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Quasistatic Distinctive Features Of Setting A Strain Wave Problems During Friction And Wear-Out

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Purpose. Development of engineering methods for calculating moving contacting parts for friction and wear, taking into account the quasi-static flow of deformation-wave processes in the zone in front of the moving part.

Methodology. To explain the effects of wave formation on contacting surfaces, a computational model is proposed in which the outer layer of interacting half-spaces is schematized in the form of two moving beams on an elastic base that have a finite length in the moving coordinate system and have hinge-fixed supports in the arrays of interacting parts. Differential equations of elastic lines of "hypothetical" beams are obtained. Practical methods for determining the number of half-waves and the height of microroughness appearing after the loss of longitudinal stability of "hypothetical" beams in the interaction of contacting parts have been developed.

Findings. Methods for determining the tangential forces in the stretched and compressed parts of the "hypothetical" beam are proposed. The results are presented to determine the degree of undulation of the deformation microroughness of a compressed beam after losing its longitudinal stability, which are largely due to the high-speed mode of interaction of the parts.

Originality of the work lies in the fact that the mechanism of formation of additional microroughnesses in details is revealed in their high-speed interaction. In addition, it has been established that the destruction of parts in friction is due primarily to the cyclic nature of the loading of the surface layers for all types of wear.

Practical value consists in achieving much greater reliability in performing calculations for friction and wear, taking into account the high-speed wave-like deformations of the surface layers (contacting parts) before and after losing their longitudinal stability. And also in more accurate prediction of the mechanism of destruction of surface layers of parts as a result of local dying of the material under the influence of tangential cyclic loads and the onset of the effect of fatigue dyeing of surface layers.

Key words: friction, wear, stretch wave, compression wave, elastic stability, compression wave crest, quasistatic problem, elastic beam, critical force, critical speed, dynamic coefficient.

Challenge problem. In the process of interaction of the two parts, performed as two half spaces with plane outer boundaries (envelopes), distortion residual waves of compression and rarefaction often occur on these planes, the height of these waves' crests is comparable to technological roughness, in certain cases [1] significantly exceeds them. The area of the parts' interaction continuously dislocates in the process of their movement. We can take two shallow beams (plates), stiffened by the elastic foundations, as design model for the interacting pair. Each of such beams, according to the nature of their interaction has two hinged immovable supports and stiffened by the elastic foundation. Length of each considered beams conditioned by length of section of the contact and the zone in front of the movable part (die block), in which the residual (or elastic) distortions are happening. (The second beam is not shown conventionally to simplify the considered model).

When the lesser part displaces in the interaction area (on the bigger part), the conventionally introduced beam starts actually moving along with hinged immovable supports, elastic foundation and the movable vertical and tangential loads. Such problem can be set and solved as quasistatic one. In other words, the static problem about loading of the moving beam on the elastic foundation along with vertical and tangential loads in the moving coordinates, which move together with the beam at a speed of the movable half space (die block) v

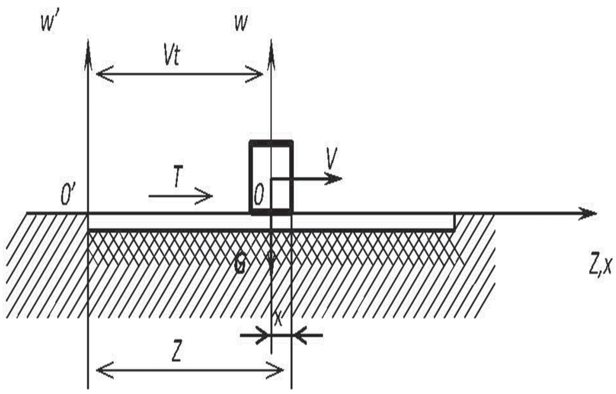

Fig.1. To assessment of the two moving parts interaction mechanism in the movable and immovable coordinates.

On the diagram (fig.1) the expression $z - vt = x$ is an abscissa of the current beam section, counted from the beginning of the movable coordinates (wox) in other words, together with the load, z is the abscissa of the current beam section concerning the immovable coordinates $\mid z \mid$).

