Application of stochastic point processes for modelling of pitting

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Abstract
The problem of modeling of distribution of pitting corrosion is considered. New stochastic model which reflects processes of origin and depth growth of pittings is suggested. Its characteristic feature is considering of mutual influence of neighbor pittings. Origin of pittings is concerned as non-uniform Poisson process, their growth in depth - as non-uniform Markov process. Distribution of pittings on sample surface is modelled by Gibbs point process with interaction function in the form of Fiksel. Comparisons between real and simulated models of damages by pitting corrosion of surface fragment are given.
Key words: PILLING CORROSION, STOCHASTIC MODEL, POINT PROCESS

Introduction

Stochastic models of pitting corrosion have advanced and became a subject of modern researches owing to occurrence of stochastic approach to the analysis of pitting corrosion, and also, as a result, more opportunities for the description of pitting corrosion development, in particular the forecast of the maximum depth of pitting in metal [1,2]. The assessment of through damage of industrial equipment in use by pittings is the main task of its reliability analysis. Depth of corrosion pittings depends on the speed of corrosion and time of their origination. Therefore, there is a need to determine the size of the deepest pitting of corrosive structure on the basis of limited quantity of samples obtained from the general population of corrosion pittings. Approaches on the basis of distributions of extreme sizes and modeling of growth of pitting depth by stochastic processes are used for this purpose [3-7].

Known models of pitting growth

In work [5] the process of pitting growth is considered as non-uniform in time Markov process. By means of this model the growth of pitting is presented as function of time taking into account discrete conditions *j (j = 1,2..., n)* of pitting depth. Such process satisfies Kolmogorov direct equation [10]:

$$\frac{dp_{ij}(t)}{dt} = -q_j p_{ij}(t) + q_{j-1} p_{ij-1}(t) \qquad (1)$$

For probabilities p_{ij} of change from *i* state to *j* state. Intensity functions of the process q_j are suggested to be chosen as follows

$$q_j(t) = jq\frac{1+qt}{1+qt^{\kappa}}, \qquad (2)$$

where *q*, *κ* – parameters of pitting corrosion system. Unfortunately the authors haven't provided physical explanation of expression (2), and also there is no instructions how to apply expression (1) for determination of the maximum depth of pitting for the set of pittings at simultaneous origination or in different timepoints.

In work [6] the model that combines two casual processes - origination of pitting and growth of its depth is presented. Origination of pitting is modelled by Poisson's process, and growth of depth - by Markov process. According to the suggested model, probability that the maximum depth of pitting corresponds to condition *i* at time *t* is possible to estimate as:

$$P_H(i,t) = exp\left(-vt^{\beta}\left(1-\frac{1}{wt^{\beta}}\sum_{j=1}^{i}\frac{\gamma(j,wt^{\beta})}{(j-1)!}\right)\right), i = 1,\cdots,n-1, \qquad (3)$$

where v– pitting origination rate, w – rate of growth and β – model parameters that should be estimated, $\gamma(i,t)$ – incomplete gamma- function.The fact that distribution of probability of the maximum depth of pitting according to the expression (3) doesn't coincide with any of the known distributions of extreme values may be referred to disadvantages of suggested model.

More perfect model, which combines processes of pittings origin and growth, is presented in [7]. For the case of one pitting, probability of its achievement of depth of a discrete condition *i* during time $(t-t_k)$ from the moment of origin t_k is defined as:

$$P(i,t-t_k) = 1-\{1-exp[-\rho(t-t_k)]\}^i, \rho(t) = \int_0^t \mu(\tau)d\tau, \qquad (4)$$

where $\mu(\cdot)$ – is the frequency of transition of value of pitting depth from certain state to the next one during one moment. For set *g* of pittings, when all pittings arise at the same time, the probability that the deepest pitting will reach in time *t* the condition of smaller or equal *i* from the moment of origination $t_k = t_0$ is equal to:

$$P_S(i,t) = \{1-\{1-exp[-\rho(t-t_0)]\}^i\}^g. \qquad (5)$$

When pitings arise in different time points t_k the probability that the deepest pitting will reach at time *t* the condition of smaller or equal *i* from the moment of origination t_k:

$$P_T(i,t) = \prod_{k=1}^{g}\left\{1-\{1-exp[-\rho(t-t_k)]\}^i\right\} \qquad (6)$$

