Distributed highly-accurate pid controller design for the systems with distributed constants

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Abstract
The possibility of expanded frequency-domain characteristics device use for the systems with distributed constants is shown. The developed method allows designing of distributed highly-accurate controller according to given oscillation index.
Key words: SYSTEM WITH DISTRIBUTED CONSTANTS, DISTRIBUTED HIGHLY-ACCURATE CONTROLLER, DESIGN, TRANSFER FUNCTION

Currently, the following solving directions of distributed systems controllers design problem are known:

1. The analytical design of optimal controllers considered in papers of T.K. Sirazetdinov, G.L. Degtyarev and others;
2. The frequency-domain method of analysis and design considered in papers of V.A. Besekerskiy, E.P. Popov, V.V. Solodovnikov, I.M. Pershin and others;
3. The controllers parametric design where distributed controller structure is given and its parameters are selected during experimental researches. The basic issues of controllers parametric design based on the structural theory are

considered in the papers of A.G. Butkovskiy, V.L. Rozhanskiy, I.M. Begimov.

Let us consider the second direction in more detail. Currently, the distributed controllers design frequency-domain methods are based on the use of ordinary frequency-domain characteristics [3, 4, 6], at that, as a rule, constraints on gain and phase stability margin are imposed. In this paper, we presented the method of calculation of distributed highly-accurate controller adjustment using the expanded frequency-domain characteristics of control object, at that, constraints on transient process oscillation rate ***m*** are imposed.

Statement of the problem

The PID controller must be synthesized for the distributed object control system (its mathematical model is given or there is opportunity to conduct experimental researches of real object). At that, the following constraints are imposed on the system stability margin:

- gain margin $\Delta L \geq Lgiv$;
- phase margin $\Delta\varphi \geq \Delta\varphi giv$;
- oscillation rate $m = m_{giv}$.

The designed control system structural diagram is shown in Figure 1.

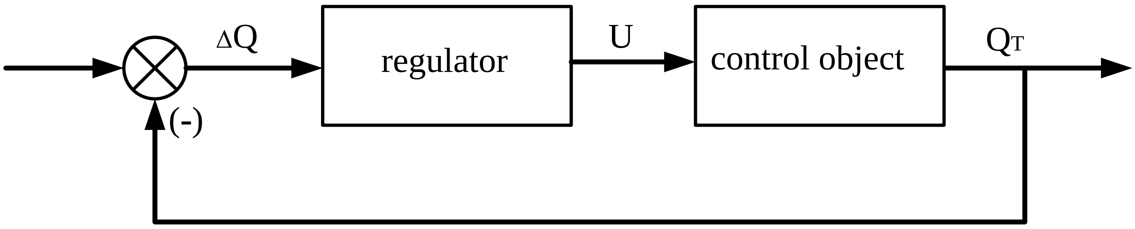

Figure 1. The system structural diagram

The distributed highly-accurate controllers (DHAC) are widely used in different technical processes control systems. In the general case, this controller transfer function consists of distributed spatial-amplifying, ideal spatial-integrating and ideal spatial-differentiating elements.

The designed controller transfer function according to [3] is of the form:

$$\boldsymbol{W(x,y,s) = E_1 \cdot \left[\frac{n_1 - 1}{n_1} - \frac{1}{n_1} \cdot \nabla^2\right] + E_4 \cdot \left[\frac{n_4 - 1}{n_4} - \frac{1}{n_4} \cdot \nabla^2\right] \cdot \frac{1}{s} + E_2 \cdot \left[\frac{n_2 - 1}{n_2} - \frac{1}{n_2} \cdot \nabla^2\right] \cdot s}$$

$$\boldsymbol{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}}$$

where *E1* , *E2* , *E4* – controller gain, *n1* , *n2* , *n4* - weight coefficients, ∇ - laplacian, s - Laplasian operator.

