Wave formation on the free surface of near-wall liquid layer in horizontal rotating cylindrical chamber

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Abstract
The effect of viscosity on the wave motions on free surface of the near-wall liquid layer in cylindrical chamber rotating around a horizontal axis at a constant speed is considered. Frequency equation of hydro system is analytically obtained.
It is shown that increase of viscosity have the damping impact on wave formation, stabilizes the established flow of near-wall layer and reduces the effect of regime hysteresis.
Key words: WAVE MOTION, NEAR-WALL LAYER, VISCOSITY, STEADY FLOW, FREQUENCY EQUATION, CYLINDRICAL CHAMBER, ROTATION AROUND THE HORIZONTAL AXIS, EFFECT OF REGIME HYSTERESIS

The boundaries of the fluid flow modes transition in a cylindrical chamber rotating around a horizontal axis are defined by the near-wall layer stability, which can be broken with waves formation on free surface. Therefore, the task of determining the wave motion has a significant practical value and is of particular interest for the dynamics of rotor systems [7].

Such a task, without taking into account the gravitational forces and liquid viscosity, was considered in [14]. Stability conditions of wave motion in the same formulation of the problem were obtained in [15]. The wave motion of the ideal fluid near-wall layer in the rotating around horizontal axis cylindrical chamber, taking into account gravitational forces, was studied in [13].

Work [1] describes analytically azimuthal waves on the surface of liquid layer in cylindrical chamber with infinite radius and builds approximate solution for the rotating around horizontal axis chamber of the end radius, taking into account gravitational forces.

The loss of stability of viscous liquid free surface motion, substantially filling the cylindrical rotating chamber, in view of the gravitational forces, was analytically studied in [5].

Forced wave motions of a thick liquid layer, without taking into account gravitational forces, were experimentally investigated in [16].

The problem of determining the position of viscous liquid free surface, the underlying definition of its wave motion with boundary-layer theory was solved analytically in [17], numerically and experimentally in [3] and algorithmically numerically using the method of markers and cells in [6,8].

Wave formation on free surface for the circulation one and mode of near-wall layer of liquid motion was experimentally investigated in [2]. Various disturbances on free surface of liquid in the rotating around horizontal axis cylindrical chamber were experimentally considered in work [4] and generalization and extrapolation of the known results was attempted for a wide range of the system parameters variations.

Stability of steady motion of the near-wall liquid layer depends greatly on the wave motions on free surface, which often cause layer fractures in slowly rotating cylindrical chamber.

Further analytical solution of the task of qualitative and approximate quantitative evaluation of the liquid viscosity effect on its wave motions involves preservation of problem formulation [13], but the chamber filling is considered to be viscous.

The cylindrical chamber of radius *R* with smooth end walls, partially filled with liquid and uniformly rotating with an angular velocity ω around horizontal axis perpendicular to the gravitational acceleration *g* is considered. At a sufficiently high angular velocity the liquid in chamber takes the form of the near-wall layer with an outer radius *R* and free surface radius cR $(0 \le c \le 1)$ (Fig. 1).

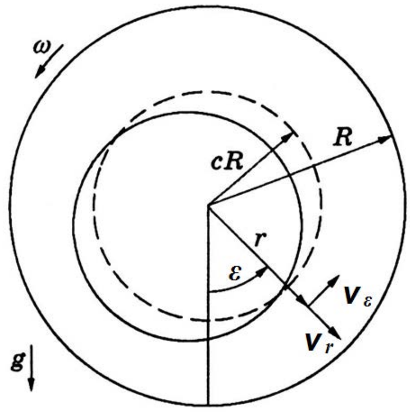

Figure 1. Computational scheme of the near-wall layer motion

The fluid motion is seen in a plane perpendicular to the axis of chamber rotation. A polar coordinate system - r, ε is introduced. The components of the fluid velocity - V_r, V_ε.

