PID hybrid control strategy optimized by the generic algorithm.

2. Modeling of vehicle semi-active suspension

To simplify dynamic modeling of vehicle semi-active suspension system, the following reasonable assumptions are provided [11]. 1) body and frame are rigid and are connected via rigid connector; 2) The vehicle structure is symmetric for the vertical plane and only vertical vibration is considered; 3) The vehicle suspension rigidity and tyre rigidity are the linear function of the shift and the suspension dampness is the linear function of the relative speed; 4) The tyre keeps contact with the road surface and does not jump. The simplified model of 1/4 vehicle semi-active suspension with 2 degrees of freedom is shown as the figure 1, m_1 is the mass of the vehicle body, m_2 is the mass of the tyre, k_1 is the rigidity of the suspension spring, C is the dampness coefficient of the suspension damper (including reference dampness C_0 and adjustable dampness C_r), k_2 is the tyre rigidity, x_1 is the body displacement, x_2 is the wheel displacement, and x_3 is the road disturbance.

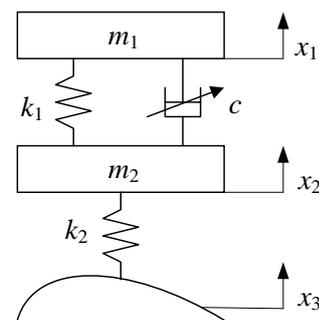


Figure 1. 1/4 vehicle semi-active Suspension model with 2 degrees of freedom

Based on Newton's second law of motion, the motion differential equation of 1/4 vehicle semi-active equation with 2 degrees of freedom can be expressed as follows:

$$\begin{cases} m_1 \ddot{x}_1 + k_1(x_1 - x_2) + C_0(\dot{x}_1 - \dot{x}_2) + f = 0 \\ m_2 \ddot{x}_2 + k_2(x_2 - x_3) - k_1(x_1 - x_2) - C_0(\dot{x}_1 - \dot{x}_2) - f = 0 \end{cases} \quad (1)$$

where f is the adjustable damping force and its value is: $f = C_r(\dot{x}_1 - \dot{x}_2)$.

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For easy computing, the state variant $X = [\dot{x}_1, \dot{x}_2, x_1, x_2, x_3]^T$ is selected. The kinetic equation (1) can be converted to the state equation:

$$\dot{X} = AX(t) + BU + Ew(t) \quad (2)$$

$$A = \begin{bmatrix} \frac{-C_0}{m_1} & \frac{C_0}{m_1} & \frac{-k_1}{m_1} & \frac{k_1}{m_1} & 0 \\ \frac{C_0}{m_2} & \frac{-C_0}{m_2} & \frac{k_1}{m_2} & \frac{-(k_1+k_2)}{m_2} & \frac{k_2}{m_2} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha v \end{bmatrix}$$

To assess control effect of the semi-active suspension, the body acceleration \ddot{x}_1 , suspension dynamic deflection x_1-x_2 and tyre dynamic load x_2-x_3 , which can indicate vehicle ride comfort, handling stability and security, are selected as the system output and the output variant Y is described as follows:

$$Y = [\ddot{x}_1, (x_1 - x_2), (x_2 - x_3)]^T$$

The output equation is:

$$Y = CX(t) + DU \quad (3)$$

where C is the output matrix and D is the feed-forward matrix.

$$C = \begin{bmatrix} \frac{-C_0}{m_1} & \frac{C_0}{m_1} & \frac{-K_1}{m_1} & \frac{K_1}{m_1} & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad D = \begin{bmatrix} \frac{1}{m_1} & 0 & 0 \end{bmatrix}^T$$

3. Control strategy of semi-active suspension

The fuzzy PID control strategy can keep the strengths of PID control such as simple and real-time parameter adjustment, feature strong adaptive fuzzy logic, realize automatic adjustment of three parameters of the PID controller, and improve the system robustness, but the fuzzy logic control rule for deciding the system control effect depends on knowledge and experiences of experts. To overcome weaknesses of this method and get optimal fuzzy control rule, this paper optimizes the fuzzy PID control rule by using the generic algorithm. The figure 2 shows the fuzzy PID control system optimized by the generic algorithm.

where A is the system matrix, B is the control input matrix, E is the disturbance output matrix, U is the adjustable damping force and $U=f$, and $w(t)$ is band-limited white noise.

