A new method for determining the weights in multi-criteria decision making based on ordinal ranking of criteria and lagrange multiplier

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Abstract
In this paper, we present a new method for determining criteria weights for resolving Multi-Criteria Decision Making (MCDM) problems. Criteria weights are determined by solving optimization problem of a function. This last is reconstituted following resolution of the Hessian matrix whose coefficients are expressed by the weighted average values of criteria as diagonal entries. The proposed method, which based on ordinal ranking of criteria and Lagrange multiplier, determines the weights for any set of criteria under consideration. A case study, relating to the choice problem of an excavator for a Company of public works, is carried out to illustrate the proposed method.
Key words: MULTI-CRITERIA DECISION MAKING, LAGRANGE MULTIPLIER, VALUED MODEL, WEIGHTS

1. Introduction

In the multi-criteria decision making (mcdm), weights are directly given by the decision maker or indirectly determined starting from computation methods. The concept of weight can be defined only in reference to one of the specific theories of preference (weber and borcherding, 1993). For the mcdm methods with a single criterion of synthesis (such as the smart method (edwards, 1971); the maut method (keeny and raiffa, 1976); the weighted sum method (timmerman, 1986); the weighted product method (Pomerol and Barba-Romeo, 1993); the TOPSIS method (Lai et al., 1994); etc.), weights represent the substitution rates between criteria under consideration. For the MCDM methods of outranking (such as ELECTRE-type methods (Roy, 1996) and PROMETHEE-type methods (Brans and Mareschal, 2002)), weights represent the relative importances of criteria. In this paper, the proposed method aims to determine the weights, without a priori, by solving an optimization problem of a function. Optimization techniques offer more possibilities, through the way in which the Hessian matrix is designated, than algebraic formulae. The weights vector $W = (W_1, W_2, W_3, \dots, W_n)^T$ minimizes the following function: $W^T\ H_F\ W$, subject to the normalization constraint $\sum_{j=1}^{n} W_j = 1$, where H_F is a diagonal square symmetric matrix. The diagonal term of the rank j is denoted by $H_{F_j}, j = 1, 2, 3, \dots, n$.

The relationship, known in literature as "criterion rank - weight", requires that the weights W_j have the same monotony as the ordinal ranking of criteria j.

For solving a multi-criteria decision making problem, we can use one of the above-mentioned MCDM methods. In the classical multi-criteria utility method, as the weighted sum method (Timmerman, 1986), the problem consists in constructing a utility function: $F(a_i)$.

$F(a_i) = \ \Sigma\, W_i \,.\, a_{ii}$, where W_j : weight of criterion j, satisfying $\sum_{i=1}^{n} W_i = 1$ and a_{ij}: partial utility of alternative a_i $(i = 1, 2, 3, \dots, m)$ associated with criterion j $(j = 1, 2, 3, \dots, n)$.

We assume that all the partial utilities a_{ij} are defined according to the same interval scale. After partial utilities aggregation of alternatives a_i, the highest final score of $F(a_i)$ indicates the best alternative. In this paper, a case study, for illustrating the proposed method, is realized. It concerns a problem of choice of an excavator for a Company of public works (see paragraph 4.2).

2. Literature review

Assigning weights to criteria under consideration is a crucial stage in the process of the decision making. Criteria weights, badly allotted by a decision maker itself or badly elicited by a technical expert, negatively influence the selection of the best solution and consequently

the decision to be taken. So many researchers, which work in the field of multi-criteria decision making, estimate that the criteria weights constitute a problem to be solved with precaution (Vincke, 1992; Schärlig, 1999; Vallin and Vanderpooten, 2002; etc.).

In the literature, there are several methods for determining criteria weights. In this paper, let us quote the most relevant methods below:

1) the method of successive comparisons (Churchman and Ackoff, 1954). This is one of the first methods to be used for weighting criteria. It is revisited later by (Knoll and Engelberg, 1978).

