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Impact of parametric resonance on the contacting surfaces fracture mechanism during friction and wear-out.

A.D. Uchitel, Doctor of Engineering Science, Professor,
Yu. A. Malinovskyi, Ph.D. in Engineering Science, Associate Professor
(Ukraine's National Metallurgical Academy)

G.V. Danilina, Ph.D. in Engineering Science, Associate Professor
S.Yu. Datsenko, lecturer,
Ye.V. Omelchenko, lecturer, (Kryvyi Rih College of National Aviation University).

Abstract.
In this article the authors researched the parametric resonance impact in the moving contacting surfaces interaction region on friction and wear-out during distortion and wave processes course in front of the moving die block. The authors state that as the parametric vibrations occur on the contacting surfaces, there is a need to reason their occurrence, and for this reason there is a physical analogue offered, in which there is nonlinear unlubricated friction between the interacting parts, which depends on velocity of their slippage – the source of parametric (auto) vibrations on the segment in front of the movable part. For this reason they used the methods, allowing to determine the length of the part surface distorted segment, the number of half waves on the segment, and we applied the specified methods of distortion microasperities altitude computation as well. As a result the researchers found out that the reason of enhanced interacting parts wear-out is the rise of the parts diffusion layers amplitude vibrations due to parametric resonance effect, and this fact is the scientific novelty of this publication. The above allows to achieve greater reliability at determining the distortion microasperities using the exact expression for the bending line of the beam curvature, and also taking into account nonlinear dependence between longitudinal and lateral distortions of the virtual beam long axis due to the fact that it lost longitudinal stability.

Key words: friction, wear-out, elastic stability, the beam on elastic foundation, parametric load, Mathieu equation, beam curvature radius, Ains-Strett diagram, Bubnov-Galerkin method, Illustration 4. Reference: 7 titles.

Challenging problem. Research of the machine parts friction and wear-out issues, conducted by big research organizations showed that if the two contacting objects interact regardless the friction hypothesis accepted for consideration on the contacting surfaces the compression-rarefaction waves occur [1] (see Fig.1). In fact our task comes down to elastic contact problem with account for the nonlinear effects of friction (with falling characteristic concerning the velocity of mutual frictional sliding on the contacting surfaces).

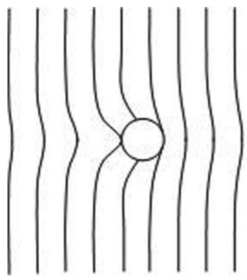

Fig. 1 Solid indenter interacts with elastic half-space: we can see compression stress in A-zone and refraction one on B-zone.

It is extremely challenging to carry out affixation and perform solution of such task in terms of elasticity theory if some characteristic properties on the contacting surfaces are available. For this reason in a first approximation in terms of qualitative consideration we will take a simpler loading schematic diagram which comes down to performance the diffusion layers of each of half-space in form of the beam on elastic foundation with hinged ends. Such beam is supported at more pliable bedding, than the diffusion layers of each half-space.

The proposed diagram enables us to visually reconstruct physical image of the process. By its length the beam is limited in one limiting point of the moving (die block) contact region and immobile half space, and in the other limiting point of attenuation sphere of elastic compression wives on length of the immobile half space (beam), Fig.2.

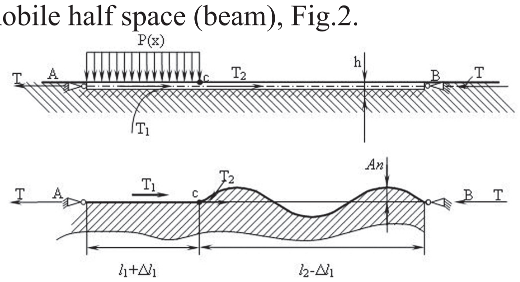

Fig.2 Virtual beam on elastic foundation on the elastic foundation in the moving coordinates

Agreed notations: h – beam thickness; An – distortion microasperities amplitude; l1, l2 – lengths of the beam segments; D l1 – longitudinal distortion of the segment l1, P(x), T, T0, T1, T2 – acting efforts

Further when we consider contact interaction of half spaces as loads on the beam we take contact stress in the region of the parts interaction (σ_x, τ_x), conditioned by normal loads and friction forces.