$$B = \begin{bmatrix} \frac{1}{m_1} & -\frac{1}{m_2} & 0 & 0 & 0 \end{bmatrix}^T \quad E = (0 \ 0 \ 0 \ 0 \ 1)^T$$

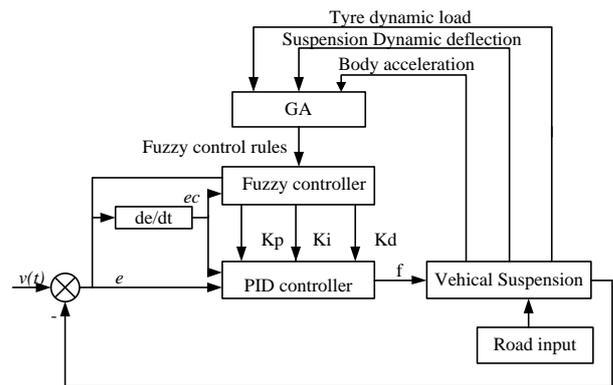


Figure 2. Fuzzy PID control system based on GA optimization

3.1 Design of fuzzy PID controller

This paper uses the difference e between the vertical speed of the body and reference (set as 0) and the vertical speed change of body (namely vertical acceleration ec of body) as the input variants of the fuzzy controller. The parameter K_p , K_i and K_d of the PID controller are the output of the fuzzy controller. The output of the fuzzy PID controller is the adjustable damping force f of the semi-active suspension system. The structural diagram of the fuzzy PID control algorithm is shown as the figure 3.

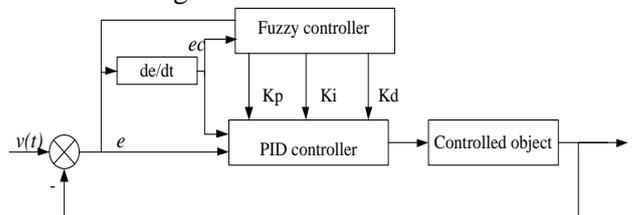


Figure 3. Fuzzy PID control theory

In practice, the vertical speed of the vehicle semi-active suspension system varies within $[-0.15, 0.15]$, so the basic domain of the input error e can be set as $[-0.15, 0.15]$. The quantification factor of the difference e is

$k_e=3/0.15=20$. The basic domain of the body's vertical acceleration is $[-3,3]$. If the fuzzy domain is set as $Ec=[-3,3]$, the quantification factor of difference change rate ec is $k_{ec}=3/3=1$. In design of the fuzzy controller, the input variant e and ec and output variant K_p , K_i and K_d are selected from 7 fuzzy subsets [NB, NM, NS, ZO, PS, PM, PB], namely negative big (NB), negative middle (NM), negative small (NS), zero (ZO), positive small (PS), positive middle (PM) and positive big (PB). The corresponding fuzzy domain of the input variants and output variants are $[-3, -2, -1, 0, 1, 2, 3]$ and the membership function of e , ec , K_p , K_i and K_d is the triangle membership function. The fuzzy domain of the input variants and output variants are $[-3, -2, -1, 0, 1, 2, 3]$ and the fuzzy control rule of K_p is shown as the table 1. The fuzzy control rule of K_i and K_d is similar to it of K_p .

Table1. Control rules of K_p

K_p		e						
		N	N	NS	ZO	PS	PM	PB
e c	NB	PB	PB	P	PM	PS	PS	ZO
	N	PB	PB	P	PM	PS	ZO	ZO
	NS	P	PM	P	PS	ZO	NS	N
	ZO	PS	PS	PS	ZO	NS	N	N
	PS	PS	PS	ZO	NS	NS	N	N
	PM	ZO	ZO	NS	N	N	N	NB
	PB	ZO	NS	NS	NS	N	NB	NB

3.2 Fuzzy control rule optimized by generic algorithm

The generic algorithm is used to optimize the fuzzy PID controller. Its fitness function need not be differentiable and its dependence on problems is small, so this generic algorithm is a global optimization method and can evaluate algorithm's effect according to the fitness function to avoid local optimization. By analyzing the fuzzy PID controller and vehicle semi-active suspension system, this paper optimizes the control rule of the fuzzy PID controller by using the generic algorithm. First the membership function of the input and output variant is specific. The rule number is identified according to the membership function and possible control rule tables listed. The fuzzy control rules are coded by using the floating number coding rule. Finally the fuzzy control rules are optimized via the generic algorithm. The steps for optimizing the control rules of the fuzzy PID controller by using the generic algorithm are described as follows:

1) The determination of objective function

The suspension system performance is measured by body acceleration \ddot{x}_1 , suspension dynamic deflection (x_1-x_2) and tyre dynamic load (x_2-x_3) . The values of three parameters should be reduced as much as possible, so the performance evaluation indicator function of the semi-active suspension system can be described as follows:

$$H = q_1h_1 + q_2h_2 + q_3h_3 \tag{4}$$

where h_1 , h_2 and h_3 indicate the root mean square (RMS) of the body vertical acceleration, suspension dynamic deflection and tyre dynamic load, respectively. q_1 , q_2 and q_3 are the weight coefficient. Suspension performance optimization in this paper aims to improve vehicle ride comfort, and consider vehicle handling stability and security, so the weight coefficient $q_1=0.5$, $q_2=0.3$ and $q_3=0.2$.

2) Coding of fuzzy control rule

The input variants of the fuzzy controller are e and ec and the output variants are K_p , K_i and K_d in the fuzzy PID-controlled vehicle semi-active suspension system. The fuzzy language set is selected from fuzzy subsets [NB, NM, NS, ZO, PS, PM, PB].

The fuzzy PID controller includes 3 control rule tables and 147 elements per table, which can be connected to a string of numbers in turn. A string of numbers indicates individuals. When the fuzzy control rule is coded, if the traditional binary coding method is used, too long individual code will lead to too large search space of the algorithm and too long search time. To improve efficiency, this paper uses the floating number coding and the number 1,2,3,4,5,6 and 7 are used to replace the language variant NB, NM, NS, ZO, PS, PM and PB in the fuzzy control rule table in the generic algorithm e.g. The K_p control rules in the table 1 can be coded as follows:

{7755554776654466654325554322554332244322214333211}

3) Generation of initial population

This paper generates the initial population randomly in an offline manner. The size of the population N is 20. Each population includes 147 individuals. The maximum evolutionary generation number of the population is regulated as 20. 20 populations are randomly generated. The first population group includes:

{634111542421647556771757446247566525155677657511746245153142122215245444146722453271112551766177776423117333451646341253717457334661231432275471664}

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4) Identify fitness function

The fitness function F is the reciprocal of the performance evaluation function, namely $F=1/H$. Bigger fitness indicates better performance of the identified control rule. After optimization ends, the code of the control rule with top fitness is obtained.

5) Selection, crossover and mutation operation

Selection operation indicates to select the individuals with the strongest vitality and generate new population. Individuals with bigger fitness will be inherited into the next population at a higher possibility. The selection probability is computed by the equation $P_i=F_i/\sum F_i$ in this paper. The selection probabilities of the initial population generated in this paper are described as follows:

$P_1=0.0515$ $P_2=0.0446$ $P_3=0.0505$
 $P_4=0.0461$ $P_5=0.0465$
 $P_6=0.0498$ $P_7=0.0457$ $P_8=0.0519$
 $P_9=0.0612$ $P_{10}=0.0441$
 $P_{11}=0.0590$ $P_{12}=0.0534$ $P_{13}=0.0504$
 $P_{14}=0.0497$ $P_{15}=0.0457$
 $P_{16}=0.0488$ $P_{17}=0.0534$ $P_{18}=0.0519$
 $P_{19}=0.0463$ $P_{20}=0.0496$

After the selection probability of each individual is identified, the uniform random numbers within $[0,1]$ will be generated to decide the mating individuals. The individuals with higher selection probability may be selected and their genetic gene will be expanded in the population. The individuals with smaller selection probability may be eliminated.

Crossover operation is also called as recombination, which will select two individuals with bigger probability from the population and exchange one or some bits of individuals to generate new individuals. Multi-point crossover method will be used in this paper. The crossover point 30, 60, 90 and 120 are selected in the first crossover and the crossover probability P_c is 0.6. Assuming that 1st group corresponds to 11th group, 2nd group corresponds to 12th group and 10th group corresponds to 20th group in the population. When crossover operation is implemented, the random numbers will be compared with the crossover probability. If the crossover conditions are met, the crossover operation is performed and the codes of the corresponding bits will be exchanged. Otherwise, crossover calculation is not performed. Finally the crossover points will reduce with growth of genetic generation number.

Mutation indicates to change one or some bits of corresponding individual code strings at a

smaller probability and generate new individuals. Some genes in individual code strings are replaced by other allelic genes in the genetic algorithm in order to generate new individuals. Single-point mutation is selected and the mutation probability P_m is 0.02 in this paper.

