Kinematic analysis of the piston-type pump mechanisms by numerical methods

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Abstract
For determination of the key geometrical and kinematic parameters of the horizontal two-cylinder double-acting piston-type pump, the analytical and numerical methods are considered by the available MathCAD software product.
The method of the closed vector path is used for investigation of mechanisms of the piston-type pump.
Key words: DRILL PUMP, KINEMATIC SCHEME, VECTOR PATH, NUMERICAL METHOD, NUMERICAL DIFFERENTIATION

Drill pumps are used as equipment for ensuring technological processes of drilling of oil and gas wells. Wide use of drill pumps in oil and gas branch imposes raise requirements to their characteristics; in turn, they affect significantly the maintenance of modes and providing continuity of technological process.

Improvement of characteristics of the drill pump leads to improvement of qualitative and quantitative characteristics of technological process that results in improvement of technical and economic indicators of branch in general.

It is possible to improve characteristics of the drill pump due to improvement of design of mechanical as well as hydraulic part, and also optimization of operating modes. For improvement of design, choice of optimum operating modes, and also the analysis of stress state of drill pump details, it is necessary to carry out kinematic, dynamic and power analyses of the mechanism; they can be carried out by three basic methods, namely graphic, analytical and numerical. Each of this methods has the advantages and disadvantages. With development of mathematical software products (*MathCAD*, *Mathematica*, *Maple*, etc.), there appeared an opportunity to apply analytical and numerical methods for investigation of mechanisms.

One of the most widespread and available software products, which is used for analysis of mechanisms, is the *MathCAD* package of *Mathsoft* company. There are a large number of publications on the subject of wide use of *MathCAD* package for the analysis of mechanisms. So, in the papers [4] and [2], the analysis crank mechanism of the pump is carried out by an analytical method. In paper [3], some ways of analytical solution of vector equations of kinematics and dynamics of second class mechanisms are traversed.

The powerful computing Given-Find block in a *MathCAD* package allows solving of nonlinear equations systems by numerical method. In papers [5] and [6], ways of carrying out the kinematic analysis by numerical method by means of the computing Given-Find block in *MathCAD* package are described.

In this article, the algorithm of kinematic analysis of mechanism of the horizontal two-cylinder double-acting piston-type pump (UNB-600 series) by numerical methods in *MathCAD* package is described. One of features of suggested algorithm is that the analysis is carried out for *n* positions of mechanism, angular coordinates of piston-rods 2 and 4 and coordinates of the centers of mass of rams 3 and 5 by means of the computing Given-Find block are determined by method of numerical solution of the equations of the closed vector paths; and kinematic characteristics are obtained by numerical differentiation of corresponding geometrical parameters. Thus, the suggested algorithm allows simplifying mathematical calculations and optimizing a program code.

The kinematic scheme of the mechanism of horizontal two-cylinder double-acting piston-type pump is shown in Figure 1.