Our contact problem can be considered as two coupled problems, on in the area under the die block, and the other in the area in front of the die block. The first problem allowed to determine that during the two parts interaction there are intervals of their cooperative and relative motions, otherwise there is nonuniformity of motion, conditioned by availability of positive difference between motion friction force and one of static friction [2].

Research method. Statement of basic materials. Therefore, friction forces on contact of two parts are not constant, but depend on the rate of their relative slippage. On the basis of [2] friction force on contacts of two parts may be presented as:

$$T(t)=T(\dot{x})=T_0 sign(\dot{x})+b\dot{x}+\tilde{c}\dot{x}^3 \quad (1)$$

Where T_0 – conservative value of static friction force; b is negative damping coefficient; c is coefficient which describes damping proportional to velocity cube; x is a longitudinal coordinate at offsetting of the two parts.

The fact of force interaction of such nature of the parts is a prerequisite to occurrence of friction vibrations in the area under the die block, and the other in the area in front of the die block.

Upon that unsteady force $T(\dot{x})$ is external load for thin coatings of half space, being in the area on front of the die block. Taking into account the local character of diffusion layers external loading, we will give form to this half space segment as "movable" beam of fixed length on elastic foundation and on the movable supports, oaded with longitudinal transient lload $T(t)=T(\dot{x})$.

As friction force depends on relative velocity of the mutual slippage of the parts, then our virtual beam will be loaded with unsteady force $T(t)=T\left[\dot{x}(t)\right]$, which due to certain parameters ratio may lead to occurrence of dynamic and static unstable state of the beam. When we consider our movable beam concerning the moving coordinates, moving concerning long axis at a velocity V (velocity of the movable part movement), we will derive a differential equation, which describes behavior of the bending line of the beam on elastic foundation of the bending line of the beam, compressed by unsteady force $T(t)=T(\dot{x})$ and which has swivel joint as follows:

$$E_n J\frac{\partial^4 w}{\partial x^4}+T(t)\frac{\partial^2 w}{\partial x^2}+\alpha w+m_0\frac{\partial^2 w}{\partial t^2}=0 \quad (2)$$

where $T(t) = T(\dot{x})$ is unsteady friction force between interacting parts, attached to virtual beam in form of longitudinal load;

E_n is an elasticity modulus of diffusion (hardened) layer of half space (virtual beam);

J is inertia moment of the virtual beam;

$w(x\ t)$ is dynamic deflection of the virtual beam under nonsteady compression force $T(t)$;

c is rigidity factor of the beam elastic foundation (coefficient of subgrade reaction);

m_0 is mass unit of the beam length.

The differential equation (2) we consider together with the boundary conditions

$$w = \frac{\partial^2 w}{\partial x^2} = 0 : x = 0; x = l \quad (3)$$

And the initial condition

$$w(x_0, 0) = w_0(x_0)$$
$$\frac{\partial w(x_0, 0)}{\partial t} = V_0 \quad (4)$$

If the nature of friction force change (1) is close to harmonic one, then it is convenient to write the friction force, presented by the expression (1) as

$$T(t) = T_0 + T_3 \sin \omega_1 t \quad (5)$$

where T_0 is steady-state value of friction force (conservative value);

T_3 is peak value of friction force variable component, if friction characteristic can be presented as quasiharmonic time function (t);

ω_1 is first natural virtual beam vibrations frequency without taking into account the friction (we take that autovibration process is happening to the first natural virtual beam vibrations frequency, which is confirmed by numerous experiments with autovibration systems).

The differential equation of longitudinal-and-flexural vibrations of the beam on elastic foundation, presented here, are pertinently the differential equations, describing beam parametric vibrations under "unlubricated" friction force.

We will write the expressions for the biggest and the smallest friction force (by friction characteristic) as the function of resultant sliding load velocity p as

$$T_3 + T_0 = f_0 p \quad (6)$$
$$T_0 - T_3 = fp \quad (7)$$

In the presented expressions $T_3 + T_0$ – the biggest value of friction force (tangential force); $T_0 - T_3$ – the least value of friction force; f_0 is static friction coefficient during interaction of two parts; f – coefficient of slippage friction.

On account of the expressions (6), (7) we can determine T_3, T_0 at changing $T(t)$ under quasiharmonic or relaxational laws.