6) Optimization rule

When the fitness error $f_{max}-f_{min} \leq 0.01$, optimization will stop, optimal individuals are obtained, and computing stops. Otherwise, genetic operation 5) is repeated till maximal genetic generation number is reached. Finally check whether the performance requirements of the suspension system are met. If the requirements are met, the fuzzy control rule table can be identified and optimization is terminated. Otherwise, the corresponding genetic operations will continue.

3.3 Optimization results

After the parameters of the genetic algorithms are identified, simulation is performed by using MATLAB according to the initial values of the randomly generated fuzzy control rule table. After multiple iterations, the optimized K_p , K_i and K_d fuzzy control rule can be obtained finally. The fuzzy control rules of K_p are shown as the table 2. The fuzzy control rules of K_i and K_d are similar to those of K_p .

Table2. Optimized Control rules of K_p

K_p		e						
		N	N	NS	ZO	PS	P	PB
e c	NB	ZO	PB	P	PB	PS	NS	PB
	N	ZO	N	NS	PS	Z	P	PB
	NS	PB	ZO	PB	NB	N	NS	PS
	ZO	ZO	PB	ZO	ZO	Z	PS	NB
	PS	P	NB	ZO	N	Z	PS	N
	PM	PS	NS	PB	NB	N	PB	N
	PB	ZO	PB	P	PB	PS	NS	PB

4. Numerical examples and analysis

This paper mainly studies vehicles of certain type and the used suspension parameters are shown as the table 3. To effectively validate the designed fuzzy PID control strategy optimized by the genetic algorithm, simulation is performed for random road surface input and different vehicle speeds on different road surface levels and the results are compared with them of passive suspension, PID control and non-optimized fuzzy PID-controlled semi-active suspension for analysis.

Table3. Vehicle suspension parameters

Suspension	m_1/kg	m_2/kg	$k_1/(\text{N}\cdot\text{m}^{-1})$	$k_2/(\text{N}\cdot\text{m}^{-1})$	$C_0/(\text{N}\cdot\text{s}\cdot\text{m}^{-1})$
Values	487.685	49	45480	604690	2546.5

4.1 Random road surface input

The uneven road surface is the main factor to affect the dynamic features of the suspension system and vehicle vibration [12]. To ensure that the proposed method is extensively applicable, this paper introduces the filtered white noise random road surface input model, which time domain is expressed as the following mathematical equation:

$$\ddot{x}(t) = -\alpha v q(t) + \omega(t) \tag{5}$$

where $q(t)$ is the road surface random excitation on the wheel, v is the driving speed of vehicles, $\omega(t)$ is the band-limited white noises (The mean is 0 and the strength is 1), and α is a constant coefficient. When the road surface grades are different, α is different. α is 0.1303 for grade B road surface and 0.12 for grade C road surface.

The figure 4 shows random road surface input wave in the time domain when a vehicle is driving

at 50km/h speed on the grade B road surface. The figure 5 shows simulation results of body's vertical acceleration, suspension dynamic deflection and tyre dynamic load when random road surface input. The table 4 shows the comparison results of passive suspension, PID-controlled, non-optimized and optimized fuzzy PID-controlled semi-active suspension.

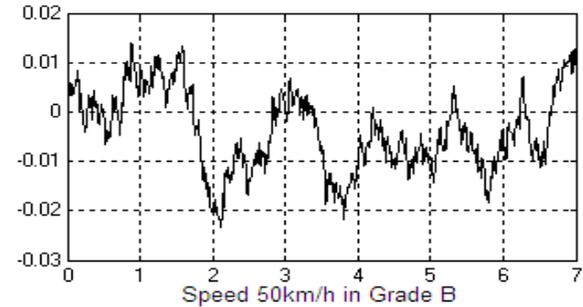
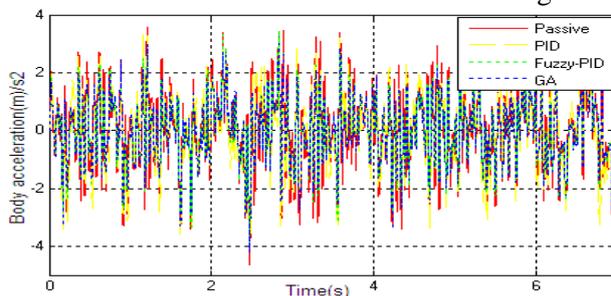
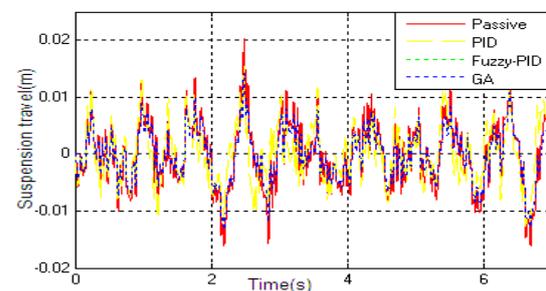