Then the equations of motion and continuity are of the form

$$\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\varepsilon}{r}\frac{\partial V_r}{\partial \varepsilon} - \frac{V_\varepsilon^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + g\cos\varepsilon +$$
$$+\nu\left(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \varepsilon^2} + \frac{1}{r}\frac{\partial V_r}{\partial r} - \frac{2}{r^2}\frac{\partial V_\varepsilon}{\partial \varepsilon} - \frac{V_\varepsilon}{r^2}\right),$$

$$\frac{\partial V_\varepsilon}{\partial t} + V_r \frac{\partial V_\varepsilon}{\partial r} + \frac{V_\varepsilon}{r}\frac{\partial V_\eta}{\partial \varepsilon} - \frac{V_r V_\varepsilon}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \varepsilon} - g\sin\varepsilon + \quad (1)$$
$$+\nu\left(\frac{\partial^2 V_\eta}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V_\varepsilon}{\partial \varepsilon^2} + \frac{1}{r}\frac{\partial V_\varepsilon}{\partial r} + \frac{2}{r^2}\frac{\partial V_r}{\partial \varepsilon} - \frac{V_\varepsilon}{r^2}\right),$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\varepsilon}{\partial \varepsilon} = 0.$$

where p - pressure, ρ - fluid density, ν - kinematic viscosity, t - time.

For the undisturbed fluid motion the uniform solid rotation in the form of near-wall layer of constant thickness in the absence of gravitational forces is accepted. At this velocity and pressure components take the values

$$V_r = 0,$$

$$V_\varepsilon = \omega r,$$

$$p = \frac{1}{2}\rho\omega^2\left(r^2 - c^2R^2\right)$$

In certain high-speed modes of camera rotation transient disturbances of the fluid velocity and pressure occur as a result of wave motions on the free surface of near-wall layer. Velocity disturbances are considered to be small as compared with ωR, and displacements of free surface - small compared with cR. The solution will be found in the area near the free surface and away from a solid wall. Therefore, the shear stresses are not taken into account, and it is considered that the pressure is determined only by gravitational and inertial forces. After replacing $\eta = r/R$ ($c \le \eta \le 1$) can be written approximately

$$V_r = \omega R V_{r0},$$
$$V_\varepsilon = \omega R\left(\eta + V_{\varepsilon 0}\right), \tag{2}$$
$$p = \frac{1}{2}\rho\omega^2R^2\left[\left(\eta^2 - c^2\right) + p_0\right] - \rho g R\eta\cos\varepsilon,$$

where V_{r0}, $V_{\varepsilon 0}$, p_0 - wave disturbances.

The boundary conditions at the solid wall are of the form

$$V_{r0} = V_{\varepsilon 0} = 0 \qquad \text{at} \qquad \eta = 1. \tag{3}$$

Suppose that on the free surface with constant pressure

$$\eta = c + \delta_0(\varphi),$$

where δ_0 – dimensionless displacement which is small compared with c. With regard to (3) after transformations and surface pressure equation to zero, the dynamic boundary condition on the free surface takes the form

$$p_0 + 2c\delta_0 = 0 \qquad \text{at} \qquad \eta = c.$$

Kinematic boundary condition on the free surface

$$V_{r0} = \frac{\partial\delta_0}{\partial\tau} + \frac{\partial\delta_0}{\partial\varepsilon} \qquad \text{at} \qquad \eta = c$$

where $\tau = \omega t$.

Permanently disturbed motion of the near-wall liquid layer was considered in [9,10,12]. Later the unsteady disturbed motion is considered.

Disturbances V_{r0} and $V_{\varepsilon 0}$ are considered to be small.