Research technique. Statement of basic materials. For this loading diagram (fig.1) we can write the differential equation, which describes distortion wave processes, happening in the lower part while the upper part goes through it in form of the so-called stationary wave. The stationary wave can be determined as the outcome of the quasistatic equation solution which describes the distortion wave processes in the section in front of the die block (the section of the beam with a length l_2), upon that we still do not account the state of the "virtual" beam in the section under the die block (the section of the beam with a length l_1) in detail.

As our "virtual" beam is hinge supported in the end-points in the interaction zone with the elastic half space and does not have opportunity for the longitudinal displacements of the supports, then such beam, laying on the elastic half space, is a statically indeterminate beam (in the longitudinal direction) concerning the longitudinal support reactions. For this reason tangential force T, which is active in the contact zone, may be divided into force T_1, stretching the section of the beam l_1, force T_2, compressing the section of the beam l_2. Also the tangential force T is "nonstationary" friction force, which is active on the parts contact section, its form and nature can be detailed as appropriate [2].

It is significant that static indetermination of the "virtual" beam can be easily breached by means of generation of the longitudinal strains compatibility equation, in the result of opening of the static indetermination of the problem we will receive:

$$T_1 = \frac{T l_2}{(l_1 + l_2)}, \quad (1)$$

$$T_2 = \frac{T l_1}{(l_1 + l_2)}. \quad (2)$$

Further we will assume that "virtual" beam on the sections l_1, l_2 has a property of the elastic line continuity, for this reason prolongation of Δl_1 of the beam section with length of l will be equal to its shortening on the section l_2.

Then we can determine absolute prolongation of the beam on the section l_1 as follows:

$$\Delta l_1 = \frac{T_1 l_1}{E_C F} = \frac{T_1 l_1}{E_C h}, \quad (3)$$

where $F = h$ is a cross-section area of the beam of the singular width $b = 1$.

On the strength of taking assumption on continuity of the beam elastic line the length of the beam axis on l_1 will enlarge on Δl_1 in response to stretching train, and the length of the beam axis on the section l_2 will decrease to the same value due to strain of the section l_2 on sine curve with the number of half waves (n) and sine curve amplitudes A_n. Upon that the sine curve amplitudes (A_n) and number of the half waves (n) describe the strain microasperities after the "virtual" beam loses the longitudinal stability. It is significant that before the longitudinal stability loss the strain microasperities also occur, but they are elastic and after removal the loads disappear (in other words, they are reversible).

When we need to describe stressfully strained state of the "virtual" beam, we turn to the differential equation, which describes behavior of the beam on the elastic foundation under vertical and horizontal load (we will take into account vertical force during determining the dynamic factor).

$$\frac{d^4 w}{dx^4} + \frac{T_2}{E_C I}\frac{d^2 w}{dx^2} + \frac{C}{E_C I} w = 0 \quad (4)$$

Introducing the notation $\frac{T_2}{E_C I} = k^2$; $\frac{C}{E_C I} = r$ we will get

$$\frac{d^4 w}{dx^4} + k^2 \frac{d^2 w}{dx^2} + r w = 0 \quad (5)$$

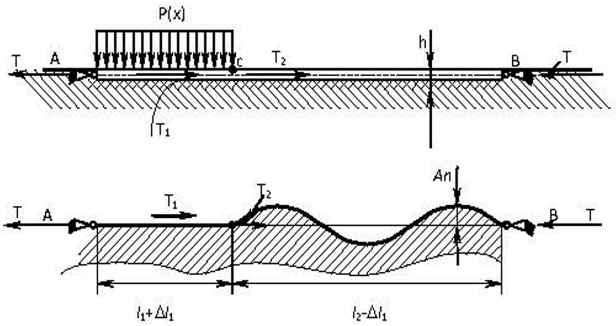

Fig. 2. The virtual beam on the elastic foundation concerning the moving coordinates

a) Mapping of two interacting parts in form of the die block and the beam on the elastic foundation (sections l_1, l_2)

б) Strains of the beam in front of the die block after it loses the elastic stability (section l_2)

The accepted notations

h is thickness of the beam;

A_n are the amplitudes of the strain microasperities;

l_1, l_2 are the lengths of the beam sections;

Dl_1 is the longitudinal strain of the section l_1;

$P(x)$, T, T_1, T_2 – applied load.