Thus, suggested models allow to establish probability of achievement by separate pitting of the maximum depth, which corresponds to certain discrete condition from final set of states. It is possible to notice that among parameters of suggested models there are no parameters, which would describe interaction of pittings during corrosion, however on the basis of experimental observation it is known that development of pittings, namely increase in their quantity, has frank logarithmic character and practically stops from a certain time point [8-10]. The researches indicating mutual influence of pittings, which leads to suppression of their growth and to passivation [11-13], are known.

It may be assumed that one of the main parameters that defines extent of pittings interaction is the distance between them. A convenient form for the description and determination of such interaction is point patterns or point fields representing the realization of certain casual process in space $R^d, d \geq 2$ in some time point t [14] fig. 1b.

General model of a piting

Pitting corrosion develops both in space and in time. To each pitting there corresponds a number of characteristics: size of visible opening, depth, stable or metastable, etc. In this case as general model, which would allow to describe completely development of pitting corrosion, it is reasonable to choose spatial-temporal marked point process, realization of which is the point pattern. Marked point processes allow to connect markers of the points describing properties of the objects presented by points and can be both dependent on a position of points and not connected with them. In other words the marked point process X_M represents the sequence of casual marked points $X_M = \{x_n(t); m(x_n(t))\}$, where $m(x_n(t))$ - is a marker of $x_n \in W$ point, W – is a limited subset R^2 or R^3, M – is the space of markers. Let us define: $\Gamma = W \times M = \{(\omega_1, \omega_2) : \omega_1 \in W, \omega_2 \in M\}$.

Except Poisson's point processes, non-uniform Markov point processes or, so-called, point processes of Gibbs are more general models [15, 16]. Such processes allow to consider interaction between objects at the different levels - from the elementary pairwise interaction to interaction between connected components that form process objects.

Let us $(x,m) = (x_1,\ldots,x_n,m_1,\ldots,m_n)$, in case of pair interaction between process elements density of the limited marked dot process of Gibbs has an appearance:

$$f(x,m) = \frac{exp[-U(x,m)]}{Z}, \quad (7)$$

where potential or function energy $U(x, m)$ is registered as follows:

$$U(x,m) = \sum_{s=1}^{n} V^{(1)}(x_s,m_s) + \sum_{s=1}^{n-1}\sum_{r=s+1}^{n} V^{(2)}(x_s,x_r,m_s,m_r),$$

and normalizing multiplier Z as:

$$Z = \sum_{n=0}^{\infty} \frac{e-|W|}{n!} \int_\Gamma \ldots \int_\Gamma exp[-U(x)]\, d(x_1,m_1)\ldots d(x_n,m_n), \quad (8)$$

$V^{(1)}(x_s,m_s) = -log[\lambda(x_s)p(m_s/x_s)]$, $\lambda(x)$– driving rate in the point x, $p(\cdot/x)$– distribution of marker values in the point x. $V^{(2)}(x_s,x_r,m_s,m_r) = \phi(x_s,x_r,m_s,m_r)$ – potential function of pair of markers (m_s,m_r) located in the points (x_s,x_r). Intensity function $\lambda(x_s)$ is depen- dent on time, as pittings are formed not simultaneously, so in general it will look as follows

