Research on Failure Mode and Effects Analysis Based on Quality and Cost

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Abstract
Traditional Failure Mode and Effects Analysis (FMEA) are determined by risk priority number (*RPN*), which is the product of three risk factors occurrence (*O*), severity (*S*), and detection (*D*). This method is unreasonable. One of the serious deficiencies existing in traditional FMEA model is focusing on the quality risk of failures and ignoring the cost risk of those. In this paper, based on the quality and cost, a new economic FMEA model was proposed for risk evaluation. In the proposed model, economy, as the 4th risk factor, was defined and its scale was formulated. Then the fuzzy belief structure and grey relational projection method were used to measure risk in terms of quality and cost to determine the risk priority order of the failure modes that had been identified. Finally, a case shows that the proposed model is rational and practical.
Key word: FMEA, ECONOMY, FUZZY BELIEF STRUCTURE, GREY RELATIONAL PROJECTION

1. Introduction

Risks exist in any project and can not be eliminated. But we can reduce risks to an acceptable level by risk assessment techniques. FMEA is a widely used engineering technique for risk evaluation. In traditional FMEA model, risk evaluation involves failure occurrence (*O*), severity (*S*) and detection (*D*), of which the product is called risk priority number (*RPN*). Failures with higher *RPN* are assumed to be given higher priorities and should be taken appropriate measures to reduce the risk. In the 1960s, the first FMEA was used in the US airline industry and now has been widely used in food, environment, healthcare and automotive industry[1-4]. However, in the application, FMEA exists some disadvantages, such as: being unable to accurately express uncertain information, high subjectivity on expert scoring, ignoring risk factor weight of *S*, *O* and *D*, ignoring the economic benefits of project.

A number of studies have been done in domestic and foreign literature to overcome some of drawbacks mentioned above and achieved certain results. For example, Bowles and Pelaez[5] proposed a fuzzy logic-based FMEA model. More understandable linguistic terms combining with fuzzy If-Then rules were applied to quantify inaccurate information to ensure the result accuracy. Chin et al.[6] developed a FMEA approach using group-based evidential reasoning. Belief structure was applied to deal with the diversity and uncertainty of assessment information. On the basis of fuzzy logic, Vencheh et al.[7] proposed a new FMEA method integrating alpha-level sets and linear programming. Risk factors were treated as fuzzy variables. Fuzzy linguistic terms and fuzzy ratings were used to assessed risk factors. Weights of risk factors were evaluated in a linguistic manner and the *RPN* was defined as fuzzy geometric means of the fuzzy ratings for *S*, *O* and *D* to determine failures. To avoid integrity constraint of belief structure in real applications, Deng[8] developed D numbers and arithmetic operations. Liu et al.[9] proposed a FMEA method using D numbers and grey relational projection (GRP). This method involved weights of risk factors and the GRP established on double base points, which effectively deal with the diversity and uncertainty of assessment information to ensure risk priority order more rigorous. Du and Vahdani[10-11] presented a FMEA approach based on evidential reasoning and technique for order of preference by similarity to ideal solution (TOPSIS) to avoid the constraint of independent decision-making preference.

Gilchrist[12] proposed an expected cost based FMEA model, which is formulated as $EC = CnP_f P_d$, where C is the average cost per failure, n the items, P_f the probability of a failure and P_d the detection probability of the failure. Ben-Daya and Raouf[13] thought that probabilities P_f and P_d were not always independent and P_f was of more importance than P_d. They revised the expected cost model by raising the ratings for the probability of occurrence to the power of 2. Rhee[14] described the life cycle cost for measuring cost risk by computing detection time, fixing time, occurrence probability, delay time, down time and complex scenarios. The proposed approach is helpful to predict life cycle failure cost, measure risk and plan preventive and ultimately improve uptime. Although a number of approaches have been suggested and obtained certain achievement, there is no rational, rigorous and complete method which can comprehensively consider quality risk and cost risk of failures. In this paper, economy, as the 4th risk factor, is introduced. The fuzzy belief structure and grey relational projection method are used to comprehensively evaluate alternatives in terms of risk factors S, O, D and E with respect to criteria.