The notations in the introduced expressions are as follows:

W is deflection of the beam on the elastic foundation in the compression region;

x is the longitudinal coordinate of the moving / die block (load) concerning the moving coordinates ($x=z-vt$).

T is total tangential force, acting in the two body's contact region;

$C = \frac{E\alpha}{2}$ – rigidity of the elastic foundation () [3];

E_c is the strip beam longitudinal elasticity modulus, laying on the elastic foundation;

I is the strip beam inertia moment ($I = \frac{bh^3}{12}$

or for the beam-strip of the singular width $I = \frac{h^3}{12}$);

E – half space deep layers elasticity modulus;

$\alpha = \sqrt[3]{\frac{E}{4E_cI}}$ is the number of the beam half waves, which lost its stability on the section with the length π.

In order to determine the loads on the "virtual" beam in the section near the die block (T_2) and the number of the beam half waves, which lost stability, (n) we will use the following considerations:

the parameter of the beam $k^2 = \frac{T_2}{E_cI}$, according to [4], is associated with the number of the beam half waves, which were formed thanks to strain, in the following ratio

$$k^2 = \frac{\pi^2}{l^2}[n^2 + (n+1)^2], \qquad (6)$$

in which two unknown values are featuring – n and l_2. We will rewrite them (6) as

$$\frac{T_2}{E_cI} = \frac{\pi^2}{l_2(2n^2+2n+1)}. \qquad (7)$$

Thanks to the known value T_2 from the equation (7) we can determine the number of half waves in the region in front of the moving die block (n) or length of the strained "virtual" beam (l_2). However (n) and (l_2) are the unknown values. In order to adjust the differences we will use the following considerations. Length of the beam, which lost its stability or before losing one, is equal to formation of the number of the half waves strain of this beam on the length of one half wave

$$l_2 = n\lambda, \qquad (8)$$

where λ is the length of one such half wave.

So when we substitute (8) to (7), we will get

$$\frac{T_2}{E_cI} = \frac{\pi^2}{(n^2\lambda^2)(2n^2+2n+1)} \qquad (9)$$

$$\left(2 - \frac{T_2\lambda^2}{E_cI\pi^2}\right)n^2 + 2n + 1 = 0. \qquad (10)$$

As we consider the compression force T_2 to be arbitrary quantity, we will receive a quadratic equation to determine the number of half waves to determine the number of the half waves in front of the moving die block in form of

$$n^2 + \frac{2}{2 - \frac{T_2\lambda^2}{E_cI\pi^2}}n + \frac{1}{2 - \frac{T_2\lambda^2}{E_cI\pi^2}} = 0. \qquad (11)$$

We are looking for the solution of the equation (11) in form of

$$n_{1,2} = -\frac{1}{2 - \frac{T_2\lambda^2}{E_cI\pi^2}} \pm \sqrt{\frac{1}{\left(2 - \frac{T_2\lambda^2}{E_cI\pi^2}\right)^2} - \frac{1}{2 - \frac{T_2\lambda^2}{E_cI\pi^2}}}. \qquad (12)$$

As a solution (11) we take only real and positive value (12).

Setting a value T_2, we will receive the following: every meaning of the compression force corresponds to the only meaning of the value (n) and value (l_2). Now we're going to link the length of the beam half wave, which lost its stability (or before it lost one) λ with rigidity of the beam elastic foundation C, which depends on not only the medium elastic properties E, but also on the stability loss form, determined by the parameter α [3]. Upon that the values C, E and α are associated with the ratio $C = \frac{E\alpha}{2}$.