$$\lambda(x_s) = \lambda_t(x_s) = \int_W \int_{t_0}^{t} \lambda(t,x_s)dtdx.$$

To determine the density of probability of conditional distribution of markers of point process $p(m_s/x_s)$ we will use the expression (4). As we see it is connected with $\mu(\cdot)$ - the frequency of transition of pitting depth from certain state to the next one during the moment. It is reasonable to connect this frequency with location of pitting, namely with properties of sample surface in the place of its formation. It is known that places of pittings origination are the inclusions of various phases on material surface that appear, for example, in thinning of protective oxide film. It is obvious that the probability of pitting origin and growth will be stronger in the area where concentration of inclusions is greater. If to define the random field $S(x)$ for the nonfailed sample, for which the value in each point is the total amount, area or volume of inclusions in area B of a radius r of point x, then:

$$S(x) = N(B+x),\ S(x) = \sum_{[x_i;m_i]\in A} 1_B(x-x_i)m_i, \quad (9)$$

where m_i – area or volume of inclusions. It is possible to suggest the following dependence between $\mu(\cdot)$ and $S(x)$ for the case when $S(x)$ – the area of inclusions around the point x:

$$\mu(t,x) = 1 - exp\left(-\sqrt{\frac{S(x)}{\pi}}\right).$$

Fiksel [17] in 1984 proposed the model of point process of pair-wise interaction, potential function for which looks as follows:

$$c(x_s,x_r) = \begin{cases} 1, \|x_s - x_r\| > \varepsilon \\ a * exp(-\kappa * \|x_s - x_r\|), h \leq \|x_s - x_r\| \leq \varepsilon, \\ 0, \|x_s - x_r\| < h \end{cases} \quad (10)$$

h – is the minimum distance of interaction, ε - indicates the range of interaction of elements if the distance is more than ε, then the elements don't interact, a – is a constant, which defines a type of process: $a = 0$ - Poisson process, $a>0$ – is a cluster process, a<0 – is a process of limited interaction, κ – is the the parameter, which defines the nature of reduction of interaction with increase in distance between process elements. As shown in work [18] application of Fiksel model allows to generate point images, which are brought closer to the laboratory ones obtained during experiments.

To show the dependence between mutual influence of pittings and their characteristics, it is suggested to express κ parameter in the expression (10) through the markers indicating depths of corresponding pittings, namely:

$$\kappa = \left(2 * (m_s + m_r)\right)^{-1}, m_s \in [0,1] \qquad (11)$$

If we consider system from two stable pittings, where depth is more than a half of metal thickness $m_s + m_r > 1$, then their mutual influence is expressed in corrosion process acceleration, then the value of interaction function will increase, and herein we consider that thickness of nonfailed metal is expressed as one. If among interacting only one of the pittings is rather deep, that is $m_s + m_r \approx 0.5$, then interaction doesn't change that is expressed in almost invariable value of interaction function of expression (10). When considering interaction of two fleet pittings when $m_s + m_r \leq 0.5$ variable κ accepts the value bigger than one, that will lead to reduction of value of interaction function. Such treatment of interaction function reflects the running process of pitting corrosion, as it is known from researches [12], pittings having reached a certain critical depth continue growing steadily that corresponds to the first case of existence of two stable pittings. The second case corresponds to existence of one stable pitting, the influence of which can lead to repassivation of neighbor metastable pittings. Also in the third case, when the depth of pittings is small, they can repassivate or turn into metastable ones that will lead to weakening of corrosion damage.

Now on the basis of expressions (4) - (11) we can obtain the following general view for density function of probability of the marked point process of pitting corrosion in some area of A sample sets of pittings x with m depths after t time from the moment of the beginning of corrosion process. Values t, x, m are the vectors of values.

$$f(x,m,t) = \left. \begin{matrix} \sum_{k=1}^{g} \left\{ \lambda(t - t_0, x_k)\left(1 - \{1 - exp[-\rho(t - t_k)]\}^{m_k}\right) \right\} * \\ * \, exp\left(a \cdot \sum_{s=1}^{g-1} \sum_{r=s+1}^{g} exp\left(-\frac{1}{2(m_s + m_r)} * \|x_s - x_r\| \right) \right) \end{matrix} \right/ Z \qquad (12)$$