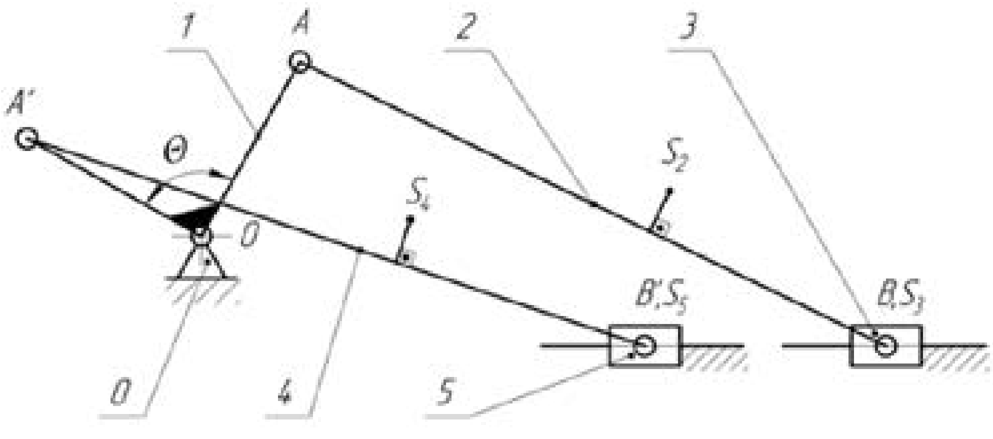

Figure 1. Kinematic scheme of mechanisms of the piston-type pump

0 - stand pipe, 1 - crank, 2, 4 - piston-rods, 3, 5 – rams

Generally, the sequence of the analysis is accepted as follows:

1) the closed vector paths are enhanced in the kinematic scheme;

2) the equations of the enhanced closed vector paths are worked out;

3) the complex vector equations for *n* mechanism positions (unknown geometrical parameters are determined), and results of calculations are registered to the corresponding arrays (each geometrical parameter is certain single-dimension arrays with dimension number *n*) are solved by numerical method;

4) coordinates of the centers of piston-rods mass are determined using results of calculations carried out according to 3rd point and are registered to the corresponding arrays;

5) the obtained dependences and values of the corresponding kinematic characteristics (analogs of speeds and accelerations) are differentiated in number, and results are registered to the corresponding arrays.

The calculation will be performed in the program medium *MathCAD*.

Let us compose the equations of these closed vector paths.

For $OABCO$ (see Figure 2), we obtain

$$\vec{l}_1 + \vec{l}_2 = \vec{d} + \vec{x}_B \cdot \quad (1)$$

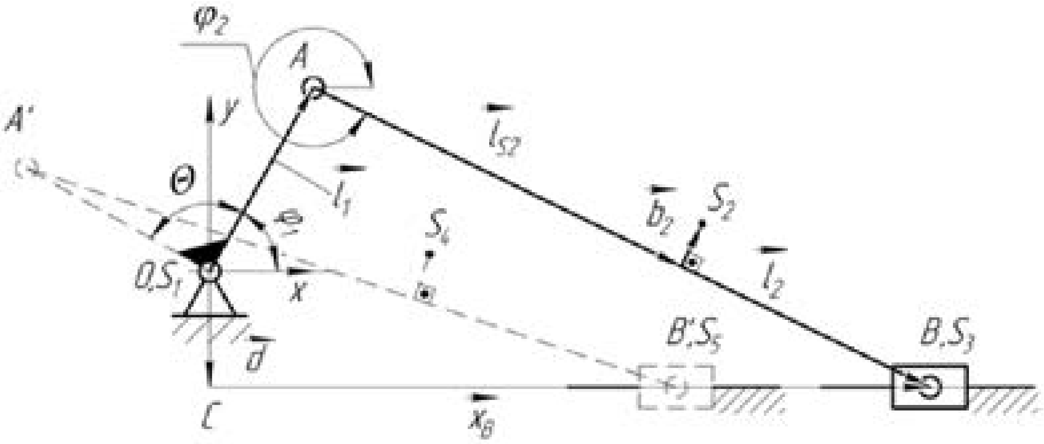

Figure 2. The calculation model for vector path $OABCO$

The vectors of equality (1) in *MathCAD* package can be presented in the form

$$\vec{l}_1 = \begin{pmatrix} l_1 \cos\varphi_1 \\ l_1 \sin\varphi_1 \\ 0 \end{pmatrix}, \vec{l}_2 = \begin{pmatrix} l_2 \cos\varphi_2 \\ l_2 \sin\varphi_2 \\ 0 \end{pmatrix}, \vec{d} = \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}, \vec{x}_B = \begin{pmatrix} x_B \\ 0 \\ 0 \end{pmatrix}.$$