2) the method of simple ranking (Kendall, 1970). It is simple and requires few calculations, but it prevents the weights to take all possible values between zero and one. According to (Pomerol and Barba Romeo, 1993), it seems enough realistic.

3) the trade off method and the princing-out method (Keeny and Raiffa, 1976).These two methods are well-known and used during resolution of the multi-criteria decisional problems with MAUT (Multi-Attribute Utility Theory) method.

4) the AHP (Analytical Hierarchy Process) method (Saaty, 1980). Although it is used in different applications, it has caused polemic in particular by (Dyer, 1990) and (Holder, 1990).

These authors see that it does not satisfy some theoretical conditions like the axiom of transitivity.

5) the method of ratio (Von Winterfeldt and Edwards, 1986). In this method, criteria weights are determined in taking into account the proportionality ratio between criteria under consideration.

6) the method of probabilistic evaluation (Rietveld, 1989). In this method, the ordinal information is applied in case of decision making under uncertainty and would refer to the weights to be attached to the alternatives.

7) the method of "resistance to change" grid (Rogers and Bruen, 1998). It is firstly used in ELECTRE-III method within the framework of weighting of the environmental criteria.

Among methods based on linear multi-attribute programming, let us note:

8) the method of Srinivasan and Shocker (1973);

9) the method of Pekelman and Sen (1974);

10) the method of Marichal and Roubens (2000);

In these methods, above-mentioned authors have developed computation programmes leading at the elicitation of criteria weights.

11) the method based on the distance between the individual partial pre-orders of criteria to aggregate

into a global ranking of alternatives (Xu, 2001).

12) the method using rank inclusion in criteria hierarchies in which the decision maker ranks a given set of criteria.

The methods based on ordinal ranking of criteria are numerous, let us mention:

13) the method of rank sum linear weights (Stillwell et al., 1981);

14) the method of reciprocal weights (Stillwell et al., 1981);

15) the method of centroid weights (Solymosi and Dombi, 1986);

16) the method of geometric weights (Lootsma, 1999).

Many works, relating to the comparison of weights, have shown that the method of centroid weights is simple to use, practical and superior in terms of accuracy (Olsen and Dorai, 1992;

Noh and Lee, 2003; etc.).

We notice that Edwards and Barron (1994) have extended SMARTS into SMATER (Simple Multi-Attribute Rating Technique Exploiting Ranks) using the method of centroid weights whose results are accurate.

17) the empiric method (Alfares and Duffuaa, 2009). In their work, these two authors have proved the linear "criteria ranks - weights" relationship while proposing an explicit linear function able to determine the weights for any number of criteria.

Among neutral methods which don't require the participation of a decision maker, let us cite:

18) the method of standard deviation (Fleiss, 1981);

19) the method of entropy (Zeleny, 1982);

20) the statistical method (Diakoulaki et al., 1992);

21) the method of total variation (Bouhedja and Pousin, 2011).

These neutral methods can be used by a technical expert to determine criteria weights if a decision maker itself can't assign weights to criteria for solving a given decisional problem.

In addition, other weight elicitation methods are recent, let us indicate:

22) the method called "Multiple Gradient Descent Algorithm" (MGDA) for multiobjective optimization (Désidéri, 2012). In this method, an original constructive solution is given by the author.

23) the method named "Q-Eval" (Eppe and De Smet, 2014). This last is an adaptive questioning procedure for eliciting PROMETHEE-II's weight parameters.

24) the Kemeny Median Indicator Ranks Accordance (KEMIRA) method (Krylovas et al., 2014). In this method, criteria weights are determined and alternatives ranking accomplished by solving optimization problem (minimization of ranks discrepancy function).

In this section, relating to the literature review, the existence of numerous methods let us to deduce that the problem of criteria weights elicitation is an open field for research. Especially if we want improve the quality of the decisions to be taken in the real world decisional problems and for advancing the knowledge.