The solution of the equation (2) can be performed by Bubnov-Galerkin method [3] as:

$$w(x,t) = \sum_{k=1}^{\infty} f_k(t) \varphi_k(x) \quad (8)$$

In our case, accounting for the boundary conditions (3) we will write the solution (2) as

$$w(x,t) = \sum_{k=1}^{\infty} f_k(t) \sin \frac{\pi k}{e} \quad (9)$$

When we substitute the expression (9) in the differential equation (2), we will receive the set of the standard second-order differential equations concerning the function $f_{\mathrm{k}}(t)$, where $k = 1,2,3,...,\infty$.

$$f_k + \left[\frac{E_n J}{m_0} \left(\frac{\pi k^k}{l} \right)^4 - \frac{T_0}{m_0} \left(\frac{\pi k^k}{l} \right)^2 + \frac{T_3}{m_0} \left(\frac{\pi k^k}{l} \right)^2 \sin \omega t + \frac{c}{m_0} \right] f_k = 0 \quad (10)$$

where l is length virtual beam in the area in front of the movable die block, can be presented as $l = n\lambda$;

n – number of half waves virtual beam, which lost elastic stability;

λ – length of such one half wave.

Number of half waves of the virtual beam, which lost stability we find in accordance with [4], as a real-valued quadratic equation positive root

$$\mathrm{n}^2 + \frac{2}{2 - \frac{T\lambda^2}{E_n J \pi^2}} n + \frac{1}{2 - \frac{T\lambda^2}{E_n J \pi^2}} = 0 \quad (11)$$

Length of one half wave of the beam we find in accordance with [4] from the expression

$$\lambda = \pi \sqrt{\frac{4 E_n J}{E}} \quad (12)$$

It should be noted that the differential equations (10) do not comprise the system and are the equations with parametric vibrations excitation.

The expression in square brackets is a square of "floating" frequency of system vibrations for each value k.

The equations (10) presented here in number of cases describe "pseudo parametric" resonances during effecting the mechanical system, as they turn out to be more dangerous than systems with ordinary excitation near resonance effects. The differential equations (10) are pretty complicated, there is no accurate solution of theirs in the elementary functions, so that is why we can write some approximate solutions for them [5]. However, we can judge about conclusion on stable (or unstable) behavior of the equations set by appearance of the differential equations (10). For further analysis it is reasonable to reduce the equations (10) to certain standard type. Further we take $\omega_k t = 2\phi - \frac{\pi}{2}$, that is we make change of the variable, we will proceed to the equations:

$$f_k'' + [a_k + 2q_k \cos 2\tau] f_k = 0 \qquad (13)$$

The equations we obtained, are called the differential equations (with variable coefficients at f_k) of Mathieu, in which

$$a_k = \frac{4}{\omega_k^2}\left[\frac{E_n J}{m_0}\left(\frac{\pi k}{l}\right)^4 - \frac{T_0}{m_0}\left(\frac{\pi k}{l}\right)^2 + \frac{c}{m_0}\right];$$

$$2q_k = \frac{4}{\omega_k^2} + \frac{T_1}{m_0}\left(\frac{\pi k}{2}\right)^2.$$

The coefficients we brought are called Mathieu coefficients.

As we need to determine dynamic stability of the vibration system, described by the equations (10), (13) without recourse to solution of the equations, we will use Ains-Strett diagram applying them to the Mathieu equations. In order to do this in each of the equations (13) we determine parameters a_k, q_k for each value k. These values a_k, q_k we will plot along Ains-Strett diagram (Fig.3), if the point with coordinates c a_k, q_k appears to be in the crosshatched region of the diagram, then vibration regime is stable and upon that loss of half space vibrating diffusion layer dynamic stability will not happen, that is the virtual beam.

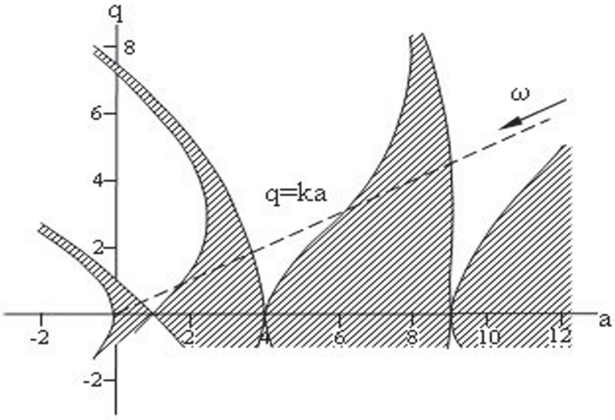

Fig. 3 Ains-Strett diagram. Dashed ray is image points geometric locus of this mechanical system; if the frequency w increases, the image point approaches to the origin of coordinates.