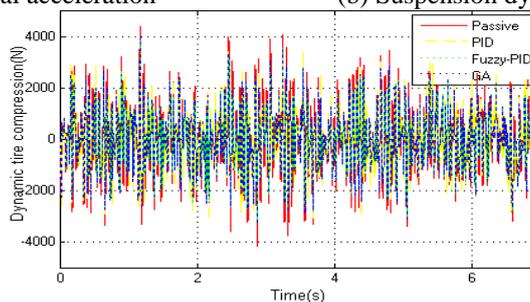
Figure 4. Random road input wave in time domain



(a) Body vertical acceleration



(b) Suspension dynamic deflection



(c) Tyre dynamic load

Figure 5. Simulation results when random road input

Table 4. Comparison of simulation results when random load input

Suspension Performance (RMS)	Passive suspension	PID-controlled suspension	Fuzzy PID-controlled suspension		performance improvement/%		
			Before optimizati	After optimizati	PID	Before optimizati	After optimizati

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Body acceleration/(m/s ²)	1.139	1.024	0.958	0.912	10.11	15.89	19.93
Suspension Dynamic deflection/mm	4.874	4.298	3.862	3.806	11.80	20.76	22.07
Tyre dynamic load/N	1156	1048	906.1	923.4	9.34	21.62	20.12

From figure 5 and table 4, when the vehicles drive through the grade B road surface at 50km/h speed, compared to a passive suspension, the RMS of the body's vertical accelerations of PID-controlled semi-active suspension system and fuzzy PID-controlled semi-active suspension before and after optimization decreases by 10.11%, 15.86% and 19.97% respectively. The suspension dynamic deflection decreases by 11.80%, 20.76% and 22.07% respectively. The tyre dynamic load decreases by 9.34%, 21.62% and 20.12% respectively. The numerical analysis results indicate that the ride comfort, handling stability and security of the fuzzy PID-controlled semi-active suspension vehicle optimized by GA is improved much, which validates effectiveness and excellence of the designed fuzzy PID controller improved by GA. For fuzzy PID-controlled semi-active suspension, the improved performance of the tyre dynamic load reduces from 21.62% before optimization to 20.12% after

optimization, but the improved performance of the body's vertical acceleration increases from 15.86% to 19.97%. This is because the weight value of the body's vertical acceleration in the target function is far bigger than it of the tyre's dynamic load by using the genetic algorithm optimization. Less decrease of tyre's dynamic load can improve the body's vertical acceleration much.

4.2 Simulation analysis on road surfaces of different grades under different vehicle speeds

Simulation is performed for the optimized fuzzy PID-controlled semi-active suspension system at low speed, medium speed and high speed (20km/h, 50km/h and 100km/h) on two road surfaces (grade B and C). The simulation results of the body's vertical acceleration, suspension dynamic deflection and tyre dynamic load are shown as the figure 6 and 7. The table 5 lists the RMS of the suspension system performance.