In this article, authors' contribution consists in proposing a new method for determining criteria weights (see section 3).

3. Presentation of the proposed method

The goal of this article consists in proposing a new method for determining criteria weights. For reaching this goal, the below steps had to be considered. The proposed method is based on ordinal ranking of criteria and Lagrange multiplier. Its bases are structured into nine steps:

*** Step 1**

Ask to a decision maker to provide ascending ordinal ranking of criteria. These latter are considered as independent.

*** Step 2**

Give value 1 to criterion of rank 1 (the least important criterion), value 2 to criterion of rank 2, value 3 to criterion of rank 3 and so on up to the value n for the criterion of rank n (the most important criterion).

*** Step 3**

Determine the weight ratio for each criterion $j\ (C_j), j = 1, 2, 3, \dots, n.$ We obtain:

$$C_1 = 1/\sum_{j=1}^{n} j, C_2 = 2/\sum_{j=1}^{n} j, C_3 = 3/\sum_{j=1}^{n} j, \dots, C_n = n/\sum_{j=1}^{n} j \quad (1) \qquad (1)$$

The weight ratio can be interpreted as a weighted average value. With the aim of eliciting the weights of criteria, we realize the following steps.

*** Step 4**

Define the Hessian matrix with the obtained values in (1) as diagonal entries. We obtain:

$$H_F = \begin{bmatrix} 1/\sum_{j=1}^{n} j & 0 & 0 & \dots & 0 \\ 0 & 2/\sum_{j=1}^{n} j & 0 & \dots & 0 \\ 0 & 0 & 3/\sum_{j=1}^{n} j & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & n/\sum_{j=1}^{n} j \end{bmatrix} \qquad (2)$$

*** Step 5**

For respecting the relationship “criterion rank – weight” and consequently to obtain a lowest weight for the criterion of rank 1, and a highest weight for the criterion of rank n in the final formula (see expression (14)), an opposite coefficient "$n+1-j$" in comparison with initial ranking of criterion j is necessary. Then, expression (2) becomes (3):

$$H_F = \begin{bmatrix} n+1-1/\sum_{j=1}^{n} j & 0 & 0 & \dots & 0 \\ 0 & n+1-2/\sum_{j=1}^{n} j & 0 & \dots & 0 \\ 0 & 0 & n+1-3/\sum_{j=1}^{n} j & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & n+1-n/\sum_{j=1}^{n} j \end{bmatrix} \quad (3)$$

*** Step 6**

Reconstitute the polynomial expression of the function $F(W)$ defined by the Hessian matrix (3). We obtain:

$$F(W) = \frac{1}{2\sum_{j=1}^{n} j}\left[(n+1-1)W_1^2 + (n+1-2)W_2^2 + (n+1-3)W_3^2 + \dots + (n+1-n)W_n^2\right] \quad (4)$$

*** Step 7**

Put $\sum_{j=1}^{n} j = [n(n+1)]/2$ (5)

n: number of criteria.

By replacing expression (5) in expression (4), we obtain:

$$F(W) = \frac{1}{n(n+1)}\left[(n+1-1)W_1^2 + (n+1-2)W_2^2 + (n+1-3)W_3^2 + \dots + (n+1-n)W_n^2\right] \quad (6)$$

*** Step 8**

Formalize the mathematical k mathematical model. In the proposed approach, problem of determination of the criteria weights is formalized like an optimization problem of the function $F(W)$. The obtained mathematical model is:

$$\min F(W) = \frac{1}{n(n+1)} \sum [(n+1)-j] W_j^2 \quad (7)$$

subject to the normalization constraint $\sum_{j=1}^{n} W_j = 1$

The function $F(W)$ is strictly convex, the subset $\{W \in \mathbb{R}^n \text{ and } \sum_{j=1}^{n} W_j = 1\}$ is closed. Then, problem $\min F(W)$ admits an unique solution. We need to calculate the solution to a minimization problem, subject to an equality constraint. As it is usual, equality constraint is accounted by a Lagrange multiplier (Hiriart-Urruty and Lemaréchal, 2009).