For this mechanical system it is highly likely that the parametric vibrations occur due to periodic changes (quasiharmonic or relaxational) of the parametric loads. Parametric ones are those types of loads, at static actions of which the virtual beam can potentially lose stability in Euler Point.

In order to determine the frequencies (variable, floating) and amplitudes of the vibrations it is reasonable to allow the set of second-order differential equations concerning the sought-for time functions $f_k(t)$. When we determine them, we put them into the trigonometric sequence (8). When we look at the equations (10) we can see that the set equations is nonlinear. As we need to make quantitative solutions of the equations (10) we will use nonlinear differential equations approximate solution method, Wentzel Kramers Brillouin method. In accordance with the method, we will present the sought-for equation as:

$$\ddot{f}_k(t) + \Omega_k^2(t) f_k(t) = 0 \qquad (14)$$

In the equations presented here (14) function $\Omega_k(t)$ then there is a point in natural frequency of the system "floating" near its average value

$$Щ_k(t) = \sqrt{\frac{E}{m_0}\left(\frac{\pi k}{l}\right)^4 - \frac{T_0}{m_0}\left(\frac{\pi k}{l}\right)^2 + T_3\left(\frac{\pi k}{l}\right)^2 \sin \omega_1 t + \frac{c}{m_0}} \qquad 15)$$

where ω_1 is one of natural vibrations frequencies of the beam on the elastic foundation, in case if there is no friction (the natural vibrations or точнее autovibrations may occur from one of natural system frequencies, for example, first or second one).

Generally the ratio $\frac{T_3}{T_0}$ at steady vibrations are no more than 10-15%. During parametric resonance vibrations are increasing and nonsteady.

Solution of each equation (15) according to Wentzel Kramers Brillouin method looks like

$$f(t) = \Omega_k^{1/2}(t)\left(c_{1k} \cos \int \Omega_k(t) dt + c_{2k} \sin \int \Omega_k(t) dt\right) \qquad (16)$$

The expression (16) can be accepted as the approximate one, provided that the following condition is carried out

$$\left| 0{,}5 \frac{\ddot{\Omega}_k(t)}{\Omega_k^3(t)} - 0{,}75 \left(\frac{\dot{\Omega}_k(t)}{\Omega_k^2(t)}\right)^2 \right| < 1 \qquad (17)$$

We determine the constants C_{1k}, C_{2k} from the initial conditions

$$f_k(0) = f_{k_0};\; f_k(0) = f'_{k_0} \qquad (18)$$

Then we will finally obtain

$$f_k(t) = \sqrt{\frac{Щ_{k_0}}{Щ_k}} f_{k_0} \cos \int_0^t \Omega_k(t) dt + \frac{1}{\Omega_k \Omega_{k_0}}\left(\dot{f}_{k_0} \frac{\dot{\Omega}_k}{\Omega_{k_0}} f_{k_0}\right) \sin \int_0^t \Omega_k(t) dt, \qquad (19)$$

Where $\Omega_{k_0} = \Omega_k(0)$; $\dot{\Omega}_{k_0} = \dot{\Omega}_k(0)$

In reality the most dangerous is the mode which corresponds to $k = 1$. The set of the equations which we described are certain equations with periodically changing frequency Ω_k. Such set possesses equilibrium, which may turn out to be unstable under certain circumstances.

We can point out the qualitative solution of the equations (10) as [5]

$$f_k(t) = C_{1k} e^{\mu t} \psi_{1k}(t) + C_{2k} e^{-\mu t} \psi_{2k}(t) \qquad (20)$$

The expression (20) is a complete solution of Mathieu equations where

$\mu = \mu(a_k, q_k)$ is a constant, which depends on the coefficients a_k, q_k,

$\psi_{1k}(t), \psi_{2k}(t)$ are some periodic time functions,

C_{1k}, C_{2k} are arbitrary constants of the desired solutions.

In our system of the movable die block and the virtual beam, the parametric vibrations may occur under impact of small perturbations. The first item (20) is vibration process with increasing amplitudes, that is why peak values of the solution (20) may reach considerable magnitudes. Such vibrations, caused by instability of the equilibrium state of the system, are called parametric resonance.

