Modeling of stress state at the start of deformation of granular filling of rotating cylindrical chamber

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Abstract
The task of determination of stress at the start of deformation of granular filling of cylindrical chamber rotating around horizontal axis is considered. By elastic approach, pressure patterns are obtained. The effect of formation of zones with minimum and maximum value of medium pressure is established.
Key words: STRESS, GRANULAR MATERIAL, START OF DEFORMATION, CYLINDRICAL CHAMBER, ROTATION AROUND HORIZONTAL AXIS, ZONE OF MINIMUM AND MAXIMUM PRESSURE

Deformation of processed granular material filling the cylindrical chamber determines technological efficiency of rather widespread class of machines of drum type. The value of moment of resistance to rotation of drum is caused by filling redistribution in the chamber [1]. Therefore, the task of determination of behavior of this material has high level of applied significance and is of special interest for dynamics of rotor systems [2].

Such task was solved analytically and numerically with use of models of lumped parameter systems in papers [3-5].

Characteristics of shear layer based on continual model of granular filling of the chamber were studied in paper [6].

Movement of granular layer in case of small rotational speed of chamber was determined analytically by the experiment in paper [7].

In paper [8], the strain-stress state of filling at the initial moment of deformation in case of slow rotation of chamber was researched numerically with use of model of limiting elastic-plastic deformation of the granular medium.

The task of determination of strain-stress state of filling in case of small rotational speed was considered analytically in plastic position in paper [9].

The further numerical solution of the task of determination of continuous medium stress at the start of deformation of granular material in the rotating chamber is found in elastic position.

The equations of two-dimensional elastic model of deformation of filling is of the form

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + F_x = 0; \qquad \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + F_y = 0; \tag{1}$$

$$F_{xv} = \sigma_x \cos(n_v \wedge x) + \tau_{xy} \cos(n_v \wedge y); \; F_{yv} = \tau_{yx} \cos(n_v \wedge x) + \sigma_y \cos(n_v \wedge y); \tag{2}$$

$$\varepsilon_x = \frac{\partial \Delta x}{\partial x}; \qquad \varepsilon_y = \frac{\partial \Delta y}{\partial y}; \qquad \gamma_{xy} = \frac{\partial \Delta x}{\partial y} + \frac{\partial \Delta y}{\partial x}; \tag{3}$$

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}; \tag{4}$$

$$\varepsilon_x = \frac{1-\nu^2}{E}\left(\sigma_x - \frac{\nu}{1-\nu}\sigma_y\right); \; \varepsilon_y = \frac{1-\nu^2}{E}\left(\sigma_y - \frac{\nu}{1-\nu}\sigma_x\right); \; \gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}; \tag{5}$$

where σ_x, σ_y and $\tau_{xy}=\tau_{yx}$ – three components of stress tensor; F_x and F_y – projections force due to mass; x and y – coordinates; F_{xv} and F_{yv} – components of intensity of surface load; $\cos(n_v \wedge x)$ and $\cos(n_v \wedge y)$ – directional cosines; n_v – external normal to angled surface; Δx and Δy – projections of displacement to coordinate axes; ε_x and ε_y – linear deformations in the directions of coordinate axes x and y; γ_{xy} – angular deformation in a coordinate plane Oxy; E – filling elasticity modulus; ν – Poisson ratio of filling.

The system (1)-(5) contains the following main equations of elastic deformation: differential equa tions of equilibrium (1), equilibrium condition on the filling surface (stresses in oblique bound) (2), differential dependences of deformations and displacement (3), continuity equation (4), Hook's law (5).

The array modeling filling in the form of segment of radius R angled to the slope of repose in the state of rest α was divided by computational grid into quadrangular cells (Fig. 1).

The algorithm of calculation of strain-stress state of filling with finite-element method was developed.

The composed function of total potential energy for two-dimensional stress state generally is of the form

$$\Pi = \frac{1}{2}\int_{\Omega}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy})d\Omega - \int_{L}(p_x u + p_y v)dL,$$

where p_x, p_y – components of vector of external loading in the direction of x and y axes respectively; u, v – displacement; $d\,\Omega$ and dL – infinitely small elements of two-dimensional area and circuit.