Now we turn our attention to the value of the coefficient α (The unknown number of the half waves on the section with length of π), which, according to [3], we find in the expression

$$\alpha = \sqrt[3]{\frac{E}{4E_c I}} . \quad (12)$$

Then we can determine the length of the rod half wave, which lost its stability, from the equality

$$\pi = \lambda \cdot \alpha .$$

$$\lambda = \frac{\pi}{\alpha} = \pi \sqrt[3]{\frac{4E_c I}{E}} . \quad (14)$$

As it follows from [3], the buckling force for the "virtual" beam on elastic foundation is determined from the expression, which is analog of the Euler formula

. (15)

We will substitute the meaning into quadratic equation and after transformation we will receive the equation to determine the number of he half waves, which occurred in the region in front of the die block from the effect of buckling force

$$n^2 - 2n - 1 = 0 . \quad (16)$$

By solving (16), we will receive

$$n_1 = 1 \pm \sqrt{1+1} = 2{,}41 . \quad (17)$$

Therefore, in the region in front of the die block while it moves, there is a moving wave front which is formed continually, and it consists of almost 2,5 half waves. In this case the length of the "virtual" beam in the region on front of the die block during its movement will be equal to

$$l = 2{,}41\pi \sqrt[3]{\frac{4E_c I}{E}} . \quad (18)$$

We are transforming the expression for the buckling force of the "virtual" beam (15) by virtue of relationship (14) to the form

. (19)

It follows from the expression (19), that the buckling force is directly proportional to thickness of the "virtual" beam h, to the beam elasticity modulus Ec in a power 1/3 and the half space deep layers elasticity modulus in a power 2/3.

It should be noted that the buckling force can be determined on the foundation [4], with taking into consideration the value of rigidity of the beam C in form

. (20)

If the real normal force T achieves its critical value, then the sinusoidal waves on the warped surface of the beam are remnant. If the compression force satisfies inequality

, (21)

then the strains will develop, but they will be elastic and after removal of the load the strain microasperities will just disappear.

For the beam of the strip with singular width (b=1) we can obtain the expression to determine he value of the buckling force, which depends on only the thickness of the diffusion layer (h) and the modules of longitudinal elasticity of the diffusion layer and internal layers of the part material. If (21) is not carried out and … , then strains of the beam are plastic (remnant) and the accepted elastic modulus (Hook's law) may give out unreliable results.

After we determined the length of the "virtual" beam compressed section (l_2) number of the half waves (n) on the compressed section, which lost its longitudinal stability, there is a question how we determine altitudes of the strain microasperities. According to what we planned earlier, on the first phase we are going to consider the case when the section of he beam under the die block (of length l_1) is located in the rarefaction zone, and the section of the beam (of length l_2) is in the compression zone. When we use the immovable supports of the beam in балки longitudinally, taking into consideration the static indetermination of the beam and continuity of the elastic line, the section of he beam (l_1) gets the elastic (plastic) extension Δl_1, the formula (3), and the section of the beam (l_2) will get the elastic (plastic) shortening Δl_1, in other words projection of the strained section of the beam (l_2) on longitudinal axis receives shortening on the value Δl_1, for this reason on the most elastic line there are some microasperities (corrugations), which are formed along with amplitudes A_n, which are, according to [4] change under effect of the law

$$W = \frac{A_n \sin \pi n}{l} , \quad (22)$$

where A_n is amplitude of (n)-harmonic of sinusoid or altitude of the strain microasperity;

n are the prime integral numbers, the meanings of which depend on the number of the half waves, formed on he camber line of the beam due to the fact that the beam lost its longitudinal stability (or right before it lost the stability;

l is length of he compressed section of the "virtual" beam (l=l_2).

When we need to determine the altitude of the strain microasperities of the beam, we start on the basis that during movement of the flat die block of the immovable half plane (half space), the intermediate bearer C of the "virtual" beam under effect of the partial tangential force T_1, performs some

work on displacement Δl_1. Now we are determining incrementation of the beam length l within the section (l_2). We are judging from the difference of the arc lengths of the camber beam and projection of the arc on the longitudinal axis (for the case of small strains). Now we can present this geometry in integral form

$$\Delta l_1 = \int_0^l [ds - dx = \int_0^{l_2} \left(dx \sqrt{1+\left(\frac{dw}{dx}\right)^2}] - dx \right) \approx \frac{1}{2}\int_0^{l_2} \left(\frac{dw}{dx}\right)^2 dx \cdot \quad (23)$$

In order to simplify we are going to use Newton binomial. We will use sinusoid (22) as form of stability loss on elastic foundation $W = \frac{A_n \sin \pi n}{l}$.