Parametric resonance is the most dangerous parametric phenomenon, it's external appearances make it look like ordinary resonance. However, in accordance with (20) the vibrational amplitude during parametric resonance increases according to exponential law, and during the ordinary resonance – by linear one. Parametric resonance can develop if there is friction available [5], which is typical for our task, upon that parametric resonance can occur in the cases when ratio of the natural system vibrations frequency to the equations coefficients alternating frequency half (10) is close to non-vanishing integer.

As the solution of the quasilinear problem (2), (3), (4) is necessary, we will use Bubnov-Galerkin method together with approximate method of nonlinear mechanics – Wentzel Kramers Brillouin method. Such solution gives consistent results in the instants of time which precede the stability loss by the virtual beam or the moments about origin during loss of stability. The mere fact of stable state of the system is determined, as it was noted before, by Ains-Strett diagram.

When the virtual beam loses stability it receives nonlinear (inelastic) wavy distortions. For this reason we accept half of distortions wave amplitude as distortion microasperity. When the beam loses stability it receives permanent distortions, which characterize the extra microasperities formed. In order to determine them it is sufficient for us to consider instead of dynamic equation (2) its static analog [6]. From the modified equation we can determine the parametric load.

$$\frac{d^4 w}{dx^4} + \frac{T}{E_n J}\frac{d^2 w}{dx^2} + \frac{C}{E_n J} w = 0 \qquad (21)$$

In this equation (21) the same notations were introduced, as in the equation (2) with the partial derivatives, only

$w = w(x)$ is static deflection of the virtual beam,

$T = T_0$ is stationary (parametric) value of the compression force.

Critical (parametric) load can be determined in accordance to [7] from the expression

$$T_0 = T_{cp} = 2\sqrt{E_n JC} = 2\sqrt{\frac{E_n}{2} J \sqrt[3]{\frac{E}{4E_n J}}} \qquad (22)$$

We consider the equation (21) with the boundary conditions (3). The solution of the equation (21) by Bubnov-Galerkin method we will write as

$$w(x) = \sum_{k=1}^{\infty} f_k \sin\frac{k\pi x}{l} \qquad (23)$$

Generally diffusion layers of the interacting parts at the intensive load are significantly flexible, the curvature of the virtual beam – $\frac{1}{\rho}$ we take as equal to $\frac{1}{\rho} = \frac{d^2 w}{dx^2}$, then a significant error slips into the calculations, so we use the precise meaning of curvature of the curve as

$$\frac{1}{\rho} = \frac{d^2 w}{dx^2}\left[1 - \left(\frac{dw}{dx}\right)^2\right]^{112} \qquad (24)$$

We will simplify it by Newton binomial (24)

$$\frac{1}{\rho} \approx \frac{d^2 w}{dx^2}\left[1 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 + \frac{3}{8}\left(\frac{dw}{dx}\right)^4 ...\right] \qquad (25)$$

When we determine the beam curvature, we will still use only two items in the expression (25).

In the expressions (24), (25) the linear coordinate x is replaced with the arc coordinate s.

In relation to the structure of the equation (21), describing the longitudinal deflection of the beam on elastic foundation instead the value $\frac{1}{\rho} = \frac{d^2 w}{dx^2}$ we consider the second derivative from the specified expression of the curvature (25). The specified expression of the beam curvature we will substitute to the first item (21) after its double differentiation.

Turning to $\frac{d^2 w}{ds^2}\left[1 + \frac{1}{2}\left(\frac{dw}{ds}\right)^2\right]$, (26)

We will carry out its double differentiation on the coordinate s.

$$\frac{d^4 w}{ds^4}\left[1 + \frac{1}{2}\left(\frac{dw}{ds}\right)^2\right] + 2\frac{d^3 w}{ds^3}\left[\frac{dw}{ds}\frac{d^2 w}{ds^2}\right] + \frac{d^2 w}{ds^2}\left[\left(\frac{d^2 w}{ds^2}\right)^2 + \frac{dw}{ds}\frac{d^3 w}{ds^3}\right] \qquad (27)$$

As we need to determine deflection bends of the beam in case of taking into consideration the precise meaning of the curvature we will carry out substitution $w = f\sin\frac{\pi s}{l}$ in the expressions (21) and (27). Without disturbing similarity of arguments in the expression for w we conventionally drop subscripts k.