Composed function of total potential energy of computational region in the compact form

$$P = \frac{1}{2}\int_{\Omega}\{\varepsilon\}_r\{\sigma\}d\Omega - \int_{L}\{p\}_r\{q\}dL.$$

The main ratios of plane stress in the complex form with matrix characters

$$\{\sigma\} = [E]\{\varepsilon\}, \quad \{\varepsilon\} = [D]\{q\},$$

where $[E]$ – elasticity matrix; $[D]$ – differentiation matrix.

The main ratios in matrix form

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}, \qquad \{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}.$$

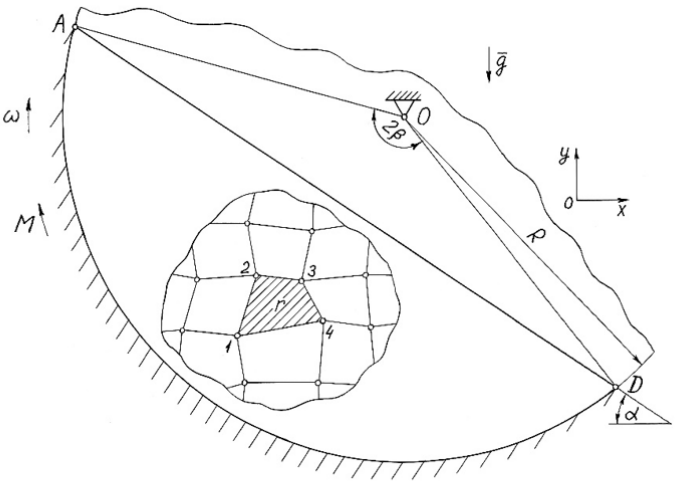


Figure 1. The computational scheme of application of finite-element method for determination of filling stress of rotating chamber

Components of vectors of displacement, external loading, stresses and deformation for the plane stress problem is of the form

$$\{q\} = \begin{Bmatrix} u \\ v \end{Bmatrix}, \qquad \{p\} = \begin{Bmatrix} p_x \\ p_y \end{Bmatrix}, \qquad \{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}, \qquad \{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}.$$

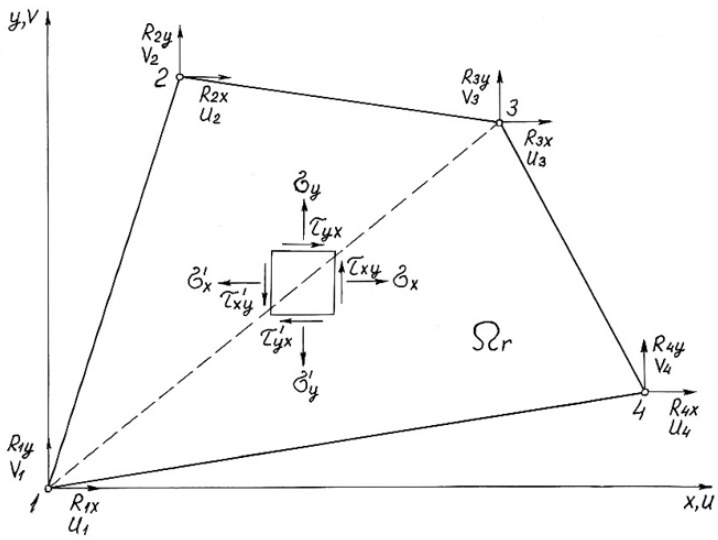


Figure 2. Diagram of quadrangular finite element r: R_{1x}, R_{1y}, R_{2x}, R_{2y}, R_{3x}, R_{3y}, R_{4x}, R_{4y} – projections of nodal reaction; u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4 – projections of nodal displacement

