Estimation of non-stationary in time stochastic signals

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Abstract
The divergence effect is one of the most important problems in KALMAN filtering. When observation model is often nonlinear and must be linearized. Inadequate linearization leads to a divergence effect that could be controlled by increasing observation noise. In this paper, we propose a modified Kalman filter by providing input in the form of the Taylor series approximation at each measuring point. The effectiveness of this procedure is illustrated by a computer simulation.
Keywords: KALMAN FILTER, GAIN, DIVERGENCE, ESTIMATION

Introduction

The construction method of modified Kalman filter and its application is considered to the tasks of filtration of non-stationary in time casual signals. Increasing of exactness of stochastic signal estimation, it is necessary to take into account the greater totality of factors, influencing on results. In this case estimated signal must be examined not as determined, but as casual process and its estimation can be received with the use of stochastic methods of treatment by the synthesis of filtration algorithms, adequate the probed signals classes.

The application of filtration theory to the technical systems, the development of mathematical theory is needed, as well as methods of design of the systems, synthesis of filters and their practical realization. For linear systems with linear measuring at Gausse noises the theory of filtration is well developed and there are exact algorithms of optimal decision. However, for engineering practice the reliable decision methods of design tasks, planning and realization of filters, adequate the probed signals are needed, in addition. In practice, as a rule, in place of determination (optimum) apply the term of (suboptimum). Because optimization of the realization process of the filter usually plugs in itself so a lot of factors, that it strongly hampers or does impossible mathematical describing, an exact optimum decision is usually used only for the estimation of descriptions of the real filter in the known mathematical model of the probed signal.

There are a few basic filtration methods of signals descriptions. A starting point for creation of filters theory were works of Kolmogorov A.N. [1] and Viner N. [2], executed in 1940-th for stationary ergodic processes.

However, much the finer filter got practical application not immediately. It is explained difficulties, at first, receipts of exact decisions of equalizations of filtration, secondly, identification of the required spectral distributing of signals and noises and, thirdly, designs of physically realized systems. Many systems, presenting practical interest, simply fallen short of suppositions, to accept in the theory of finer filter.

Using estimations of kind $X = \sum S_n$ requires the set of statistics on great number of realizations, while; we frequently a dispose only one. Application of autoregressive methods assumes preliminary researches of processes with the purpose of construction of adequate stochastic models for the probed temporal rows and identification of signal characteristics. Thus, it is also necessary to have a representative sampling of realization of the probed processes in the order.

Development of Kalman-Bucy filter [3, 4] in 1960th removed the necessity of supposition about the stationary of the system and about the presence of information on an endless time domain. In addition, a decision, got as a recurrent computational algorithm, makes possible the direct synthesis of evaluation chart by computer and allows to get the real-time estimations of signal.

Construction of the modified Kalman

Receiving an evaluation of the current value in form of dependence from previous values allows easy replacement of the n_th assessments for $n+k$ making the forecast process in k steps forward. In the case of discrete measuring of signal S_n, will present him as additive mixture:

$$S_n = X_n + Q_n \tag{1}$$

Where X_n a useful signal, Q_n an additive noise with the universal mean $M[Q_n] = 0$.

R it is the dispersion of noise, determined as:

$$R = \frac{1}{n-1}\sum_{i=1}^{n}(x_n - \bar{x})^2; \quad \bar{x} = \frac{1}{n}\sum_{n=1}^{n} x_i$$

For the signal model S_n (equation 1) Kalman filter equations can be written as:

$$\begin{aligned} \hat{X}_n &= F_n \hat{X}_{n-1} + K_n \left[S_n - H_n F_n \hat{X}_{n-1} \right] \\ K_n &= A_n H_n^T \left[H_n A_n H_n^T + R \right]^{-1} \\ A_n &= F_n P_{n-1} F_n^T \\ P_n &= A_n - K_n H_n A_n, \end{aligned} \tag{2}$$

Where: $\hat{X}_n$: is the estimated state vector, F_n: is the matrix of transition from the state $n-1$ to n, K_n: is the Kalman filter gain, S_n: is the measured value of signal, P_n: is the estimated state covariance matrix, H_n: is the matrix of measurement conditions, The index «T» means the transpose of matrix.