To assess the effect of viscosity on wave formation, in the terms associated with viscous diffusion $\partial V_r/\partial\varepsilon$, $\partial V_\varepsilon/\partial\varepsilon$ и V_r/r^2 are taken into account. Then the equations of disturbed motion on the basis of (1) with (2) take the form

$$\frac{\partial V_{r0}}{\partial\tau} + \frac{\partial V_{r0}}{\partial\varepsilon} - 2V_{\varepsilon 0} = -\frac{1}{2}\frac{\partial p_0}{\partial\eta} + \frac{1}{\mathrm{Re}_c}\left(-\frac{2}{\eta^2}\frac{\partial V_{\varepsilon 0}}{\partial\varepsilon} - \frac{V_{r0}}{\eta^2}\right),$$

$$\frac{\partial V_{\varepsilon 0}}{\partial\tau} + \frac{\partial V_{\varepsilon 0}}{\partial\varepsilon} + 2V_{r0} = -\frac{1}{2}\frac{\partial p_0}{\partial\varepsilon} + \frac{1}{\mathrm{Re}_c}\left(\frac{2}{\eta^2}\frac{\partial V_{r0}}{\partial\varepsilon}\right), \tag{4}$$

$$\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta V_{r0}\right) + \frac{1}{\eta}\frac{\partial V_{\varepsilon 0}}{\partial\varepsilon} = 0,$$

where $\mathrm{Re}_c = \omega\eta^2R^2/\nu$ – Reynolds number at the free surface of near-wall layer.

The solution will be sought in the form

$$V_{r0} = \chi(\eta)\exp i(n\tau + m\varepsilon),$$
$$V_{\varepsilon 0} = \xi(\eta)\exp i(n\tau + m\varepsilon), \tag{5}$$
$$p_0 = P(\eta)\exp i(n\tau + m\varepsilon),$$
$$\delta_0 = \Delta\exp i(n\tau + m\varepsilon),$$

where n and m – constants.

After substitution (5) the equation (4) can be written as

$$i(n+m)\chi + 2\xi = -\frac{1}{2}P' + \frac{1}{\mathrm{Re}_c}\left(-\frac{2im}{\eta^2}\xi - \frac{\chi}{\eta^2}\right),$$

$$i(n+m)\xi + 2\chi = -\frac{im}{2\eta}P + \frac{1}{\mathrm{Re}_c}\left(\frac{2im}{\eta^2}\chi\right), \tag{6}$$

$$\frac{\chi}{\eta}+\chi'+\frac{im}{\eta}\xi=0,$$

and boundary conditions in the form

$$\chi=0,\qquad \xi=0\quad \text{at}\quad \eta=1,$$
$$P+2c\Delta=0,\quad \chi=i(n+m)\Delta\quad \text{at}\quad \eta=c. \tag{7}$$

From the first two equations (6) after the exclusion of ξ

$$\chi=\frac{-\frac{i(n+m)}{2}\mathrm{Re}_c P'-im(\mathrm{Re}_c-im)\mathrm{Re}_c\frac{P}{\eta}}{4(\mathrm{Re}_c-im)^2+i(n+m)[i(n+m)\mathrm{Re}_c+1]\mathrm{Re}_c}. \tag{8}$$

After the transformation (6) and exclusion of ξ and χ

$$P''+a\frac{P'}{\eta}+b\frac{P}{\eta^2}=0, \tag{9}$$

where $a=1,\ b=m^2\left[\frac{i}{(n+m)\mathrm{Re}_c}-1\right]$.

The solution (9) has the form

$$P(\eta)=\eta^q[A_1\cos(S\ln\eta)+A_2(S\ln\eta)]+ +\eta^{-q}[A_1\sin(S\ln\eta)+A_2\cos(S\ln\eta)], \tag{10}$$

where $q\pm iS=\gamma,\lambda;\ \gamma+\lambda=1-a;\ \gamma\lambda=b$; in which case

