Analysis of fractal characteristics of mining and geological parameters of minerals

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Abstract
The basic properties of fractals were studied and the main ways of their possible use in justification of mineral deposits characteristics were analyzed.
Key words: FRACTAL ANALYSIS, CRACKS, DEPOSITS, MINERALS, MINERAL FORMATION

Introduction

Fractal geometry and fractal analysis are relatively new branch of science, that declared itself after the publication of fundamental works of B. Mandelbrot, who has shown that the geometry of the most natural objects allows, in addition to the characteristics of traditional geometrical concepts, describing by fractal objects the parts of which are in some extent similar to the whole. In 1975, Benoit Mandelbrot coined the term "fractal" from the Latin word fractus, which meant "broken". But only after the publication in 1983 of "The fractal geometry of nature" by B. Mandelbrot the concept of fractals had entered into other sciences, and became the basis for the consideration of a variety of natural forms. It turned out that fractals provide extremely compact way of describing objects and processes [4].

Analysis of recent researches and publications

The fractal characteristics of geological objects and processes were studied by such scholars as Bulat A. F., Dyrda V. I. [1], Vadkovsky V. N., Zakharov V. S. [2], Pozdnyakov O. V. [5], Koptikov V. P. [3], Goryainov P. M., Ivanyuk G. Yu. [7].

Fractal characteristics of large-scale geological structures and objects of seismic zones and unworked coals were studied in most of the papers from fractal analysis in mining. In turn, insufficient attention is paid to the problem of possibility of fractal analysis application to the study of characteristics of natural stone deposits and researches on the subject are not carried out.

The work objective is to analyze the experience of application of fractal analysis methods in geology and mining and to determine the prospects of using it for researches of natural stone deposits.

Presentation of the main material of the article

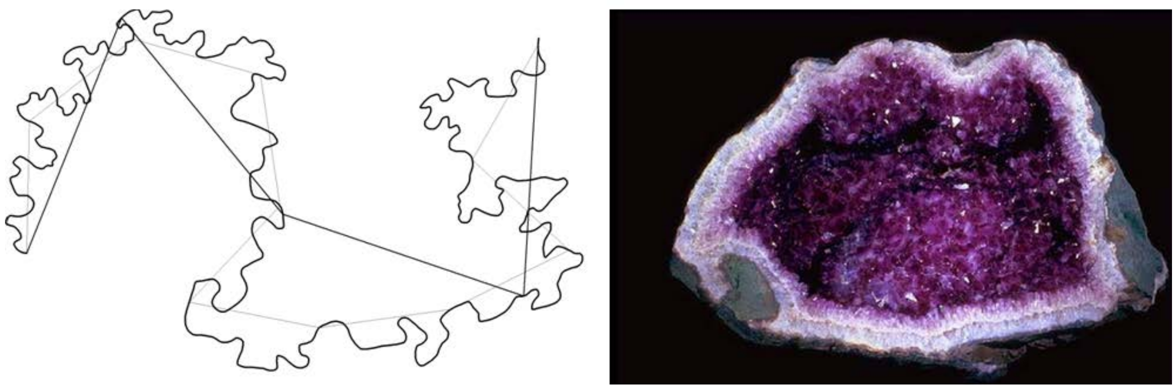
Traditional methods of mining geometry are based on a close approximation of mining and geological objects complex forms by geometrical figures: points, lines, segments, planes, polygons, etc., which metric and topological dimension are equal to each other. At the same time, when it comes to geometrization of geological structures (geometrization of mineral deposits), part of the information about the objects is lost, because the distribution of qualimetric characteristics of the deposits is averaged and shape of the geological structure is simplified artificially. Fractal geometry, which operates fractional metric dimensions of the studied objects, characterizes not only their geometric image, but also reflects the processes of formation and evolution. Many complex structures have a fundamental property of the geometric regularity – scale invariance, or "self-similarity". When considering such structures with different magnifications it turns out that the same structural elements are repeated at each scale level. These repetition laws define the fractional or fractal dimension of the structure. In this case, it is clear that fractal geometry describes natural objects more accurately and easier than Euclidean geometry.