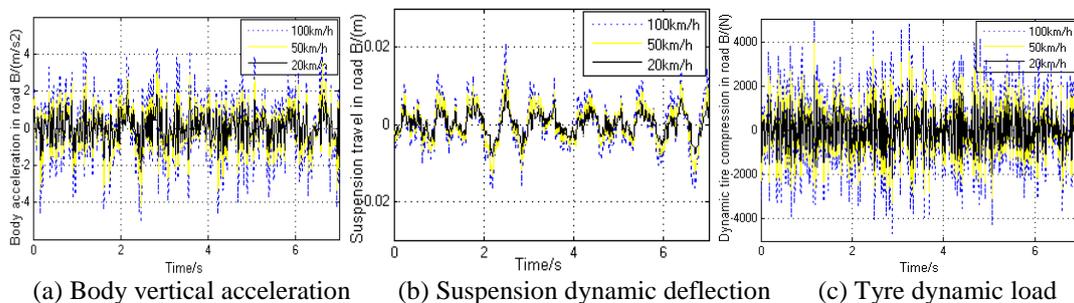


Figure 6. Simulation results of different speeds on the class B road

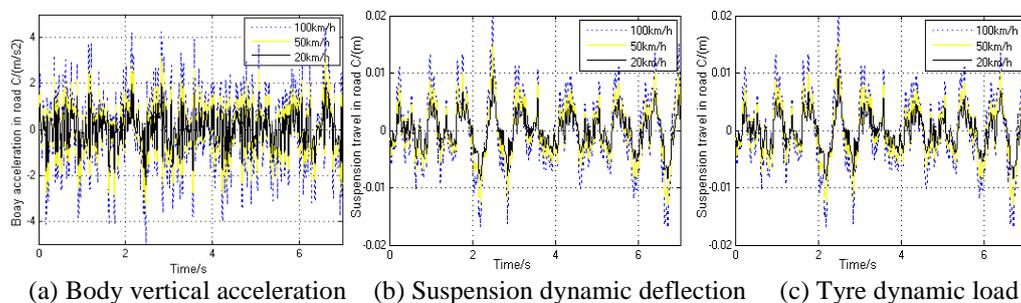


Figure 7. Simulation results of different speeds on the class C road

Table 5. Comparison of simulation results when different speeds on the different class road

Suspension Performance (RMS)	Grade B road surface			Grade C road surface		
	20km/h	50km/h	100km/h	20km/h	50km/h	100km/h
Body acceleration / (m/s^2)	0.583	0.912	1.318	0.588	0.915	1.322
Suspension dynamic deflection /mm	2.372	3.806	5.063	2.472	3.851	5.217
Tyre dynamic load /N	598.8	923.4	1318	601.2	924.1	1327

From the figure 6 and 7 and table 5, when the vehicle drives at the constant speed on the grade B road surface, if the speed increases from 20km/h to 50km/h and 100km/h, the RMS of the body's vertical acceleration of the suspension system will increase by 56.43% and 126.07% respectively, the RMS of the suspension dynamic deflection will increase by 60.46% and 113.44% respectively, and the corresponding tyre dynamic load will increase by 54.21% and 120.11% respectively. when the vehicle drives at the constant speed on the grade C road surface, if the speed increases from 20km/h to 50km/h and 100km/h, the body's vertical acceleration of the suspension system will increase by 55.61% and 124.83% respectively, the suspension dynamic deflection will increase by 55.78% and 111.04% respectively, and the corresponding tyre dynamic load will increase by 53.71% and 120.73% respectively, so the body's vertical acceleration, suspension dynamic deflection and tyre dynamic load of the suspension system will increase with growth of the vehicle speed on the grade B and C road surface, namely the suspension performance will deteriorate with growth of the vehicle speed on the same grade road surface. It indicates that the vehicle speed has remarkably influences on the performance of the suspension system.

From the table 5, when a vehicle is driving at the constant 20km/h on the grade B and C road surface, the RMS of the body's vertical acceleration, suspension dynamic deflection and tyre dynamic load of the suspension system will increase by 0.86%, 4.22% and 0.40%. When a vehicle is driving at constant 50km/h speed on the grade B and C road surface, the RMS of three performance indicators will increase by 0.33%, 1.18% and 0.08% respectively. When a vehicle is driving at constant 100km/h speed on the grade B and C road surface, the value of three performance indicators will increase by 0.33%, 1.18% and 0.08% respectively, so when a vehicle is driving

at same speed on the road surface of different grades, the body's vertical acceleration, suspension dynamic deflection and tyre dynamic load of the suspension system will change little, namely the change of the road surface grade will have smaller influence on vehicle ride comfort, handling stability and security.

5. Conclusions

This paper improves the dynamic performance of the suspension system via the designed fuzzy PID controller. To overcome the dependency of the traditional fuzzy PID controller on expert experiences, the genetic algorithm is used to optimize the control rules of the fuzzy PID controller. For the road surface random input, the comparison results of the passive suspension, PID-controlled semi-active suspension and fuzzy PID-controlled semi-active suspension before and after optimization validate excellence of the fuzzy PID controller optimized by the genetic algorithm. Simulation results for different road surfaces and vehicle speeds validate that the suspension system performance is influenced by the vehicle speed much and is affected by the road surface grade little.

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