Equation 2 can be applied for the receipt of estimation directly, if the matrix of transition fin is known, that essentially, analytical type of the processed dependence. In practice, as a rule, an analytical expression for the estimated signal is unknown, and determination of the coefficients of a matrix is the difficult and labor intensive process of identification. The proposed modification of Kalman filter (equation 2), consists in the following. It is known that the signal of free-form can be presented as decomposition in a row, for example, Teylor row. Limited to the members not higher than m order, for every element of F matrix can write down:

$$F_{ij} = \begin{cases} \dfrac{j!}{i!(j-i)!} - (j-i), 0 \le i \le m, i \le j \le m, \\ \qquad 0, \quad j < i. \end{cases} \tag{3}$$

Putting for definiteness $m = 2$, will get:

$$F = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} \tag{4}$$

As measured only the signal itself, and not its derivatives, the matrix of measurement conditions $H = [1 \quad 0 \quad 0]$. Then substitution **Equation 4** in **Equation 2** gives a procedure of estimation of signal for the accepted approximation in a scalar form, as:

$$\begin{aligned} \hat{x}_n &= \hat{x}_{n-1} + \hat{y}_{n-1} + \hat{z}_{n-1} + \alpha_n C_n \\ \hat{y}_n &= \hat{y}_{n-1} + 2\hat{z}_{n-1} + \beta_n C_n \\ \hat{z}_n &= \hat{z}_{n-1} + \gamma_n C_n \\ C_n &= S_n - \hat{x}_{n-1} - \hat{y}_{n-1} - \hat{z}_{n-1}, \end{aligned} \tag{5}$$

Where: $\alpha_n = P_n^{11} R^{-1}$, $\beta_n = P_n^{21} R^{-1}$ and $\gamma_n = P_n^{31} R^{-1}$ are elements of the matrix Kalman filter gain $K_n = P_n H_n^T R_n^{-1}$ determined through the elements P_{ij} of the matrix P_n equation 2, y_n and z_n make sense according to the first and second derivatives.

Known from the theory of filters causes of divergence (bias estimations), associated with this case, mainly to the ultimate value of m there is a need for more control divergence. For these aims procedure which is taken to the count of statistics of the kind is offered:

$$B_M = \sum_{l=1}^{M} b_l, \quad B_0 = 0, \quad l = 1, 2, ..., M \tag{6}$$

$$b_l = \text{sgn}(S_l - \hat{X}_l) = \begin{cases} +1, & S_l - \hat{X}_l \ge 0 \\ -1, & S_l - X_l < 0 \end{cases} \tag{7}$$

On an interval $[n-M, \quad n]$. Defined on the interval values $[B_M - MinB_M]$ and $[MaxB_M - B_M]$ compared to the threshold of h. When exceeding the values h one of the values of the decision on the divergence, the filter parameters assign initial values, and filtering lasts from the moment of $(n-M)$. Particular interests are issues of forecasting future values of a time series. Such task can be decided on the base of the offered modified method of Kalman filtration. For this pur-