Then the vector equation (1) is registered as follows

$$\begin{pmatrix} l_1 \cos\varphi_1 \\ l_1 \sin\varphi_1 \\ 0 \end{pmatrix} + \begin{pmatrix} l_2 \cos\varphi_2 \\ l_2 \sin\varphi_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} + \begin{pmatrix} x_B \\ 0 \\ 0 \end{pmatrix}, \quad (2)$$

where l_1, l_2, d – the known geometrical parameters of the pump (design parameters), namely, length of crank 1, length of rod 2 and shift of ram axis about the center of crank rotation respectively; φ_1 – independent variable, angular coordinate of a crank 1; φ_2 – unknown geometrical parameter, angular coordinate of rod 2; x_B – unknown geometrical parameter, coordinate of ram 3.

Let us do the same for vector path (see Figure 3).

Consequently,

$$\vec{l}_1' + \vec{l}_4 = \vec{d} + \vec{x}_{B1}.$$

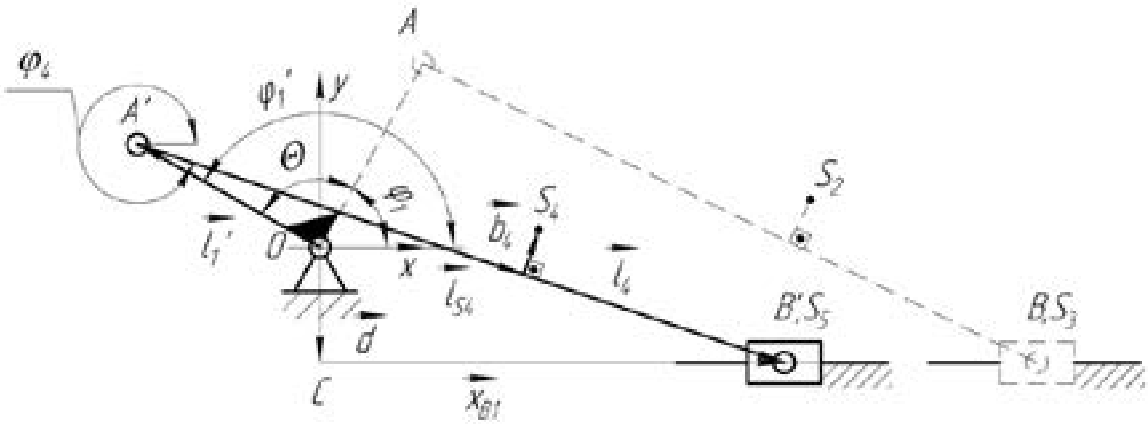

Figure 3. The calculation model for vector path $OA'B'CO$

As well as in case of vector path, the corresponding vectors in *MathCAD* package can be presented in the form

$$\vec{l}_1'=\begin{pmatrix} l_1'\cos\varphi_1' \\ l_1'\sin\varphi_1' \\ 0 \end{pmatrix}, \vec{l}_4=\begin{pmatrix} l_4\cos\varphi_4 \\ l_4\sin\varphi_4 \\ 0 \end{pmatrix}, \ \vec{d}=\begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}, \vec{x}_{B1}=\begin{pmatrix} x_{B1} \\ 0 \\ 0 \end{pmatrix}.$$

Thus, vector equality (3) will be the following:

$$\begin{pmatrix} l_1'\cos\varphi_1' \\ l_1'\sin\varphi_1' \\ 0 \end{pmatrix}+\begin{pmatrix} l_4\cos\varphi_4 \\ l_4\sin\varphi_4 \\ 0 \end{pmatrix}=\begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}+\begin{pmatrix} x_{B1} \\ 0 \\ 0 \end{pmatrix} \quad (4)$$

For design of the horizontal two-cylinder double-acting piston-type drill pump, we can write down the following

$$\begin{cases} \left|\vec{l}_1\right|=\left|\vec{l}_1'\right|=l_1 \\ \left|\vec{l}_2\right|=\left|\vec{l}_4\right|=l_2 \\ \theta=\dfrac{\pi}{2}, \varphi_1'=\varphi_1+\theta=\varphi_1+\dfrac{\pi}{2} \end{cases} \quad (5)$$