Vector of stresses

$$\{\sigma\}=[E]\{\varepsilon\}=[E][D]\{q(x,y)\},$$

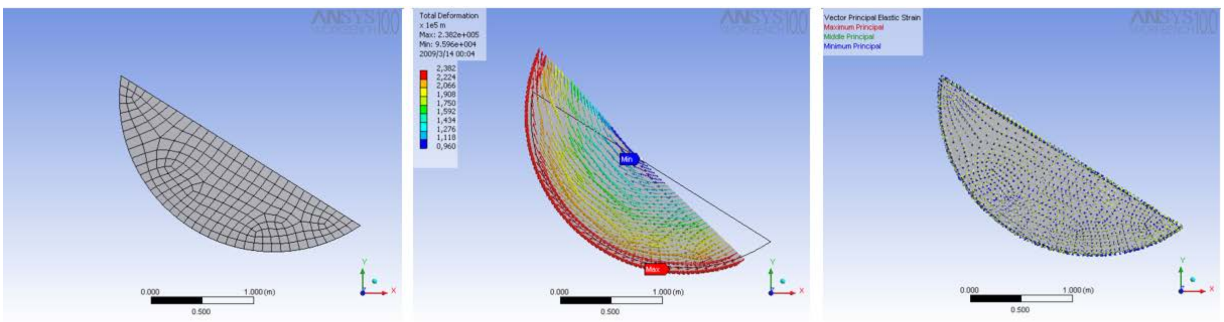
where $\{q(x,y)\}$ – vector of approximating functions consisting of components $u(x,y)$ and $v(x,y)$. Vectors of stress in regions of triangular components of finite element (Fig. 2).

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\frac{E}{(1-\nu^2)\Delta}\begin{bmatrix}(y_2-y_3) & \nu(x_3-x_2) & (y_3-y_1) & \nu(x_1-x_3) & (y_1-y_2) & \nu(x_2-x_1)\\ \nu(y_2-y_3) & (x_3-x_2) & \nu(y_3-y_1) & (x_1-x_3) & \nu(y_1-y_2) & (x_2-x_3)\\ \frac{1-\nu}{2}(x_3-x_2) & -\frac{1-\nu}{2}(y_3-y_2) & -\frac{1-\nu}{2}(x_3-x_1) & \frac{1-\nu}{2}(y_3-y_1) & \frac{1-\nu}{2}(x_2-x_1) & -\frac{1-\nu}{2}(y_2-y_1)\end{bmatrix}\begin{Bmatrix}u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\end{Bmatrix},$$

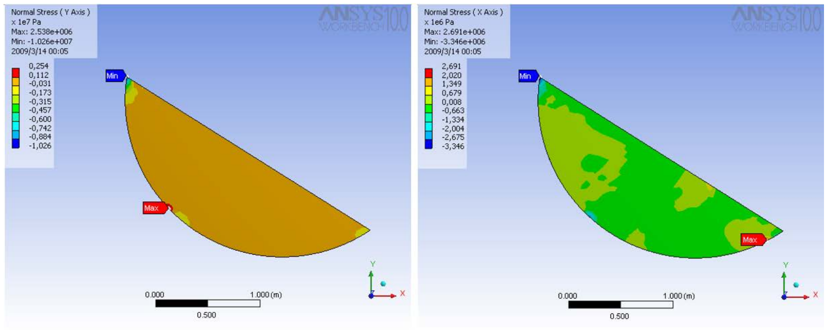
$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\frac{E}{(1-\nu^2)\Delta}\begin{bmatrix}(y_3-y_4) & \nu(x_4-x_3) & (y_4-y_1) & \nu(x_1-x_4) & (y_1-y_3) & \nu(x_3-x_1)\\ \nu(y_3-y_4) & (x_4-x_3) & \nu(y_4-y_1) & (x_1-x_4) & \nu(y_1-y_3) & (x_3-x_4)\\ \frac{1-\nu}{2}(x_4-x_3) & -\frac{1-\nu}{2}(y_4-y_3) & -\frac{1-\nu}{2}(x_4-x_1) & \frac{1-\nu}{2}(y_4-y_1) & \frac{1-\nu}{2}(x_3-x_1) & -\frac{1-\nu}{2}(y_3-y_1)\end{bmatrix}\begin{Bmatrix}u_1\\ v_1\\ u_3\\ v_3\\ u_4\\ v_4\end{Bmatrix},$$

where Δ – double area of triangle.

According to the developed algorithm, calculation of steady-stress state of working medium of filling with finite-element method was conducted.