The rest of this paper is organized as follows. The traditional FMEA method is presented in the next section. In Sect. 3, the 4th risk factor, economy, is introduced. Its concept, formula and ratings are presented. In Sect. 4, the procedure of proposed new FMEA model based on quality risk and cost risk is described. In Sect. 5, a numerical case study about the preference of cause failures of cylindrical sealer production process is provided to illustrate the process of the proposed method. Compared with the traditional FMEA model, the proposed model is of higher rationality and practicality by comprehensively consider the quality and cost of alternatives. The last section is devoted to conclusion.

2. FMEA

FMEA is a rigorous analysis approach for risk assessment and is implemented by a team of representative multidisciplinary experts. FMEA has six procedures: determining customer needs, identifying potential failure modes, identifying potential consequences, identifying potential causes, evaluating risk and developing measures, of which evaluating risk plays a very important role in rigorous analysis of failures. The traditional FMEA determines the risk priorities of failures with the RPN, which is the product of S, O and D of a failure. That is

$$RPN = S \times O \times D \tag{1}$$

where S is the severity of failure effects, O the probability of occurrence and D the probability of detection.

Usually, RPN includes the three factors above. The three factors S, O and D are evaluated by using numeric scales (ratings) from 1 to 10, as described in FMEA manual. 1 is the lowest level and devotes respectively no effect, nearly impossible and almost certain. 10 is the highest level and devotes respectively catastrophic effect, extremely high possibility and absolutely impossible to detect. Traditional FMEA method has been widely used for its simpleness and easy to understand, but it suffers from several shortcomings such as follows:

(1) Much information of the three factors is diversity, incomplete and uncertain and is difficult to evaluate in precise numerical values.

(2) The three factors are evaluated by experts who have different opinions for their different professional knowledge, experiences and departments. The evaluation information is of strong subjectivity and the evaluation result is far from that in real process.

(3) The relative importance among S, O and D is not taken into consideration in a practical application. It's not obviously rational that the three factors are given the same importance.

(4) Risk includes quality risk and cost risk. However,in traditional FMEA, there are only three risk factors in terms of quality to be considered, other important factors such as economical aspects are ignored.

Therefore, in order to effectively overcome the weaknesses existing in the traditional FMEA, this paper proposed a economic FMEA model by integrating quality and cost in the project. In the proposed model, the economy (E), as the fourth risk factor, was defined as the ratio of output and input in terms of cost in the life cycle of the project. Fuzzy belief structure and grey relation projection method were applied to comprehensively assess the risk priority of potential failures in terms of both quality and cost.

3. Economy

3.1. The Risk Factor E

Economic benefits must be taken into consideration when it comes to the production elements inputs and corresponding outputs of products or services. Therefore, it is clearly unreasonable to only consider severity, occurrence and detectability of failures in the conventional FMEA process. The economy (E) is defined as the ratio of total outputs and the inputs of all production elements in the life cycle of the project. With times going on, the market is volatile. So the inputs of all production elements should be divided into two parts: the cost in construction period and that in production period. The economy (E) is expressed as:

$$E = \frac{O_1}{I_0 + I} \qquad (2)$$

where I_0 is the total input cost of all production elements in construction period, I the total input cost of all production elements in production period, O_1 the total output cost.

(1) Input cost of production elements

In general, the main production elements in construction period include staff salaries, bonuses, energy, equipment and maintenance. In production period the main production elements are wages, bonuses, raw materials, energy and maintenance. Suppose there are M production elements in construction period of T_0 months, the total input cost is determined by:

$$I_0 = \sum_{t_0=1}^{T_0}\sum_{m=1}^{M} I_{0t_0}^{m}\,(m = 1, 2, \cdots, M; t_0 = 1, 2, \cdots, T_0) \qquad (3)$$

where $\sum_{m=1}^{M} I_{0t_0}^{m}$ is the all elements investment costs of the t_0th month of construction period.

Similarly, suppose there are N production elements in production period of T_1 months, the total input cost is determined by:

$$I = \sum_{t_1=1}^{T_1}\sum_{n=1}^{N} I_{t_1}^{n}\,(n = 1, 2, \cdots, N; t_1 = 1, 2, \cdots, T_1) \qquad (4)$$

where $\sum_{n=1}^{N} I_{t_1}^{n}$ is the all elements investment costs of the t_1th month of production period.