The solution is thus characterized by the Optimality Conditions of Euler-Lagrange. The Necessary Conditions of Optimality are given by:

$$\nabla F(W) = \lambda(1,1,1,\dots,1)^T \text{ and } \sum_{j=1}^{n} W_j = 1 \quad (8)$$

Where λ: Lagrange multiplier (due at the equality constraint), n: number of criteria, and $j = 1, 2, 3, \dots, n$.

We obtain:

$$\frac{2[(n+1)-j]W_j}{n(n+1)} = \lambda(1,1,1,\dots,1)^T \quad (9)$$

$$\frac{\lambda[n(n+1)]}{2}\left(\sum \frac{1}{[(n+1)-j]}\right) = 1 \quad (10)$$

Then:

$$\lambda = \frac{2}{[n(n+1)] \cdot \left(\sum \frac{1}{[(n+1)-j]}\right)} \quad (11)$$

*** Step 9**

Starting from expression (9), we have:

$$W_j = \frac{\lambda[n(n+1)]}{2[(n+1)-j]} \quad (12)$$

By replacing Lagrange multiplier "λ" given by expression (11) in expression (12), we obtain:

$$W_j = \frac{2}{[n(n+1)] \cdot \left(\sum \frac{1}{[(n+1)-j]}\right)} \cdot \frac{[n(n+1)]}{2[(n+1)-j]} \quad (13)$$

After simplification, the weight of criterion j (W_j) is determined according to the expression below:

$$W_j = \frac{1}{\left(\sum \frac{1}{[(n+1)-j]}\right) \cdot [(n+1)-j]} \quad (14)$$

Where n: number of criteria, and $j = 1, 2, 3, \ldots, n$.

3.1 Usage, advantages and inconveniences of the proposed method

3.1.1 Usage

1) Ask to a decision maker to provide ascending ordinal ranking of criteria.

2) Determine the weight of criterion j (W_j) according to the expression (14).

In accordance to the expression (14), criterion of rank 1 (the least important criterion) denotes the lowest weight, whereas criterion of rank n (the most important criterion) denotes the highest weight. In the proposed method, the relationship "criterion rank - weight" is thus respected and verified.

3.1.2 Advantages

1) It is straightforward and simple to use.

2) It gives normalized weights whose their sum is equal to one ($\Sigma_{j=1}^{n} W_j = 1$).

3) It does respect the axiom of transitivity

$(W_j < W_{j+1}; j = 1, 2, 3, \ldots, n-1)$.

3.1.3 Inconveniences

1) It works with a decision maker during ordinal ranking of criteria.

2) It requires a rather long calculation.

4. Comparison of weights

In multi-criteria analysis, many researchers work on comparison of weights. Let us quote: Schoemaker and Waid, 1982; Bana e Costa, 1986; Mareschal, 1988; Lootsma, 1996; Bottomley et al., 2000; Olsen, 2004; etc. In this section, we consider a case study (see paragraph 4.2) for which we compare the proposed model with five methods below:

1) Method of rank sum linear weights (method 1) (Stillwell et al., 1981):

$$W_r = \frac{100\,(n + 1 - r)}{n} \quad (15)$$

2) Method of reciprocal weights (method 2) (Stillwell et al., 1981):

$$W_r = \frac{100}{r} \quad (16)$$

3) Method of centroid weights (method 3) (Solymosi and Dombi, 1986):

$$W_r = \frac{100\, \Sigma_{i=r}^{n}\, 1/i}{\Sigma_{i=1}^{n}\, 1/i} \quad (17)$$

4) Method of geometric weights (method 4) (Lootsma, 1999):

$$W_r = \frac{100}{\left(\sqrt{2}\right)^{r-1}} \quad (18)$$

5) Method of empiric weights (method 5) (Alfares and Duffuaa, 2009):

$$W_r = 100 - \left(3{,}19514 + \frac{37{,}75756}{n}\right).(r-1) \quad (19)$$

Where r: criterion rank, and n: number of criteria under consideration.