In this we are supposed to write the expression Δl_1

$$\Delta l_1 = \frac{1}{2}\int_0^{l_2} A_n^2 \frac{\pi^2 n^2}{l_2^2}\cos^2\frac{\pi n}{l_2}dx = A_n^2\frac{\pi^2 n^2}{l_2^2}\int_0^{l_2}\cos^2\frac{\pi n}{l_2}dx = A_n^2\frac{\pi^2 n^2}{l_2^2\left(\frac{l_2}{2}+\frac{l_2}{4\pi^2 n^2}\sin 2\pi n\right)}. \quad (24)$$

Then we have

$$\Delta l_1 = A_n^2\frac{\pi^2 n^2}{l_2^2\left(\frac{l_2}{2}+\frac{l_2}{4\pi^2 n^2}\sin 2\pi n\right)},$$

if n is a prime integral number целое, then $\sin 2\pi n = 0$, and then

$$\Delta l_1 = \frac{A_n^2}{2}\frac{\pi^2 n^2}{l_2^2}\cdot\frac{l_2}{2} = \frac{\pi^2 n^2}{4 l_2}A_n^2. \quad (25)$$

Therefore, from the expression (25), having the known Δl_1, we can find A_n.

Now we equate the expression (3) and (25)

$$\frac{T_1 l_1}{E_c h} = \frac{\pi^2 n^2}{4 l_2}A_n^2, \quad (26)$$

from which we find

$$A_r^2 - \quad (27)$$

or

$$A_n = \frac{2}{\pi n}\sqrt{\frac{T_1 l_1 l_2}{E_c h}}. \quad (28)$$

It follows that the altitude of the biggest, strain microasperity is directly proportional to the geometric mean of the section lengths l_1 and l_2 and directly proportional to the square root of relation of partial effort T_1 to thickness of the beam h.

There are cases when the condition of continuity of the "virtual" beam is not done, in other words when we implement the moving die block into some array, the beam discontinues in the zone on front of the die block (on the section l_2). Here as Δl_1 we take the value equal to the discontinuity length on the beam section (in the place where it contiguous to the half space), with taking into consideration stretching of the beam section l_1 under effect T_1. And as an effort, that compresses the beam we take the full effort T (or in the limiting case $T_{кр}$).

We write the performance standard of the beam-strip caused by the compressive load

, (29)

or along with taking into account the geometrical and physical specifications we will write as:

$$T < 2\sqrt{\frac{E_c E}{2} I \sqrt[3]{\frac{E}{4E_c I}}}. \quad (30)$$

The normal running of the interacting pair of the parts requires the following condition – the tangential force (full) should not exceed bucking force for the top layer of the half space. In order to determine the bulking force we need to conduct measurements of the half space hardness on different depth, the layers with extreme hardness describe the "virtual" beam, on the ground of it we can determined by experiment thickness of the "virtual" beam and calculate the inertia moment of cross-section view. After that we have to calculate by virtue of the known methods the elasticity modules for the "virtual" beam and inner layers of the half space.

In some cases when we determine the parts' performance, which are subject to wear-out, it is reasonable to determine direct stresses in the beam-strip, which should not exceed their critical value. If the effective stress is significantly less than the critical ones and do not exceed endurance limit of the "virtual" beam-strip material, then there are the following inequalities to be performed

(31)

and

(32)

The expression (31) for the beam-strip we will write with taking into account .

Then the expression (31) we will write as

$$F = \mathbf{h};\, I = \frac{h^3}{12} \quad (33)$$

The results considered above, were obtained during quasistatic setup, in which case the mutual displacement of the parts can happen at different linear speeds, which significantly influences the "virtual" beam deflection on elastic foundation caused by weight of the die block G. In the source [5] we presented expression of the "movable" beam deflection on elastic foundation, displaced concerning the half space at a speed v. To be definite we will go with the point in the balance center of the die block, during its movement (together with the movable beam) on the half-plane, in which case deflection of the beam under balance center of the die block looks like

$$y(0) = -\frac{G}{E_c I d\sqrt{2(d - av^2)}}, \quad (34)$$

where

$$d^2 = \frac{C}{E_c I}\quad a = \frac{m}{E_c I};$$

are the agreed notations of the system constants;

C is the modulus foundation (bed) for the beam

$$\left(C = \frac{E_c}{2}\sqrt[3]{\frac{E}{4E_c I}}\right);$$

m – mass of the beam length unit (wash);

v – speed of the die block displacement.