Taking into account (5), vector equation (4) will be of the form

$$\begin{pmatrix} l_1\cos\left(\varphi_1+\dfrac{\pi}{2}\right) \\ l_1\sin\left(\varphi_1+\dfrac{\pi}{2}\right) \\ 0 \end{pmatrix}+\begin{pmatrix} l_2\cos\varphi_4 \\ l_2\sin\varphi_4 \\ 0 \end{pmatrix}=\begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}+\begin{pmatrix} x_{B1} \\ 0 \\ 0 \end{pmatrix}.$$

After elementary transformations we obtain

$$\begin{pmatrix} -l_1\sin\varphi_1 \\ l_1\cos\varphi_1 \\ 0 \end{pmatrix}+\begin{pmatrix} l_2\cos\varphi_4 \\ l_2\sin\varphi_4 \\ 0 \end{pmatrix}=\begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}+\begin{pmatrix} x_{B1} \\ 0 \\ 0 \end{pmatrix},$$

where φ_4 – unknown geometrical parameter; angular coordinate of piston-rod 4; x_{B1} – unknown geometrical parameter, coordinate of ram 5. As a result of transformations, the system of vector equations will be of the form

$$\begin{cases} \begin{pmatrix} l_1\cos\varphi_1 \\ l_1\sin\varphi_1 \\ 0 \end{pmatrix}+\begin{pmatrix} l_2\cos\varphi_2 \\ l_2\sin\varphi_2 \\ 0 \end{pmatrix}=\begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}+\begin{pmatrix} x_B \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} -l_1\sin\varphi_1 \\ l_1\cos\varphi_1 \\ 0 \end{pmatrix}+\begin{pmatrix} l_2\cos\varphi_4 \\ l_2\sin\varphi_4 \\ 0 \end{pmatrix}=\begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}+\begin{pmatrix} x_{B1} \\ 0 \\ 0 \end{pmatrix} \end{cases}$$

Block of calculation of geometrical parameters

$l_1 := 0.2$ $\quad l_2 := 1.190$ $\quad l_{S2} := 0.125\cdot l_2$ $\quad l_{S4} := 0.125\cdot l_2$ $\quad b_2 := 0$ $\quad b_4 := 0$

$d := 0.0$ $\quad \omega_1 := \dfrac{\pi\cdot 65}{30}$ $\quad \phi_0 := \pi$ $\quad n1 := 12$ $\quad n := 0,1..n1$ $\quad \phi01(n) := \phi_0 - n\cdot\dfrac{\pi}{6}$

$$\phi_1(n) := \begin{vmatrix} \phi01(n) \ \text{if} \ \phi01(n) > 0 \\ \phi01(n) + 2\cdot\pi \ \text{otherwise} \end{vmatrix}$$

$\phi_2 := 0$ $\quad \phi_4 := 0$ $\quad x_B := 0.4$ $\quad x_{B1} := 0.4$

Given

$$\begin{pmatrix} l_1\cdot\cos(\phi_1(n)) \\ l_1\cdot\sin(\phi_1(n)) \\ 0 \end{pmatrix}+\begin{pmatrix} l_2\cdot\cos(\phi_2) \\ l_2\cdot\sin(\phi_2) \\ 0 \end{pmatrix}=\begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix}+\begin{pmatrix} x_B \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{-\pi}{2} \le \phi_2 \le \frac{\pi}{2} \qquad x_B \ge 0$$

$$\begin{pmatrix} -l_1 \cdot \sin(\phi_1(n)) \\ l_1 \cdot \cos(\phi_1(n)) \\ 0 \end{pmatrix} + \begin{pmatrix} l_2 \cdot \cos(\phi_4) \\ l_2 \cdot \sin(\phi_4) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -d \\ 0 \end{pmatrix} + \begin{pmatrix} x_{B1} \\ 0 \\ 0 \end{pmatrix}$$