$$q=d\cos\left(\frac{\alpha}{2}\right),$$

$$S=d\sin\left(\frac{\alpha}{2}\right),$$

$$d=m\left[1+\frac{1}{(n+m)^2\mathrm{Re}_c}\right]^{\frac{1}{4}},$$

$$\alpha=arctg\left[1-\frac{1}{(n+m)\mathrm{Re}_c}\right].$$

Considering Re_c>>m (8) is simplified

$$\chi=\frac{-\frac{i(n+m)}{2}P'-im\frac{P}{\eta}}{4-(n+m)^2}. \tag{11}$$

From the first condition (7), (10) and (11) it follows

$$\frac{A_1}{A_2}=-\frac{(n+m)(S-q)+2m}{(n+m)(S+q)+2m}. \tag{12}$$

Then, on the basis of (10), (11), (4) and (12) after transformations the frequency equation can be written as

$$\begin{aligned}&(n+m)^4\{(S+q)-(S-q)B+[(S+q)B-(S-q)]c^{2q}\}+\\&+(n+m)^3 2m(1-B)(1-c^{2q})-\\&-(n+m)^2\{(S+q)[(4+SB+q)+(4B-S-qB)c^{2q}]-\\&-(S-q)[(4B-S+qB)+(4+SB-q)c^{2q}]\}-\\&-(n+m)4m\{[2-(2-S+q)B]-[2-(2-S-q)B]c^{2q}\}+\\&+4m^2(1-B)(1-c^{2q})=0,\end{aligned} \tag{13}$$

where $B=tg(S\ln c)$..

According to (11), the equation (13) has radicals at $(n+m)^2\neq 4$.

At $v\to 0$ ($Re_c\to\infty$) the equation (13) takes the form obtained in [13]

$$n^2(1+c^{2m})+2n[(1+m)-(1-m)c^{2m}]+ +m[(1+m)-(1-m)c^{2m}]=0. \tag{14}$$

In Fig. 2 in axes *c* and *n* the solutions (13) are shown at R=0,075 m, m=1 and at $\nu=10^{-6}$-10^{1} m^2/s the solution (13) is almost the same as (14).

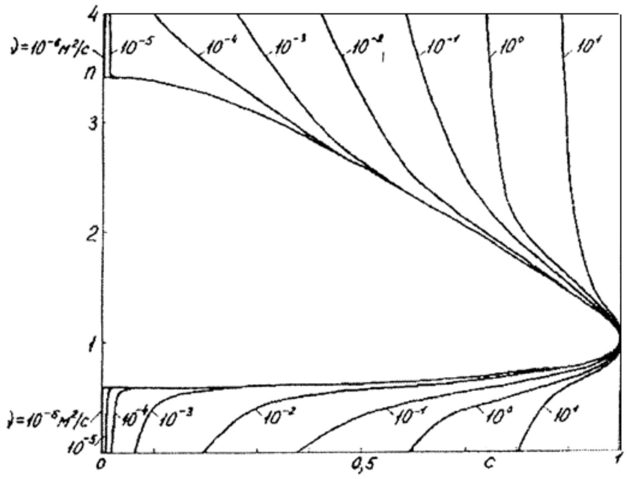

Figure 2. The solution of equation (13) at m=1

With the increase in viscosity or decrease in the Reynolds number Re_c, the frequency of low-frequency waves, which accompany the destruction of near-wall layer is reduced in the limit of zero, and the frequency of high-frequency, relatively rarely implemented [13], is greatly increased in the limit to infinity, and the process of wave formation fades. In this case, the disruption of liquid layer, while reducing the chamber rotation speed is not due to wave formation, but due to significant deformation of free surface in a steady motion, caused by the development of secondary circulation currents in the form of roller *ab* on the free surface (Fig. 3) [5].