(2) The total output cost

It is assumed that the inventory is zero in industrial production, so the total output is the product of the number and the unit price of products, namely:

$$O_1 = NC \qquad (5)$$

Thus, the economy (E) is described as:

$$E = \frac{NC}{\sum_{t_0}^{T_0}\sum_{m=1}^{M} I_{0t_0}^{m} + \sum_{t_1}^{T_1}\sum_{n=1}^{N} I_{t_1}^{n}} \qquad (6)$$

3.2. The Ratings of E

It's known from Eq.(2) that the project has no benefit when the total outputs are lower than or equal to the inputs of all production elements, which is set the lowest rating-rating 1. E has larger difference among different industries, thus the E_{max} in history of the industry is set the highest rating-rating 10 and E_{max} is often higher than 1 in real process. The ratings of E is formulated by the proportion of the E_{max}, as shown in table 1.

4. Fundamental Concepts

4.1. Fuzzy Belief Structure

Since the evaluation scales for four factors of S, O, D and E are fuzzy, and there is no strict distinction between adjacent scales, ordinary sets can not be used to precisely evaluate risk of failures. In this paper, fuzzy belief structure in evidential reasoning are applied for evaluation of failures. Considering the relative importance among S, O, D and E for a failure, the ratings of risk factors and their weights are described in linguistic terms, as shown in table 2.

Table 1. Ratings for economy of a failure

Rating	Criteria
1	The total outputs are lower than or equal to the inputs
2	The ratio of total outputs and the inputs is（1%-10%）E_{max}
3	The ratio of total outputs and the inputs is（11%-25%）E_{max}
4	The ratio of total outputs and the inputs is（26%-40%）E_{max}
5	The ratio of total outputs and the inputs is（41%-55%）E_{max}
6	The ratio of total outputs and the inputs is（56%-70%）E_{max}
7	The ratio of total outputs and the inputs is（71%-85%）E_{max}
8	The ratio of total outputs and the inputs is（86%-95%）E_{max}
9	The ratio of total outputs and the inputs is（96%-100%）E_{max}
10	The ratio of total outputs and the inputs is E_{max}

Table 2. Ratings for S, O, D and E and their relative importance

Linguistic rating	Traditional rating for *S*, *O*, *D* and *E*	Weight for *S*, *O*, *D* and *E*
Very low	1	The importance of risk factor is very low
Low	2/3	The importance of risk factor is low
Moderate	4/5/6	The importance of risk factor is moderate
High	7/8	The importance of risk factor is high
Very high	9/10	The importance of risk factor is very high

In this paper, linguistic variables is applied to evaluate the risk factors from FMEA team members, and the individual evaluation grade is expressed as:[15]

$$\{H_{ii}\} = \{H_{11}, H_{22}, H_{33}, H_{44}, H_{55}\}$$
$$= \{Very\ Low, Low, Moderate, High, VeryHigh\} \quad (7)$$

where $\{H_{ii}\}$ (i=1,2,…5) is the ith grade defined for risk assessment.

Definition 1 Let $\{H_{ij}, \beta_{ij}^{x}(FM_y, RF_l)\}$, i=1,2,…5; j=1,2,…5; x=1,2,…X; y=1,2,…Y; l=1,2,…4} be the belief structure provided by TM_x (the xth team member) on FM_y (the yth failure mode) with respect to RF_l (the lth risk factor). The group belief structure is described as:[15]

$$\tilde{Z}_y(l) = \{H_{ij}, \beta_{ij}(FM_y, RF_l)\} \quad (8)$$

where $\{H_{ij}\}$ (i=1,2,…4; j=i+1,…5) is the interval between $\{H_{ii}\}$ and $\{H_{jj}\}$. $\beta_{ij}(FM_y, RF_l)$ is a group belief degree and $\beta_{ij}(FM_y, RF_l) = \sum_{x=1}^{X} \mu_x \beta_{ij}^{x}(FM_y, RF_l)$ i=1,2,…5; j=1,2,…5; x=1,2,…X; y=1,2,…Y; l=1,2,…4. μ_x (μ_x>0) is the weight of TM_x satisfying $\sum_{x=1}^{X} \mu_x = 1$.