$$\frac{-\pi}{2} \le \phi_4 \le \frac{\pi}{2} \qquad x_{B1} \ge 0$$

$$Rez\_Geom(n) := Find(\phi_2, \phi_4, x_B, x_{B1})$$

$$\phi_2(n) := Rez\_Geom(n)_0 \qquad \phi_4(n) := Rez\_Geom(n)_1$$

$$x_B(n) := Rez\_Geom(n)_2 \qquad x_{B1}(n) := Rez\_Geom(n)_3$$

Figure 4. Document fragment of MathCAD with code for calculation of geometrical parameters

The result of the numerical solution are registered to the array $Rez\_Geom(n)_i$. Here, n – position of crank; the index i = 0 corresponds to angular coordinate φ_2; i = 1 – corresponds to angular coordinate φ_4; i = 2 – to angular coordinate x_B , i = 3 – to coordinate x_{B1} respectively.

Supporting by the 4th point of sequence of the analysis for determination of coordinates of the centers of S_2 and S_4 rods mass, the following vector equations are worked out:

$$\vec{r}_{S2} = \vec{l}_1 + \vec{l}_{S2} + \vec{b}_2 , \tag{7}$$

$$\vec{r}_{S4} = \vec{l}_1{}' + \vec{l}_{S4} + \vec{b}_4 . \tag{8}$$

The vectors of the equations (7), (8) in *MathCAD* package are presented in the form

$\vec{r}_{S2} = \begin{pmatrix} x_{S2} \\ y_{S2} \\ 0 \end{pmatrix}, \vec{r}_{S4} = \begin{pmatrix} x_{S4} \\ y_{S4} \\ 0 \end{pmatrix}$ – radiuses-vectors of the centers of rods 2 and 4 mass respectively;

$$\vec{l}_1 = \begin{pmatrix} l_1 \cos\varphi_1 \\ l_1 \sin\varphi_1 \\ 0 \end{pmatrix}, \vec{l}_1{}' = \begin{pmatrix} -l_1 \sin\varphi_1 \\ l_1 \cos\varphi_1 \\ 0 \end{pmatrix}, \vec{l}_{S2} = \begin{pmatrix} l_{S2} \cos\varphi_2 \\ l_{S2} \sin\varphi_2 \\ 0 \end{pmatrix}, \vec{l}_{S4} = \begin{pmatrix} l_{S4} \cos\varphi_4 \\ l_{S4} \sin\varphi_4 \\ 0 \end{pmatrix};$$

$$\vec{b}_2 = \begin{pmatrix} b_2 \cos\left(\varphi_2 + \frac{\pi}{2}\right) \\ b_2 \sin\left(\varphi_2 + \frac{\pi}{2}\right) \\ 0 \end{pmatrix} = \begin{pmatrix} -b_2 \sin\varphi_2 \\ b_2 \cos\varphi_2 \\ 0 \end{pmatrix};$$

$$\vec{b}_4 = \begin{pmatrix} b_4 \cos\left(\varphi_4 + \frac{\pi}{2}\right) \\ b_4 \sin\left(\varphi_4 + \frac{\pi}{2}\right) \\ 0 \end{pmatrix} = \begin{pmatrix} -b_4 \sin\varphi_4 \\ b_4 \cos\varphi_4 \\ 0 \end{pmatrix}.$$