Analysis of the control object

The control object is given in Figure 2. A mathematical model is of the form:

$$\frac{\partial Q(x,y,z,t)}{\partial t} = a \cdot \left(\frac{\partial^2 Q(x,y,z,\tau)}{\partial x^2} + \frac{\partial^2 Q(x,y,z,\tau)}{\partial y^2} + \frac{\partial^2 Q(x,y,z,\tau)}{\partial z^2}\right),$$

$$0 < x < L_x, \ \ 0 < y < L_y, \ \ 0 < z < L_z. \qquad (1)$$

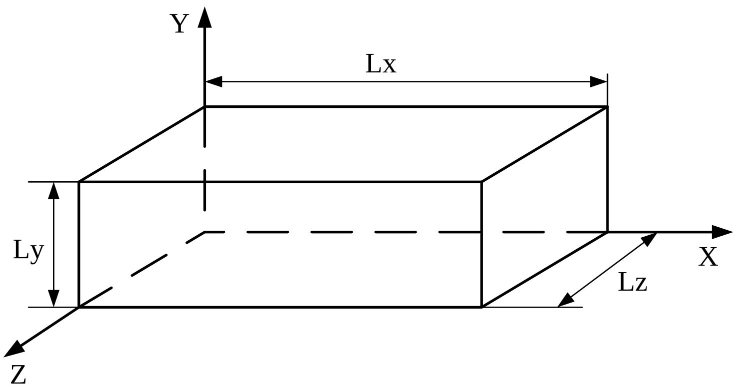

Figure 2. The control object

The boundary and initial conditions are given by the following equations:

$$Q(x,0,z,\tau) = Q(x, L_y, z, \tau) = 0 \quad (2)$$

$$\frac{\partial Q(0,y,z,\tau)}{\partial x} = \frac{\partial Q(L_x,y,z,\tau)}{\partial x} = 0 \quad (3)$$

$$\lambda \cdot \frac{\partial Q(x,y,z,\tau)}{\partial z} = U(x,y,\tau) \quad (4)$$

$$\frac{\partial Q(x,y,0,\tau)}{\partial z} = 0 \quad (5)$$

$$Q(x,y,z,0) = 0 \quad (6)$$

The object mathematical model is described by differential equations system, particularly derived ones. The transfer functions of such control objects may be presented in the form transfer functions set for each spatial mode [4, 5]. It is known [1] that for thermal control objects, the transfer functions for each input action mode may be approximated by transfer functions of the form:

$$W_\eta(s) = \frac{K_\eta}{T_\eta s+1} \cdot e^{-\tau_\eta s}, \eta = 1,2 \dots. \quad (7)$$

The parametersK_η, T_η, τ_η values are determined according to numerical modeling results.

For this purpose, let us provide the input action of the following form to the system input:

$$\alpha(x,y,s) = C_{\eta,\gamma}(s)\sin(\psi_\eta, x)\sin(\overline{\psi_\gamma}, y),$$

where $C_{\eta,\gamma} = e^{j\omega\tau}$, $\psi_\eta = \frac{\pi\cdot\gamma}{x_l}$, $\overline{\psi_\gamma} = \frac{\pi\cdot\gamma}{r_l}$.

Let us determine the parameters K_η, T_η, τ_η values by the numerical modeling results. The calculated values are: $K_1 = 0{,}7$; $T_1 = 70$; $\tau_1 = 6$; $K_3 = 0{,}5$; $T_3 = 67$; $\tau_3 = 6$.

When this parameters determining for1, 3, the dynamic properties of the thermal field distribution process within the object were considered. For the object frequency-domain analysis, let us suppose that

$$s = \omega\,(j - m)$$

where ω - circular frequency, m - oscillation rate. Let as determine the module $M_\eta$and phase φ_η[1, 2]:

$$M_\eta(m,\omega) = \frac{K_\eta}{\sqrt{(1-T_\eta m\omega)^2 + T_\eta^2\omega^2}} \cdot e^{m\tau\omega}, \quad (8)$$

$$\varphi_\eta(m,\omega) = -\omega\tau_\eta - \frac{\pi}{2} - arctg\left(\frac{T_\eta m\omega - 1}{T_\eta\omega}\right).$$

The design process

Let us suppose that phase deviation, which is caused by controller, is zero. Then we obtain the following equation for determining of the open-circuit system module cutoff frequencies:

$$-\pi + \Delta\varphi_\eta = W(m, j\omega, \eta, \gamma),$$

$$-\pi + \Delta\varphi_\eta = -\omega\tau_\eta - \frac{\pi}{2} - arctg\left(\frac{T_\eta m\omega - 1}{T_\eta\omega}\right). \quad (9)$$

where $W(m, j\omega, \eta, \gamma)$ – complex transfer coefficient of control object determined by numerical method.