We signal that the proposed method and five above-mentioned methods are all based on ordinal ranking of criteria what facilitates their comparison.

During performances outranking of excavators (see Table 1), the weights obtained by proposed method and above-mentioned methods are compared in PROMETHEE method (Brans and Mareschal, 2002).

The comparison of criteria weights consists in analyzing the complete pre-orders provided by this MCDM method and checking reliability of the proposed model.

4.1 Presentation of promethee method (Brans and Mareschal, 2002).

The PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation) method is presented as follows:

PROMETHEE-I

1) Have decision matrix A, $A := (a_{ij})$, where a_{ij} is the evaluation (performance) of alternative (action) a_i $(i = 1, 2, 3, \ldots, m)$ associated with criterion j $(j = 1, 2, 3, \ldots, n)$.

2) For each considered criterion j, the decision maker is asked to choose one of six forms of criteria curves (usual criterion, quasi criterion, criterion with preference, level criterion, criterion with preference and indifference, and Gaussian criterion).

The parameters, relating to each curve, represent indifference and/or preference thresholds. In this paper, we specify usual criterion with a strict preference. Then the function of preference, for each pair (a_{ij}, a_{kj}), is:

$$F_j(a_{ij}, a_{kj}) = \begin{cases} 1 \text{ if } a_{ij} > a_{kj} \\ 0 \text{ if } a_{ij} \leq a_{kj} \end{cases} \quad (20)$$

$$i = 1, 2, 3, \ldots, m-1;\ k = i + 1,\ k = 2, 3, 4, \ldots, m;\ j = 1, 2, 3, \ldots, n$$

3) Define a vector containing the weights, which are a measure for the relative importance of each criterion, $W^T = [W_1, W_2, W_3, \ldots, W_n]$. If all the criteria are of the same importance in the opinion of

the decision maker, all weights can be taken as being equal.

The normalization of the weights $\Sigma_{j=1}^{n} W_j = 1$ is not necessarily required, but facilitates an uniform representation and comparison of different evaluations. No specific approach for the setting of weights is proposed, because the aim of PROMETHEE (and outranking in general) is seen in the explanation of the weighting factors spontaneously expressed by the decision maker.

4) Define, for all alternatives $a_i, a_k \in A$, the outranking relation P:

$$P\left(a_i, a_k\right) = \sum_{j=1}^{n} W_j \cdot F_j\left(a_{ij}, a_{kj}\right) \qquad (21)$$

The preference outranking $P\left(a_i, a_k\right)$ is a measure for the intensity of preference of the decision maker for an alternative a_i in comparison with an alternative a_k for the simultaneous consideration for all criteria. It is basically a weighted average of the preference function $F_j\left(a_{ij}, a_{kj}\right)$ and can be represented as a valued outranking graph (see Figure 1).

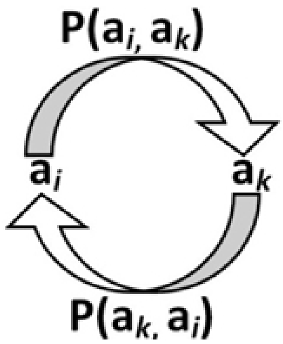

Figure 1. Outranking graph

5) As a measure for the strength of the alternatives $a_i \in A$, the leaving flow is calculated:

$$\Phi^{+}\left(a_i\right) = \frac{1}{m}\sum P\left(a_i, a_k\right) \qquad (22)$$

The leaving flow is the sum of the values of the arcs which leave node a_i and therefore yield a measure of the outranking character of a_i.The normalization using the total number of considered alternatives "m" is not a necessary precondition, but both for the leaving and entering flows, the same approach has to be chosen. As through the normalization of the weights, a comparison of different evaluations is made easier.