Thus, the vector equations (7), (8) in *MathCAD* package will be of the form:

$$r_{S2}=\begin{pmatrix} l_1\cos\varphi_1 \\ l_1\sin\varphi_1 \\ 0 \end{pmatrix}+\begin{pmatrix} l_{S2}\cos\varphi_2 \\ l_{S2}\sin\varphi_2 \\ 0 \end{pmatrix}+\begin{pmatrix} -b_2\sin\varphi_2 \\ b_2\cos\varphi_2 \\ 0 \end{pmatrix}$$

$$r_{S4}=\begin{pmatrix} -l_1\sin\varphi_1 \\ l_1\cos\varphi_1 \\ 0 \end{pmatrix}+\begin{pmatrix} l_{S4}\cos\varphi_4 \\ l_{S4}\sin\varphi_4 \\ 0 \end{pmatrix}+\begin{pmatrix} -b_4\sin\varphi_4 \\ b_4\cos\varphi_4 \\ 0 \end{pmatrix}$$

From formulas (9) and (10), unknown coordinates of the centers of mass of rods 2 and 4 x_{S2}, y_{S2}, x_{S4} and y_{S4} are determined.

The fragment document *MathCAD* with code is presented in Figure 5.

$$r_{S2}(n):=\begin{pmatrix} l_1\cdot\cos(\phi_1(n)) \\ l_1\cdot\sin(\phi_1(n)) \\ 0 \end{pmatrix}+\begin{pmatrix} l_{S2}\cdot\cos(\phi_2(n)) \\ l_{S2}\cdot\sin(\phi_2(n)) \\ 0 \end{pmatrix}+\begin{pmatrix} -b_2\cdot\sin(\phi_2(n)) \\ b_2\cdot\cos(\phi_2(n)) \\ 0 \end{pmatrix}$$

$$r_{S4}(n):=\begin{pmatrix} -l_1\cdot\sin(\phi_1(n)) \\ l_1\cdot\cos(\phi_1(n)) \\ 0 \end{pmatrix}+\begin{pmatrix} l_{S4}\cdot\cos(\phi_4(n)) \\ l_{S4}\cdot\sin(\phi_4(n)) \\ 0 \end{pmatrix}+\begin{pmatrix} -b_4\cdot\sin(\phi_4(n)) \\ b_4\cdot\cos(\phi_4(n)) \\ 0 \end{pmatrix}$$

$$x_{S2}(n):=r_{S2}(n)_0 \qquad x_{S4}(n):=r_{S4}(n)_0$$

$$y_{S2}(n):=r_{S2}(n)_1 \qquad y_{S4}(n):=r_{S4}(n)_1$$

Figure 5. Document fragment of MathCAD with code for calculation of coordinates of the centers of rods 2 and 4 mass

According to the 5th point of sequence of analysis, we obtain kinematic parameters by numerical differentiation of the corresponding geometrical parameters of the mechanism.

Analogs of angular and linear speeds of the pump mechanism links will be equal to

$$\varphi_2'=\frac{d\varphi_2}{dn}\cdot\frac{1}{2\pi},\quad \varphi_4'=\frac{d\varphi_4}{dn}\cdot\frac{1}{2\pi},$$

$$x_{S2}'=\frac{dx_{S2}}{dn}\cdot\frac{1}{2\pi},\quad y_{S2}'=\frac{dy_{S2}}{dn}\cdot\frac{1}{2\pi},$$

$$x_{S4}'=\frac{dx_{S4}}{dn}\cdot\frac{1}{2\pi},\quad y_{S4}'=\frac{dy_{S4}}{dn}\cdot\frac{1}{2\pi},$$

$$x_B'=x_{S3}'=\frac{dx_B}{dn}\cdot\frac{1}{2\pi},$$

$$x_{B1}'=x_{S5}'=\frac{dx_{B1}}{dn}\cdot\frac{1}{2\pi},$$

where φ_2' φ_4' – analogs of angular speeds of rods 2 and 4 respectively; x_{Sj}', y_{Sj}' – projections of speed analog of the mass center of *j*-th link on an axis of coordinates x, y.