After substitution of w into the expression (27) we will obtain (28)

$$f\frac{\pi^4}{l^4}\sin\frac{\pi s}{l}\left[1+\frac{1}{2}f^2\frac{\pi^2}{l^2}\cos^2\frac{\pi s}{l}\right]-\frac{\pi^3}{l^3}f\cos\frac{\pi s}{l}\left[f\frac{\pi}{l}\cos\frac{\pi s}{l}\frac{\pi^2}{l^2}f^2\sin^2\frac{\pi s}{l}\right]$$

$$-f\frac{\pi^3}{l^3}\cos\frac{\pi s}{l}\left[f\left(-\frac{\pi}{l}\right)\cos\frac{\pi s}{l}-\frac{\pi^2}{l^2}f^2\sin\frac{\pi s}{l}\right]-\frac{\pi^2}{l^2}f\sin\frac{\pi s}{l}\left[-\frac{\pi^2}{l^2}f\sin\frac{\pi s}{l}-\left(-f\cos\frac{\pi s}{l}\right)\left(\frac{\pi^3}{l^3}f\cos\frac{\pi s}{l}\right]\right.$$

When we substitute the function of the curvature (27) into the equation (21), and also the expression w into the rest of the differential equation items (21) and require the selected functional orthogonal property result with the function $\sin\frac{\pi s}{l}$.

As a result of the selected functional integration in the range from 0 to l we will obtain, in order to determine, the deflection bends of the virtual beams, which possess big flexibility (and nonlinear elasticity)

$$f\frac{\pi^4}{l^4}\frac{l}{2}+f^3\frac{\pi^6}{l^6}\frac{l}{8}-\frac{T}{EI}f\frac{\pi^2}{l^2}\frac{l}{2}+\frac{T}{EI}f\frac{l}{2}=0 \qquad (29)$$

Ignoring the trivial value $f=0$, we will obtain the expression to determine the deflection bends

$$f=+\sqrt{\frac{8l^2}{\pi^2}\left(\frac{T}{EI}\frac{l^2}{\pi^2}-\frac{T}{EI}\frac{l^4}{\pi^4}-1\right)} \qquad (30)$$

Alongside this there is an influence on development of the lateral deflection by shifting motion of the virtual beam exposed (movable) end and account for longitudinal rigidity of the beam E_nF (Fig.4).

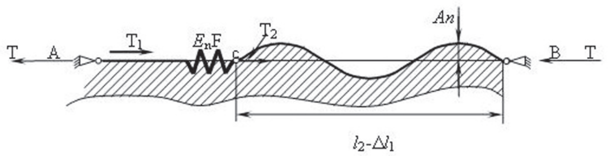

Fig.4 The virtual beam on elastic foundation with account for longitudinal rigidity E_nF

During the effect of the longitudinal force T in the beam (in front of the die block) the displacement of its movable end is taking place on the value $l-v$, upon that the second end of the beam remains immovable. Then between the longitudinal displacement of the beam v and the lateral deflection w the nonlinear dependence is being established. The longitudinal movement of the beam movable end we will find as difference between the original length l and projection of the beam curved axis

$$v=l-\int_0^l\cos\varphi ds=l-\int_0^l\sqrt{1-\left(\frac{dv}{d}\right)^2}d \qquad (31)$$

In the presented expression φ-angle, consisting of the tangent to the arc of the beam deflected axis with long axis $0x$.

We will expend the expression under integral sing in the series by formula of Newton binomial.

$$\sqrt{1-\left(\frac{dv}{d}\right)^2}=1+\frac{1}{2}\left(\frac{dv}{d}\right)^2+\frac{1}{8}\left(\frac{dv}{d}\right)^4+...+... \qquad (32)$$

Substituting the expression (28) into (32) and performing integration in the range from 0 to l and taking into account the value of the integrals

$\int_0^l\cos^2\frac{\pi s}{l}ds=\frac{l}{2}$; $\int_0^l\cos^4\frac{\pi s}{l}ds=\frac{3}{8}l$, we will receive the meaning of the longitudinal displacement v through the deflection bend f and length of the beam l as

$$v=\frac{\pi^2f^2}{4l}+\frac{3}{64}\frac{\pi^4f^4}{l^3}+...+... \qquad (33)$$

At longitudinal displacements of the beam, besides the variable force of friction $T(t)=T(\dot{x})$, increment of the longitudinal force ΔT occurs, conditioned by longitudinal rigidity of the beam material under the die block

$$\Delta T=-vG \qquad (34)$$

Then the differential equation of the longitudinal stability (static or dynamic stability), if turning to the expression (2), looks like

$$\frac{d^4w}{ds^4}+\frac{T}{E_nJ}\frac{d^2w}{ds^2}+\frac{C}{E_nJ}w-\frac{vG}{E_nJ}=0 \qquad (35)$$