6) As a measure for the weakness of the alternatives $a_i \in A$, the entering flow is calculated, measuring the outranked character of a_i (analogously to the leaving flow):

$$\Phi^{-}\left(a_i\right) = \frac{1}{m}\sum P\left(a_k, a_i\right) \qquad (23)$$

7) Graphical evaluation of the outranking relation. Basically, the higher the leaving flow and the lower the entering flow, the better alternative. The PROMETHEE partial pre-order is determined by a comparison of the leaving and entering flows by a set intersection in a manner that also allows the representation of weak preferences and incomparabilities of alternatives. In the valued outranking graph, an arc leads from alternative a_i to a_k, if a_i is preferred to a_k (see Figure 2: e.g. a_4 outranks a_3, a_2 and a_4 are indifferent, a_5 and a_6 are incomparable to each other, but are outranked by (i.e. worse than) a_6).

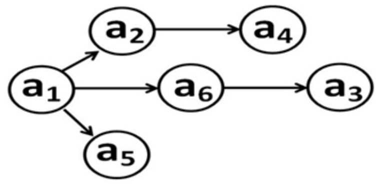

Figure 2. Partial pre-order as an example for a graphical representation of the result on an outranking model

PROMETHEE-II

1) In case a complete pre-order is requested, PROMETHEE-II yields the so-called net flow $\Phi\left(a_i\right)$ as the difference of the leaving flow $\Phi^{+}\left(a_i\right)$ and entering flow $\Phi^{-}\left(a_i\right)$:

$$\Phi\left(a_i\right) = \Phi^{+}\left(a_i\right) - \Phi^{-}\left(a_i\right) \qquad (24)$$

For any pair of alternatives $\left(a_i, a_k\right) \in A$, we have one of the following relations:

a) the rank of alternative a_i is better than the rank of alternative a_k, if $\Phi\left(a_i\right) > \Phi\left(a_k\right)$.

b) the rank of alternative a_k is better than the rank of alternative a_i, if $\Phi\left(a_i\right) < \Phi\left(a_k\right)$.

c) the rank of alternative a_i has the same rank as alternative a_k, if $\Phi\left(a_i\right) = \Phi\left(a_k\right)$.

2) Rank the alternatives $\left(a_i\right)$ according to the descending order of the net flow $\Phi\left(a_i\right)$.

3) Establish the complete pre-order.

4) Select the best solution

NB/ For a detailed presentation of the PROMETHEE methods, we invite the reader to Brans and Mareschal (2002).

4.2 Case study

A Company of Public Works, which intends acquire an excavator having a bucket capacity equal to 2.5 m^3, launches an invitation to tender. The tenderers have presented financial and technical offers. These latter are shown in the decision matrix A (see Table 1). For this case study:

1) Determine criteria weights according to the proposed method and five above-mentioned methods.

2) Select best solution (best excavator) according to the PROMETHEE method.

Table 1. Decision matrix A for choosing an excavator

Excavators a_i	Criteria			
	Criterion of motor power (horsepower)	Criterion of energy consumption (liter / hour)	Criterion of acquisition price (Euro)	Criterion of equipment mass (ton)
a_1	162	34	260 000	14.9
a_2	163	35	265 000	14.6
a_3	160	33	243 000	12.8
a_4	151	32	235 000	14.0
a_5	150	30	225 000	12.3
a_6	186	38	250 000	13.9

4.2.1 Results

1/ the final results of criteria weights are presented in Table 2.