Analogs of angular and linear accelerations of pump mechanism links are

$$\varphi_2''=\frac{d^2\varphi_2}{dn^2}\cdot\left(\frac{1}{2\pi}\right)^2,\quad \varphi_4''=\frac{d^2\varphi_4}{dn^2}\cdot\left(\frac{1}{2\pi}\right)^2,$$

$$x_{S2}''=\frac{d^2x_{S2}}{dn^2}\cdot\left(\frac{1}{2\pi}\right)^2,\quad y_{S2}''=\frac{d^2y_{S2}}{dn^2}\cdot\left(\frac{1}{2\pi}\right)^2,$$

$$x_{S4}''=\frac{d^2x_{S4}}{dn^2}\cdot\left(\frac{1}{2\pi}\right)^2,\quad y_{S4}''=\frac{d^2y_{S4}}{dn^2}\cdot\left(\frac{1}{2\pi}\right)^2,$$

$$x_B''=x_{S3}''=\frac{d^2x_B}{dn^2}\cdot\left(\frac{1}{2\pi}\right)^2,\quad x_{B1}''=x_{S5}''=\frac{d^2x_{B1}}{dn^2}\cdot\left(\frac{1}{2\pi}\right)^2,$$

where φ_2'' φ_4'' - analogs of angular accelerations of rods 2 and 4 respectively; , x_{Sj}'', y_{Sj}'' – projections of acceleration analog of the mass center of *j*-th link respectively on an axis of coordinates x, y.

Having analogs of speeds and accelerations of the pump mechanism links and projection of these analogs to axes of coordinates, the vectors of speeds and accelerations in MathCAD package can be presented in the form

$$\vec{\omega}_j = \omega_1 \begin{pmatrix} 0 \\ 0 \\ \varphi_j' \end{pmatrix}, \vec{V}_{Sj} = \omega_1 \begin{pmatrix} x_{Sj}' \\ y_{Sj}' \\ 0 \end{pmatrix},$$

$$\vec{\varepsilon}_j = \begin{pmatrix} 0 \\ 0 \\ \varphi_j''\omega_1^2 + \varphi_j'\varepsilon_1 \end{pmatrix}, \vec{a}_{Sj} = \begin{pmatrix} x_{Sj}''\omega_1^2 + x_{Sj}'\varepsilon_1 \\ y_{Sj}''\omega_1^2 + y_{Sj}'\varepsilon_1 \\ 0 \end{pmatrix},$$

where $\vec{\omega}_j$ – vector of angular speed of *j*-th link; ω_1 – value of angular speed of crank 1; $\vec{V}_{Sj}$ – vector of speed of the mass center of *j*-th link; $\vec{\varepsilon}_j$ – vector of angular acceleration of *j*-th link; ε_1 – value of angular acceleration of crank 1; $\vec{a}_{Sj}$ – vector of acceleration of the mass center of *j*-th link.

Conclusion

Regardless of the fact that at big (more than six) quantity of links, the convergence deteriorates, and it is necessary to introduce a number of additional restrictions into system for improvement of convergence, there is a number of advantages in this algorithm, namely:

- the use of vector equations makes it possible to reduce quantity of the equations by three times; that allows more effective use of possibilities of *MathCAD* package for the achievment of objective;

- the number of positions of the mechanism under investigation is limited by hardware opportunities of the computer and system opportunities of a software package;

- possibility of calculation of kinematic parameters for *n* (*n* > 1) mechanism positions gives the opportunity to analyze the obtained parameters for the full period of the mechanism movement (or for its part), and also in case of enough quantity of *n* makes it possible to analyze these kinematic parameters as continuous quantities and build their graphic dependences;

- determination of kinematic parameters (speeds and accelerations of the mechanism links) by numerical differentiation does not require introduction of the additional equations to the computing block *Given-Find*. These additional equations are obtained by differentiation of the equations of closed vector paths in a coordinate form; usually they are rather lengthy and require introduction of additional restrictions and initial approximations, and therefore, convergence of results is improved significantly and process of calculations is optimized.

The results obtained by the described algorithm will be used further for the power, dynamic analysis and the analysis of the strain-stress state of mechanisms of the piston-type pump.

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