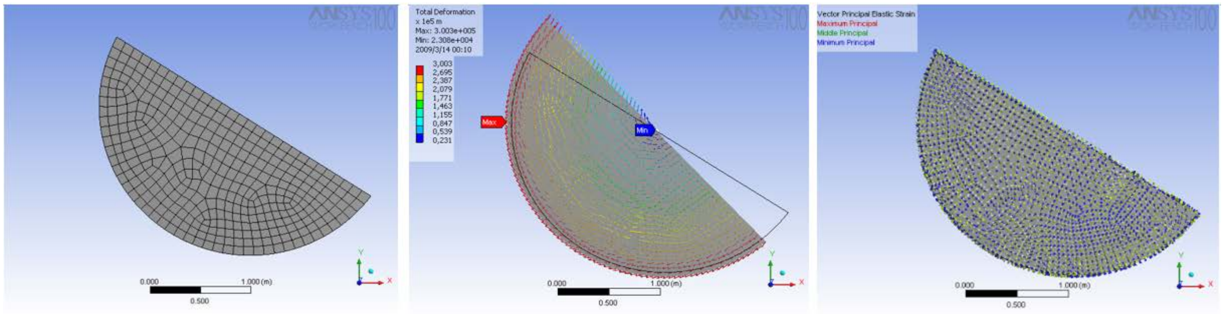


a) b) c)



d) e)

Figure 3. Results of calculation of filling steady-stress state at the start of deformation in case of κ=0.25: a - computational grid, b – displacement region, c – region of vectors of principal stresses, d - pressure field along vertical axis, e – pressure field along horizontal axis



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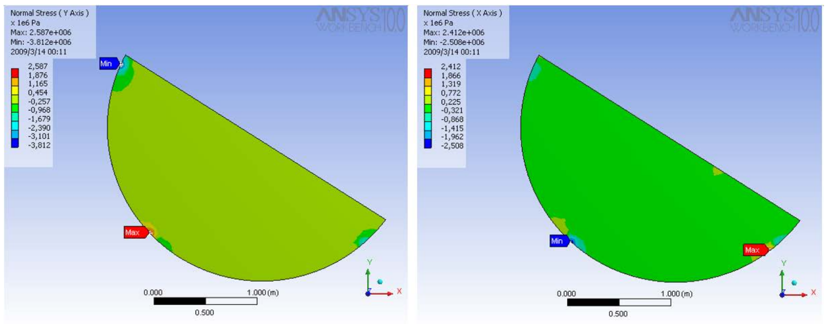

d) e)

Figure 4. Results of calculation of steady-stress state of filling at the start of deformation in case of κ=0.45 (designations according to Fig. 3)

In Figures 3-4, visual representation of calculation results in the form of computational grid, displacement region, regions of vectors of principal stresses and pressure field along vertical and horizontal axis for two patterns of filling motion is shown.

Computational patterns correspond to state of relative rest of filling for rational technological value of minimum and maximum level of chamber filling κ of drum mill. As initial parameters for averaged granular medium of filling, E = 2·10^5 MPa, ν = 0.5, bulk density of ρ = 4600 kg/m^3, R =1.5 m, average relative size of granular elements d in the chamber with diameter of D – d/D = 0.01were accepted.

The analysis of obtained fields of steady-stress state allowed confirming the proposed hypothesis of zone occurrence in the upper part cross section of filling near point A (Fig. 1), where under certain conditions, separation of granular elements from chamber surface can take place with subsequent shift or unfree falling under the influence of gravity forces. It is caused by reaching of stress in zone of the minimum value. The maximum value of stress in the central part of supporting surface of chamber causes crushing impact on the processed medium confirmed by our experiments in this zone.

Appearance of registered effect localization of zones of minimum and maximum value of pressure field had steady character and was implemented in case of different levels of chamber filling.

Conclusions

Thus, the effect of behavior of granular filling at the start of deformation in the horizontal axis of the rotating cylindrical chamber is established. It consists in occurrence of filling zone in the upper part of section of chamber with the minimum value of pressure field, where separation of elements from chamber surface and also from zone in the central part of chamber surface the maximum value of pressure takes place, while processed medium is affected by crushing.

With deformation increase, the applied elastic model becomes unsuitable for description of further behavior of medium due to instantaneous reversible breaking of rigid structure of filling after overcoming of initial ultimate stress of shift in zone near the upper part of free surface when small elastic deformations are transformed in finite inelastic ones.

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