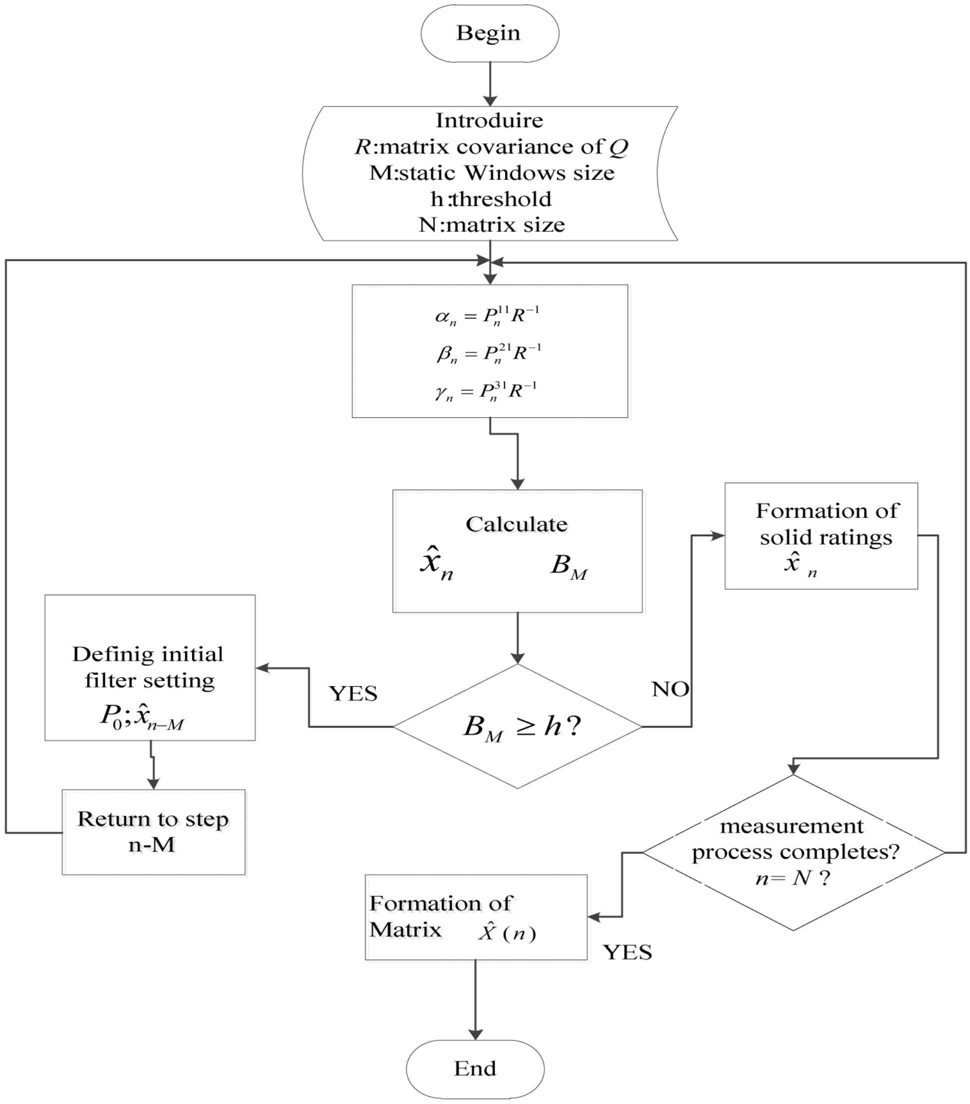

Figure 1. Block diagram of algorithm

pose sufficiently in equation 2 to put $H_n = [0 \quad 0 \quad 0]$, and to present the index of n as $n = n + k$, where k is a number of the steps set in advance. With respect to the proposed modification (**Equation 5** and **6**}), consider a block diagram of the software implementation suboptimal filter is presented in **(Figure 1)**. The program's algorithm estimation of the measured signal is the following. First produced in the initial installation. As matrix RA, as it is known from the theory of filtration, accepted the diagonal matrix of size 3 x 3 (in obedience to the accepted approximation). Moreover, the elements on the main diagonal are assigned the values of a priori found dispersion of the investigated process. The required number of measuring is set. As the mathematical expectation for non-stationary processes, it is expedient to choose the measurement value S_1. After starting the program calculates the gain of the filter and the current value X_n of estimation of the object state. Executes control the bias estimations and, if necessary, adjustments to the initial values of the filter. The forming of an array of estimations of the state of the object and the calculation of his prognosis values is further made. All of process repeats oneself then. Under reaching the set number of measuring, the output of information is made. In connection with the fact that the gain Kalman filter presents an iterative ratio and, therefore, it cannot be expressed as universal stationary solutions, efficiency of application of the modified algorithm will demonstrate a model example. With this purpose, as the useful signal X_n of Equation **1** take a Gaussian signal type:

$$X_n = e^{-(n-n_0)^2/2} \tag{8}$$

for which a transition matrix is determined as:

$$F_n = \frac{X_n}{X_{n-1}} = \frac{e^{-(n-n_0)^2/2}}{e^{-(n-1-n_0)^2/2}} = e^{-(2n-2n_0-3)} \tag{9}$$

As $H = 1$, the system of **Equalizations** 2 can be written in a kind:

$$P_{(n)} = \frac{F_n^2 P_{n-1} R}{F_n^2 P_{n-1} + R} \qquad (10)$$

$$\hat{X}_n = F_n \hat{X}_{n-1} + P_{(n)} R^{-1} S_n - P_{(n)} R^{-1} F_{(n)} \hat{X}_{n-1} \qquad (11)$$

Thus, as the error estimation of filtration process for the known matrix of transition $P_{(n)}$ is accepted from **Equation** 10, and for the modified algorithm of element P_{11} matrices of $P_{(n)}$ from **Equation** 2. On **Figure 2** the result of the work of the modified algorithm of the Kalman filter performance is presented. In **Figure 3** the change of rationed error of estimation is resulted for the known and modified transition matrices.

Conclusion

The developed method of filtration, based on modification of Kalman filter, allows to get suboptimum in sense of minimum dispersions of estimation of signals, characterized a substantial unstationarity in time and not having exact analytical description (the mathematical model).

Paper results can be used for further development of researches with the purpose of development of optimum methods of measurement and management in the real scale of time.

Acknowledgements

The authors are gratefully acknowledging the Laboratory of Automation and Signals (LASA) of Badji Mokhtar Annaba University and Information Security and Telecommunications Department, Dnipropetrovsk National Mining University, for their support to carry out this research work.

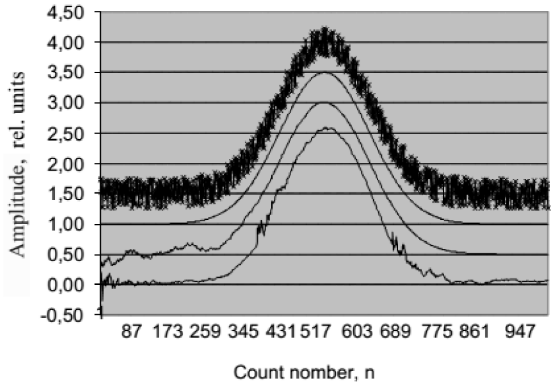

Figure 2. Result of the work of the modified algorithm of Kalman filter performance, where from top to bottom measurable signal of Sn, useful signal of Xian, estimations of signal of Xn(1) with the known matrix of transition and Xn(2) with the modified matrix of transition, accordingly

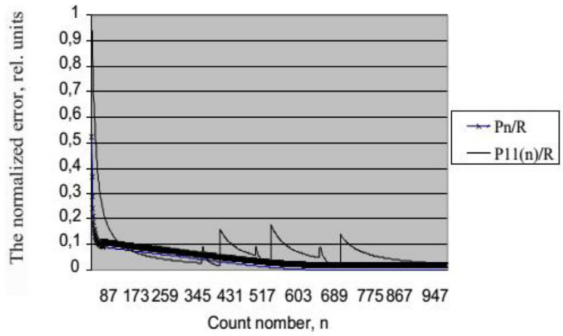

Figure 3. Change the rationed error of estimation for the known and modified matrices of transition

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