In the presented expressions (34), (35) $G=E_nF$ – longitudinal rigidity of the virtual beam; F is its cross-sectional area. As it follows from (31) longitudinal displacement of the exposed end of the beam in front of the die block connected nonlinearly with lateral displacement (33).

When we substitute the expression (33) into the differential equation (35) with taking into account (34) and requiring orthogonal property of the obtained functional with the function $\sin\frac{\pi s}{l}$ we come to the expression, containing the beam deflection bend f.

$$E_nJ\frac{\pi^4}{l^4}f+T\frac{\pi^2}{l^2}f+fC-\frac{\pi^2f^2}{4l}-\frac{3}{64}\frac{\pi^4f^4}{l^3}E=0 \qquad (36)$$

If the deflection f, included into the equation (36) takes not too big meaning, which in reality corresponds to viscoelastic behavior of the materials (at operating temperatures approximately $450\div600^0C$), then in the expression to determine the approach of the ends (33) it would be sufficient to use only the first item, while the deflection bend of the virtual beam can be determined from the expression

$$\frac{\pi^2}{4l}E_nF=E_nF\frac{\pi^4}{l^4}f+T\frac{\pi^2}{l^2}f+fC \qquad (37)$$

Altogether the deflection bend is calculated from the expression

$$f = \frac{4J}{F}\frac{\pi^2}{l^2} + \frac{4T}{F} + \frac{C}{E}\frac{4l}{\pi^2} \qquad (38)$$

In case if the beam loses stability and the deflections f will be significant, which corresponds to fluidity of the material (at $t^0 \geq 600^0 C$), then in the expression for the approach of the ends (33) we will use only the first two items. Then, carrying out the modifications with the differential equations (35) also by Bubnov-Galerkin method, keeping in the expression (33) for v two items, we come to the next expression to determine the deflection bend f in the plastic domain

$$f^3 + \frac{6}{3}\frac{l}{\pi^2}f - \frac{4}{3l}\frac{J}{F} - \frac{4}{3}\frac{T}{\pi^2 E} - \frac{4}{3}\frac{C^3}{\pi^4 E} = 0 \qquad (39)$$

The cubic equation which we obtained, is an incomplete Cardano equation:

$$f^3 + pf + q = 0 \qquad (40)$$

where $p = \frac{6}{3}\frac{l}{\pi^2}$; $q = -\frac{4}{3l}\frac{J}{F} - \frac{4}{3}\frac{T}{\pi^2 E} - \frac{4}{3}\frac{C^3}{\pi^3 E}$ are the coefficients of the incomplete Cardano equation.

We will calculate the roots of the equation (39) by Cardano formulas

$$f_1 = A + B;\; f_{2,3} = -\frac{A+B}{2} \pm i\frac{A-B}{2}\sqrt{3} \qquad (41)$$

where $A = \sqrt[3]{-\frac{q}{2} + \sqrt{Q}}$; $B = \sqrt[3]{-\frac{q}{2} - \sqrt{Q}}$; $Q = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2$

Moreover, only the real numbers we take as the roots (40). For solution of the equation (40) it is enough to separate one real positive root, if the rest of the roots are complex-conjugate or the real ones and the negative ones, then the first root will be the solution of the equation (40). In case if all the roots are real, then we choose the biggest positive root as the solution.

When we further specify the solution of our problem on determining the deflections of the beam (the biggest meaning of the microasperity) in the plastic region we will receive by using the specified meaning of the beam curvature, the formula (25), and also the longitudinal movement of the virtual beam front support v in the formula (33) and using algorithm of solution in Bubnov-Galerkin form, we will obtain

$$\frac{\pi^2 f}{4l}\frac{E}{E} + \frac{3}{4}\frac{\pi^4 f^3}{l^3} = \frac{\pi^4}{l^4} + T\frac{\pi^2}{l^2 E} + \frac{C}{E} + f\frac{t^2\pi^6}{8l^4};$$

$$\frac{3}{4}\frac{\pi^3}{l^3}f^3 - f^2\frac{\pi^6}{8l^4} + f\frac{\pi^2}{4l}\frac{F}{J} - \frac{\pi^4}{l^4} - T\frac{\pi^2}{l^2 E} - \frac{C}{E} = 0 \qquad (42)$$

This expression is the general-typed cubic equation. For its solution we need to introduce it to the type (40), or separate one real root, after which, to determine the other roots by resolution (40) into factors. If the roots are real, then we choose the biggest positive root as f, if one root is real (positive), and the other two ones are complex-conjugate, then as the solution we take the real positive root.