The concept of fractals was first used to measure the shore lines of England, Australia and Norway. Measurements of their approximate length have shown that the length of the coast is a power function, which depends on the scale (length unit).

Any topographical profile is continuously formed by dynamic processes of destruction and creation, so we can assume that it is also fractal. This question has been studied according to the data of Vadkovsky V. N. and Zakharov V. S.: Himalaya-Tibet region has a dimension of 1,67, the southern regions of Lake Baikal - 1,674, Central Africa - 1,704, the North Atlantic - 1,484, etc.

High roughness means intensive endogenous feeding in the area. This is an important consequence of the fractals theory allowing geologists to compare quantitatively the tectonic regimes in the region.

Like the determination of sea shoreline structures it is possible to use fractal analysis for pegmatites contours or amethyst geodes shown in Fig. 1. In the latter case, the calculated value is equal to the fractal dimension $d = 1.14$.



a) b)

Figure 1. Minerals contouring structure using fractal analysis

Recently, fractal analysis began to apply not only in mathematics, but also in many branches of physics, materials science, biology, geology and geophysics. It has been proved theoretically and experimentally that using fractal models as mathematical and natural objects as inanimate and living objects can be studied. Various researchers of natural objects fractality have revealed the presence of ordering in the structures that appeared to be disordered.

When studying the processes of rock destruction researchers have come to the conclusion that the process is non-linear. Hooke's law has the following form

Hooke's law was only at the initial stage of deformation. When a certain threshold value was reached, linear law ceased to operate, and there was a redistribution of stresses (disconnection), resulting in an extensive system of cracks. In cases of rock destruction in tension and shear, ypical configuration of cracks have a fractal structure with the dimension d = 1.12...1.65.

In theory of destruction, the theory of fractal structures is actively applied [8, 9]. The spread of cracks in the rock mass has a number of features – the fractal nature of the fracture process [9] and stochastic trajectory [10]. The trajectory of crack occurring at the same time can be curved; the crack propagation can be chaotic. The microstructure has a significant influence on the crack propagation stability, especially near bifurcation points. In some modes of crack propagation the process of self-sustaining destruction occurs when crack propagation becomes avalanche, self-supporting nature. The sufficient voltage level for such explosive crack propagation is much less than critical one.

The residual stresses have essential character [10]. Instability can be caused by non-uniform field of dislocations, inhomogeneous mechanical properties of the continuum, random fluctuations in the applied stress [10].

Usually it is assumed that the density W of microcracks has a fractal nature of hyperbolic distribution:

$$W(a_c) = \frac{N(a \geq a_c)}{V} = \lambda \cdot a_c^{-d},$$

where a_c - critical crack size; λ - constant, d - fractal dimension, $0 \leq d \leq 3$; $N(a \geq a_c)$ - a number of microcracks, which are greater than the critical length value [9].

According to the fractals theory for destruction, one crack is enough. Therefore, only one main crack determines the tensile strength of the material. Then the probability of failure is $P = 1 - e^{-nv}$, where P - the probability of finding at least one critical crack; V - volume of the sample; n - a number of critical cracks per volume unit. The transition from the dispersed destruction to the main crack [10] and emergence of bifurcation points corresponds to the phenomena of "catastrophe". Therefore, the classical destruction objects can be considered from the perspective of "catastrophe theory".

Fractal patterns have found their application in the study of logic and information descriptions of mineral deposits. The methods of fractal geometry can estimate the morphology of ore bodies on a quantitative basis, the complexity of the ore bodies is reflected in a regular increase in the fractal dimension *D*. With regard to the geology such an object, which structure when considered at different scale levels shows a fundamental similarity, at the same time the number of the structure fixing elements when transition from one level to another scale changes in the power ratio is called self-similar fractal.