Substituting the values T_η, τ_η and $\Delta\varphi_\eta = \frac{\pi}{6}$ into (9), let us determine the module cutoff frequencies value: $\eta = 1$, $\gamma = 1$, $\omega_1 = 0{,}15347$; $\eta = 3$, $\gamma = 3$, $\omega_3 = 0{,}15408$.

The controller parameters will be determined on the basis of the conditions that frequencies value ω_η belongs to the knuckle line. For the frequencies ω_η, which belongs to the knuckle line, the phase deviation, which is caused by controller, is zero.

Substituting $\omega = \omega_1$, $\omega = \omega_3$ into equation

$$lg\omega = 0.5[lgK_4(G) - lgK_2(G)],$$

we obtain the combined equation:

$$lg\omega_1 = 0.5lg\left(\boldsymbol{E_4} \cdot \left[\frac{\boldsymbol{n_4 - 1}}{\boldsymbol{n_4}} - \frac{\boldsymbol{G_1}}{\boldsymbol{n_4}}\right]\right) - 0.5lg\left(\boldsymbol{E_2} \cdot \left[\frac{\boldsymbol{n_2 - 1}}{\boldsymbol{n_2}} - \frac{\boldsymbol{G_1}}{\boldsymbol{n_2}}\right]\right), \quad (10)$$

$$lg\omega_3 = 0.5lg\left(\boldsymbol{E_4} \cdot \left[\frac{\boldsymbol{n_4 - 1}}{\boldsymbol{n_4}} - \frac{\boldsymbol{G_3}}{\boldsymbol{n_4}}\right]\right) - 0.5lg\left(\boldsymbol{E_2} \cdot \left[\frac{\boldsymbol{n_2 - 1}}{\boldsymbol{n_2}} - \frac{\boldsymbol{G_3}}{\boldsymbol{n_2}}\right]\right), \quad (11)$$

Subtracting (10) from (11), we obtain the following result:

$$lg\Delta\omega^2 = lg\left(\frac{\boldsymbol{n_4 - 1 + G_3}}{\boldsymbol{n_4 - 1 + G_1}}\right) - lg\left(\frac{\boldsymbol{n_2 - 1 + G_3}}{\boldsymbol{n_2 - 1 + G_1}}\right), \quad (12)$$

Using (12), let us determine the values n_2, n_4. As $\Delta\omega^2 > 1$, let us suppose that $n_2 = \infty$ in (10). then n_4 is determined by equation:

$$n_4 = \frac{(\Delta\omega^2 - 1 + \boldsymbol{G_3} - \Delta\omega^2\boldsymbol{G_1})}{(\Delta\omega^2 - 1)}, \quad (13)$$

where $G_1 = \Psi_1 + \overline{\Psi_1} = \frac{\pi\times 1}{X_l} + \frac{\pi\times 1}{Y_L}$, $G_3 = \Psi_3 + \overline{\Psi_3} = \frac{\pi\times 3}{X_l} + \frac{\pi\times 3}{Y_L}$

Considering the simultaneous equation

$$lg\omega_1 = 0.5lg\left(\boldsymbol{E_4} \cdot \left[\frac{\boldsymbol{n_4 - 1}}{\boldsymbol{n_4}} - \frac{\boldsymbol{G_1}}{\boldsymbol{n_4}}\right]\right) - 0.5lg\left(\boldsymbol{E_2} \cdot \left[\frac{\boldsymbol{n_2 - 1}}{\boldsymbol{n_2}} - \frac{\boldsymbol{G_1}}{\boldsymbol{n_2}}\right]\right)$$

and equations

$$lg\omega_1 = \lg\left(\frac{1}{K_2(G_1)}\right),\ lg\omega_2 = \lg\left(K_4(G_1)\right)$$

we obtain E_4 and E_2.