The overall conclusions and the research objectives

If the parametric effects occur in outer layers of the parts during friction and wear-out, it is necessary that characteristics of friction $T = T\left[\dot{x}(t)\right]$ $T = T[\dot{x}(t)]$ as function of the parts mutual movement velocity had a "falling" nature and the negative damping was observed in the system, then in the diffusion layers of the parts autovibration process develops and these layers have unstable equilibrium state, caused by tangential forces $T \geq T_{kp}$.

This is reason why there are prerequisites for autovibrations and parametric resonance to occur in the system. The nature of the process itself is conditioned with the view of the solution of Mathieu equations and the ratio of the coefficients $a_k a_k$, $q_k q_k$, in which there are increasing and transient components available.

The set forth arguments allow to outline the following scenarios of the transient processes development in the diffusion layers of the half space:

1. Explicitly parametric vibration regime, when vibration process is self-excite (unstable) at longitudinal load close to critical $T = T_{kp}$ $T = T_{кр}$ and small disturbing effects. In this case the parametric resonance develops with increasing amplitudes and further quick destruction of the diffusion layers of the virtual beam (the cleavage for the materials with high hardness $HRC \geq 40$, or plastic collapse and the following cut of the distortion microasperities.

2. Pseudo autovibration process with floating frequency and the limited amplitudes. If upon that each pair of meanings a_k, q_k get the stability zone on Ains-Strett diagram and $T = T_{кр}$ $T = T_{kp}$, then loss of stability by diffusion layers in Euler Point may occur, but the parametric resonance against the background of loss of stability will not develop and quick destruction of the diffusion layers will not happen. In this case there will be wear-out of distortion microasperities occurred together with engineering microaspetities (along with this the quantities of the engineering and distortion microasperities will be commensurable). In consequence of cyclical nature of loading the further fatigue failure of all the microasperities will happen, upon that intensiveness of wear-out increases.

3. The stationary longitudinal wave process occurs if $T(t) = T\left[\dot{x}(t)\right]$ and upon that the characteristics

of friction does not have the falling segment or negative damping, then in the area in front of the die block the stationary wave is formed moving (with the die block velocity, which is considered in the moving coordinates). If wavy distortions of the diffusion layers are final, then wear-out process is similar to wear-out of the engineering microasperities.

4. The immovable lateral wave process (in the moving coordinates) occurs at the liquid friction, when we can ignore friction force $T(t)T(t)$ due to its smallness. Upon that virtual beam will test distortion of the flexion from weight effect of the displacing die block P. In this case displacement of the die block and the flexural wave of distortion in the finite case leads to fatigue failure of the interacting flats of the surfaces even at small weight value of the die block P.

Turning to the version 1, as to the one most unfavourable, which correspond to parametric resonance, we can calculate the biggest distortions for static loading by distortion to determine the biggest stress, like in Euler problem. Upon that when we take into account nonlinear dependence of the elasticity modulus from the distortions $E_n = E_n(\varepsilon)$. We preliminarily calculate the stresses in the virtual beam at the biggest lateral distortions without taking into account the dependence $E_n(\varepsilon)$, calculating $E_n = const$, an then with account for amendment for alteration $E_n(\varepsilon)$, we will receive the real stress at which the beam loses stability with its further destruction.

The considered physical phenomenon of the wave generation in front of the moving flat die block can be generalized and for the case of displacement of the rotating cylinder on the half space. The proposed information may be useful at calculations of engineering processes on the pressure metal treatment (rolling, drawing, blacksmithing and others).

Calculation of such processes is necessary for choosing correctly the capacities of the rolling and the drawing mills. In many cases they allow to determine the billets dimensions correctly, calculate the optimal conditions of distortion, which provides manufacture of high-grade products.

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