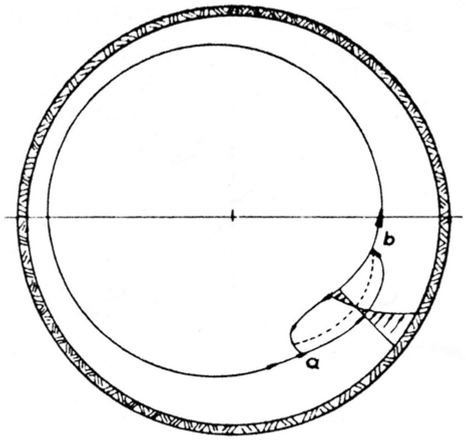

Figure 3. The scheme of the secondary circulation flow development in the form of roller at the free surface of near-wall layer

Wave formation on the free surface of near-wall liquid layer affects the display of registered effect of the steady flow regime hysteresis in the cylindrical chamber rotating around a horizontal axis [11]. This effect is in the excess of the rotation speed ω1 chamber magnitude (Fig. 4), at the transition of flow circulation mode in the near-wall layer mode during rotation acceleration, with the velocity magnitude *ω2*, at the reverse transition of modes, during rotation deceleration.

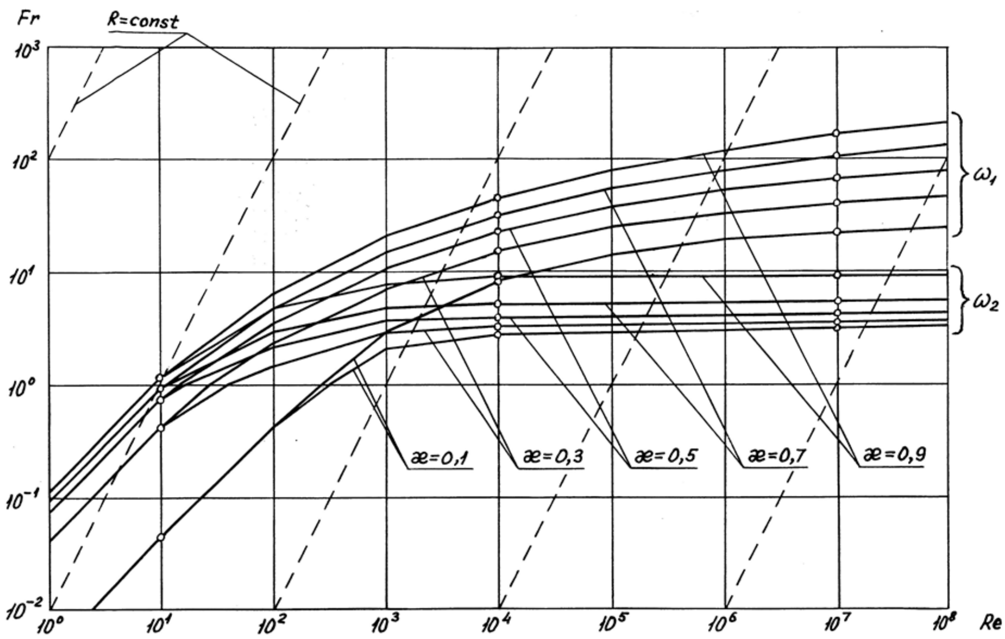

Figure 4. Universal diagram (in axes the Reynolds number $Re=\omega R^2/\nu$ and Froude number $Fr=\omega^2 R/g$ on the cylindrical surface of chamber) of fluid flow modes transition when the degree of chamber filling κ=0.1, 0.3, 0.5, 0.7 and 0.9:
1 - border of the circulation mode transition into the near-wall layer one during rotation acceleration; 2 - border of the reverse transition of modes during rotation deceleration

The effect of regime hysteresis appears at high Reynolds numbers Re, when the flow regimes transition is pronounced, and the destruction of near-wall layer, at the slowing of camber rotation - abrupt in nature and caused by wave formation at free surface. At the same time, this effect does not occur at low Re, when flow modes transitions are implicitly marked, - smooth in nature and caused by secondary flows at the absence of wave formation.

Conclusions

Thus, the increase in viscosity has a significant damping effect on the wave formation on free surface, and stabilizing effect on the steady flow of near-wall liquid layer, which reduces the effect of mode hysteresis.

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