Let us introduce the parameter *Δ(G1)*, which value selection is associated with the ability of control object parametric disturbance compensation:

$$\lg\left(\omega(G_1)\right) = 0.5[lgK_4(G_1) - lgK_2(G_1)] \quad (14)$$

For this purpose let us add to the equation (14) the equation connecting parameters $K_4(G_1)$ and $K_2(G_1)$ with parameter $\Delta(G_1)$. The equation of connection may be presented in the form:

$$\Delta(G_1) = lg\omega_1 - lg\omega_2$$

$$lg\omega_1 = \lg\left(\frac{1}{K_2(G_1)}\right),\ lg\omega_1 = \lg\left(K_4(G_1)\right)$$

$$\Delta(G_1) = \lg\left(\frac{1}{K_2(G_1)}\right) - \lg\left(K_4(G_1)\right)$$

Substituting the values $\omega = \omega_1$, $\omega = \omega_3$ into equation:

$$M_\eta(m,\omega) = \frac{K_\eta}{\sqrt{(1 - T_\eta m\omega)^2 + {T_\eta}^2\omega^2}} \cdot e^{m\tau\omega},$$

Let us determine the module M_1, M_3 value.

As $\omega = \omega_1$, $\omega = \omega_3$ are the module cutoff frequencies of the open-circuit system, the controller gain values in these points are:

$$\overline{M_1} = (M_1)^{-1},\ \overline{M_3} = (M_3)^{-1}$$

$$\overline{M_1} = \boldsymbol{E_1} \cdot \left[\frac{\boldsymbol{n_1 - 1}}{\boldsymbol{n_1}} - \frac{\boldsymbol{G_1}}{\boldsymbol{n_1}}\right] \quad (15)$$

$$\overline{M_3} = \boldsymbol{E_1} \cdot \left[\frac{\boldsymbol{n_1 - 1}}{\boldsymbol{n_1}} - \frac{\boldsymbol{G_3}}{\boldsymbol{n_1}}\right] \quad (16)$$

Let us determine the parameters n_1, E_1. Dividing (15) to (16), we obtain the following results:

$$n_1 = \frac{\Delta M - 1 - \Delta M \cdot G_1 + G_3}{\Delta M - 1}$$

where $\Delta M = \frac{\overline{M_3}}{\overline{M_1}}$= *1,40771.*

Substituting the values n_1, $\overline{M_1}$, G_1 into (15), we obtain E_1.

Conducting the controller parameters calculations using the foregoing method, we obtain the following results: n_1 = *246,245;* n_4 = *15707,189;* E_1 = *5,55197;* E_2 = *3,664;* E_4 = *0,085964.*

Let us write the transfer function of distributed highly-accurate controller:

$$\boldsymbol{W(x,y,s)} = 5.99197 \times \left[\frac{246.245 - \boldsymbol{1}}{246.245} - \frac{\boldsymbol{1}}{246.245} \times \boldsymbol{\nabla^2}\right] + 0.085964 \times$$
$$\times \left[\frac{15707.189 - \boldsymbol{1}}{15707.189} - \frac{\boldsymbol{1}}{15707.189} \times \boldsymbol{\nabla^2}\right] \times s^{-1} + 3.664 \times \left[1 - 0 \cdot \boldsymbol{\nabla^2}\right] \times \boldsymbol{s}$$

After transformation we obtain

$$\boldsymbol{W(x,y,s)} = 5.99197 \times \left[0.995939 - 0.00406 \times \boldsymbol{\nabla^2}\right] + 0.085964 \times$$
$$\times \left[0.999936 - 0.000063 \times \boldsymbol{\nabla^2}\right] \times s^{-1} + 3.664 \times \left[1 - 0 \cdot \boldsymbol{\nabla^2}\right] \times \boldsymbol{s} \quad (17)$$

A special software has been developed for obtained data checking and closed-cycle control system modeling, which the equations (1-5) and (17) describe. The modeling results showed that the transient processes possess specified quality indicators.

Conclusions

The distributed highly-accurate controllers were calculated for different control objects with the help of developed method. The closed-cycle control system modeling results indicate the control advantage in full concordance with requirements applicable to the systems. From this we can conclude that the developed system allow adjustments calculating of distributed controller implementing a proportional-integral-derivative control law by the given value of oscillation rate and, consequently, the required degree of transient process attenuation.

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