Along with taking into consideration the coefficient values d, a we write expression for deflection under load during movement

$$y(0) = -\frac{G}{E_cI\sqrt{\frac{C}{E_cI}}\sqrt{2\left(\sqrt{\frac{C}{E_cI}}-\frac{m}{2E_cI}v^2\right)}}. \quad (35)$$

Taking into account meaning of the elastic foundation rigidity for the beam we write the expression for deflection under load during its movement

$$y(0) = -\frac{G}{E_cI}\sqrt{\frac{\frac{E}{2}\sqrt[3]{\frac{E}{4E_cI}}}{E_cI}}\sqrt{2\left(\sqrt{\frac{\frac{E}{2}\sqrt[3]{\frac{E}{4E_cI}}}{E_cI}}-\frac{m}{E_cI}v^2\right)}. \quad (36)$$

If the speed of the die block displacement v=0, then deflection $y_{ст}(0)$ of the beam is called static, if v¹0 then the beam deflection will be dynamic.

$$\frac{y(0)}{y_{CT}(0)} = \mu$$

Relation of the beam deflection is called dynamic magnification factor, which is presented here to show that this dynamic magnification factor can be calculated by formula [5]

$$\mu = \sqrt{\frac{1}{1-\frac{mv^2}{2\sqrt{CE_cI}}}} \quad (37)$$

$$\mu = \sqrt{\frac{1}{1-\frac{mv^2}{2\sqrt{\frac{E_cEI}{2}\sqrt[3]{\frac{E}{4E_cI}}}}}}. \quad (38)$$

As we can see from the formulas (37), (38), along with die block displacement speed increase, dynamic magnification factor increases and when the speed gets the value $v=v_{кр}$, dynamic magnification factor achieves endlessly large meaning, for this purpose we determine the critical speed $v_{кр}$, on account of the denominator (37) from the expression

(39)

Or with taking into account the meaning of the rigidity foundation C we will have

. (40)

Insights and development potential of the area. In the result of the performed researches we come to the conclusions that during quasistatic problem setting the applied load (weight of the die block, friction force), stress and strain in the virtual beam, and also the strain microasperities depend on speed of mutual displacement of the parts (or the die block movement speed v).

For this reason first before calculation of the parts on friction and wear-out we need to calculate dynamic magnification factor μ and calculate applied loads G, T , taking into consideration the coefficient μ, and then to calculate the parameters of the beam l_2, n, A_n, and also the beam deflection $y_{CT}(0)$ under effect of the die block G (on the length l).

After the required calculations are performed, one of the three following controlling cases are happening:

1. Applied loads G, T are such that $T=T$кр and wash of the half space before the die block receives non-elastic strains and this layer loses longitudinal stability, for this purpose strains of the layer are irreversible. We can measure the crests of the received strain waves and frequency of their formation. After the die block goes again through on the strain projections their tops can be partially or completely cut – here is the case of ductile failure of the part material.

2. Applied load T does not achieve its critical value ($T<T_{кр}$) so the wash receives the elastic reversible strains, which disappear after effect of the loads G and T stops. Failure of the half space wash occurs only as a result of fatigue (more frequently fracture) failure of the half space outer layers. For this purpose the number of the loading cycles must be significant (for instance, 10^8 cycles and more).

3. Applied load T, during obtaining a qualitative lubrication in combination with antifriction materials is such that strains from the effect of longitudinal force T will be significantly smaller than the beam deflections under effect of weight of the die block G. For this reason the tangible strains (deflections) will be happening under effect of the force G. These strains look like progressive waves, which during the parts interaction are cyclical, and after the parts go through basic number of cycles, for example, 10^8 cycles and more, the outer layers fatigue failure is happening under effect of the die block G weight.

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