The goethite mineral is a classic example of the fractal organization of mineral individuals (Fig. 2.). Moving up the hierarchy of the lithosphere the fractal property of structures and textures of rocks can be noted. Some self-similarity reveals itself in nodule of minerals and other geometric selection derivatives.

Figure 2. The goethite mineral

Such complex objects as breccia can be self-similar in the scale range of at least five orders of magnitude, and behind the apparent randomness of their structure, the strict laws can be viewed. The latter is well illustrated by the classic fractal "Apollonian gasket" analyzed in detail by B. Mandelbrot [4]. Its construction starts with three circle of arbitrary diameter in contact with each other, between which the round triangle with angles 0° will be located. The circle with the most possible diameter fits in this triangle and forms another three smaller rounded triangle. Endless repetition of this procedure creates "Apollonian gasket" (Fig. 3).

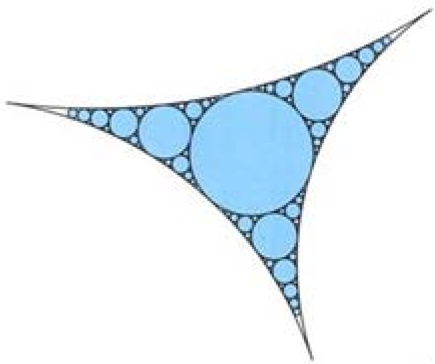


Figure 3. Apollonian gasket

It is interesting that large areas of ocean ice are fragmented like the Apollonian cascade. Even "effect of deliquescent floe" was formulated: each piece, despite its size can be with sufficient degree of confidence associated with its neighbors even with enough distant fragments. According to the theory of non-equilibrium fragmentation, the formation of fractal divisibility of rocks due to the fact that destruction probability of an inhomogeneous medium increases when same size fragments are interacting (i. e. the probability that a large fragment crushes the smaller and, especially, on the contrary is much lower than the probability of mutual destruction of similar in size rocks). As a result, similar in size neighboring fragments are destroyed and two identical pieces do not occur beside each other.

In order to self-similar fractal model will suit to the characteristics of the studied formation, there should be a proportional relationship between the number logarithm of n-dimensional cubes with a side δ $(N(\delta))$, covering n-dimensional fractal and with side size of δ. It is obvious that the natural object when increasing or decreasing the scale will not have an infinite self-similarity. Furthermore, similar may be only certain large-scale levels. Therefore, assessing the conformity of the self-similar fractal model explore some scale interval and determine the fractal dimension of the set, which would have been if the self-similarity of the object had been infinite.

Usually, to determine compliance of the fractal model with the real geological object the graph of dependence $lnN(\delta)$ from $ln(\delta)$: $\ln N(\delta) = a \cdot \ln(\delta) + b$ is built. Logarithms on any basis can be used, but the same for both axes. In the case of matching the graph points should be substantially approximated by a straight line.

The rate of growth of the elements number while increasing the scale is the fractal dimension D. It is defined as tangent of the angle of this straight line to the abscissa axis. The fractal dimension of investigated object is a quantitative characteristic of the filling degree of plane or space. The essence of fractal analysis is to study the dynamics of changes in the geometry of the studied object when scale changing.

In this regard, the scheme of fractal analysis is as follows:

1. By methods of fractal geometry the various geotechnical parameters of mineral deposits development are assessed.

2. The correlations between the fractal dimension of the geotechnical parameters and coefficients of variation statistics characterizing by variability are established.

3. The variable nature of fractal dimensions for various geotechnical parameters due to the complexity of geological conditions is determined.

Conclusions

The analysis of the information allows drawing a conclusion of the extensive use of fractal analysis methods in various branches of modern science. On the basis of applying fractal analysis in geology, we can make a guess about its use in the mineral deposits geometrization, in particular in the study of fracture parameters, and especially in dealing with assessment of the variability of geotechnical parameters for the purpose of further planning the mining operations conducting directions.

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