Table 2. Final results of criteria weights

Weight elicitation methods	Weights			
	Criterion weight of equipment mass	Criterion weight of energy consumption	Criterion weight of motor power	Criterion weight of acquisition price
Proposed method	0.1200	0.1600	0.2400	0.4800
Method 1	0.1000	0.2000	0.3000	0.4000
Method 2	0.1200	0.1600	0.2400	0.4800
Method 3	0.0626	0.1454	0.2706	0.5214
Method 4	0.1392	0.1948	0.2764	0.3896
Method 5	0.1915	0.2305	0.2695	0.3085

2/ the net flows obtained by the excavators, according to the PROMETHEE method, are shown in Table 3.

Table 3. Net flows obtained by the excavators

Excavators a_i	Weight elicitation methods					
	Proposed method	Method 1	Method 2	Method 3	Method 4	Method 5
	Net flow	Net flow	Net flow	Net flow	Net flow	Net flow
a_1	- 1,2400	- 0,8000	- 1,2400	- 1,0856	- 0,9520	- 0,9220
a_2	- 2,2800	- 1,8000	- 2,2800	- 2,0032	- 1,8465	- 1,8440
a_3	+ 0,5200	+ 0,4000	+ 0,5200	+ 0,4588	+ 0,4472	+ 0,4610
a_4	+ 1,5600	+ 1,2000	+ 1,5600	+ 1,3764	+ 1,3416	+ 1,3830
a_5	+ 2,6000	+ 2,0000	+ 2,6000	+ 2,2940	+ 2,2360	+ 2,3050
a_6	- 1,1600	- 1,2000	- 1,1600	- 1,0404	- 1,2264	- 1,4310

3/ the ranks obtained by the excavators, according to the PROMETHEE method, are presented in Table 4.

Table 4. Ranks obtained by the excavators

Excavators a_i	Weight elicitation methods					
	Proposed method	Method 1	Method 2	Method 3	Method 4	Method 5
	Rank	Rank	Rank	Rank	Rank	Rank

a_1	5	4	5	5	4	4
a_2	6	6	6	6	6	6
a_3	3	3	3	3	3	3
a_4	2	2	2	2	2	2
a_5	**1**	**1**	**1**	**1**	**1**	**1**
a_6	4	5	4	4	5	5

4.2.2 Discussion of results

The two complete pre-orders, obtained by PROMETHEE method, are:

1) $a_5 \rightarrow a_4 \rightarrow a_3 \rightarrow a_6 \rightarrow a_1 \rightarrow a_2$

This first complete pre-order is given by the weights resulting from proposed method, method of reciprocal weights (Method 2) and method of centroid weights (Method 3).

2) $a_5 \rightarrow a_4 \rightarrow a_3 \rightarrow a_1 \rightarrow a_6 \rightarrow a_2$

The second complete pre-order is given by the weights emanating from method of rank sum linear weights (method 1), method of geometric weights (method 4), and method of empiric weights (method 5).

By analyzing the complete pre-order provided by the PROMETHEE method for each weight elicitation method used (see Table 4), we notice that there is stability of ranks obtained by excavators $a_2, a_3, a_4,$ and $a_5,$ whereas excavators a_1 and a_6 have changed position.

In all cases, excavators a_5 and a_2 are respectively designated as best and bad solution and this, whatever the weight elicitation model used. The Company of public works, as decision maker, can select excavator a_5.

The proposed method, whose results are conclusive for the case study, can be used in other multi-criteria decision making problems.

5. Conclusion

In this paper, authors' contribution consists in proposing a new method for determining criteria weights. The proposed method, which is based on ordinal ranking of criteria and Lagrange multiplier, is straightforward and easy to use. It gives normalized weights $(\sum_{j=1}^{n} W_j = 1)$ and does respect the axiom of transitivity $(W_j < W_{j+1}; j = 1, 2, 3, \ldots, n-1)$.

The use of the proposed method, on a great number of multi-criteria decisional problems, will permit it to show its assets vis-a-vis the other methods of determination of the criteria weights. The extension of this research work is to elaborate a new theory of preference ranking.

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