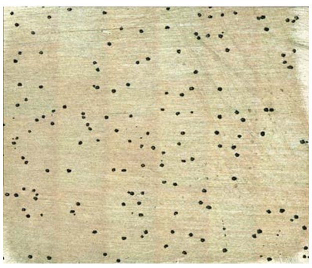
Experimental results

The software on the basis of the Monte-Carlo method was developed for verification of the suggested model (12). Generally it is impossible to use directly Monte-Carlo method through non integrated normalization constant Z in the expression (8). A number of ways, which solve this problem by approximating such multiplier using additional random variables that belong to the same probabilistic space as the main value of distribution function[19], is suggested.

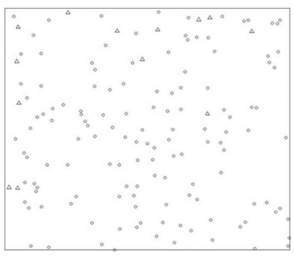
For effective application of model (12) it is necessary to define or construct the parameters estimators, which will allow to verify the constructed model. For this purpose a series of corrosion experiments has been carried out, each of which contained n_c samples of material that was affected by corrosion environment during certain time (Fig. 1a). After that depth of all pittings was measured and the maximum ones were defined. Distribution of maximum sizes was modelled by Gumbel distribution, and distribution of their arrangement - by distribution of Fiksel (Fig. 1c). Determination of parameters for expression (12) occurred due to minimization of function of the general error which can be expressed as:

$$E_T = \sum_{i=1}^{N_t} \left(\sqrt{\left(\mu_e^i - \mu_p^i\right)^2} + \sqrt{\left(\sigma_e^{2i} - \sigma_p^{2i}\right)^2} \right), \qquad (13)$$

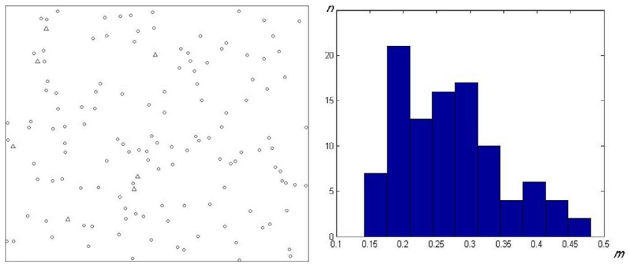
where (μ_e^i, σ_e^{2i}) and (μ_p^i, σ_p^{2i}) are the average value and dispersion of values for experimental and estimated according to (12) distributions obtained during the i-th experimen. The minimum of expression (13) was determined by Monte-Carlo method with the number of iterations N_{MC}= 999.



a)



b)



c) d)

Figure 1. Results of modeling of pitting on the base of the parameters obtained from real data: a) image of surface; b) point image of surface; c) model point image; d) distribution of the maximum depths received on the base of expression (12)

The carried-out modeling allowed to establish correspondence of the results obtained on the basis of suggested model, with real images of surface damaged by pitting corrosion with known depths. In figure 1b and 1c point images for real and model damage by pitting are presented, where depths of pittings are spesified by different markers. For pittings of up to 100 microns in depth – rings, and more than 100 microns – triangles. As is seen from the given example, the model suggested allows to form images which rather precisely correspond to real. Constructed empirical distribution of the maximum pitting depths obtained on the basis of the suggested model and presented in Fig.1d corresponds to Gumbolt distribution according to Kolmogorov-Smirnov criterion [18].

Conclusions

Development of stochastic models for an assessment of maximum depth of pittings with the use of information on their relative position is suggested. On the base of established correspondence between places of pitting origin in the area and occasional point process with function of interaction in the form of Fiksel, there specified the function of density of distribution of occasional process, which describes origin and development of pitting corrosion on the surface and allows to estimate its maximum depth.

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