The belief structure allows FMEA members to provide their subjective judgments and is of following characters[6]:

(1) A precise rating such as 4, which can be written as $\{(H_{44}, 1.0)\}$ and it is referred to as a precise belief structure.

(2) A distribution such as Low to 0.4 and Moderate to 0.6, which can be written as $\{(H_{22}, 0.4), (H_{33}, 0.6)\}$ and is referred that a *FM* is assessed with respect to the *RF* to grade 2 to the degree of 0.4 and to grade 3 to the degree of 0.6. 0.4 and 0.6 represent the belief degrees of the FMEA team member.

(3) An interval such as 2-3, which can be written as $\{(H_{23}, 1.0)\}$ and means that the grade of a *FM* with respect to the risk factor under consideration is between 2 and 3.

(4) If the rating could be anywhere between Very Low and Very High, which is can be written as $\{(H_{15}, 1.0)\}$, it means that the team member is not willing to or cannot provide an assessment for a *FM* with respect to the risk factor under evaluation.

4.2. Grey Relation Projection

Grey Relation Projection (GRP) is a tool for quantitative analysis, which can effectively deal effectively with various uncertainties and incompleteness and imprecision of evaluation information. GRP ranks the *FMs* by determining the difference in contribution between a reference sequence (ideal solution) and each comparative sequence (alternatives).

(1) Comparative matrix of *RFs*

Evaluation information provided by TM_x on the FM_y with respect to RF_l is defuzzified by eq 9:

$$Z_y(l) = \sum_{i=1}^{5} H_{ij} \frac{\beta_{ij}}{j - i + 1} \quad (9)$$

An information sequence with l *RFs* can be expressed as $Z_y = (Z_{y1}, Z_{y2}, \cdots, Z_{yl}) \in Z$, where Z_{yl} denotes the lth factor of Z_y. The matrix constituted by evaluation vectors of risk factors is called risk factors comparison matrix, which is expressed as:

$$Z_y(l) = \begin{bmatrix} FM_1 \\ FM_2 \\ \vdots \\ FM_y \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1l} \\ Z_{21} & Z_{22} & \cdots & Z_{2l} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{y1} & Z_{y2} & \cdots & Z_{yl} \end{bmatrix} \quad (10)$$

(2) Reference matrix of RFs

In this paper, the GRP established on double base points (the positive ideal alternative and the negative ideal alternative) is employed. In the FMEA method, failures with higher risk are ranked preferentially. Thus, the highest levels of all the risk factors are the positive ideal alternative and the lowest levels of those are the negative ideal alternative. They are respectively expressed as:

$$Z_0^+ = (z_{01}^+, z_{02}^+, \cdots, z_{0l}^+) \quad (11)$$

$$Z_0^- = (z_{01}^-, z_{02}^-, \cdots, z_{01}^-) \quad (12)$$

(3) Grey relation coefficient

Grey relation coefficients are referred to the correlation values between vectors of comparative matrix and those of reference matrix. γ_{yl}^+ is the relation coefficient between Z_{yl} and Z_{0l}^+. γ_{yl}^- is the relation coefficient between Z_{yl} and Z_{0l}^-. The lower the grey relation coefficient γ_{yl}^{+} is, the closer the alternative is to the positive ideal alternative. The higher the grey relation coefficient γ_{yl}^{-} is, the further the alternative is from the negative ideal alternative. The grey relation coefficient is determined by:

$$\gamma_{yl}^{+}=\frac{\min\limits_{y}\min\limits_{l}\left|z_{0l}^{+}-z_{yl}\right|+\xi\max\limits_{y}\max\limits_{l}\left|z_{0l}^{+}-z_{yl}\right|}{\left|z_{0l}^{+}-z_{yl}\right|+\xi\max\limits_{y}\max\limits_{l}\left|z_{0l}^{+}-z_{yl}\right|} \quad (13)$$

$$\gamma_{yl}^{-}=\frac{\min\limits_{y}\min\limits_{l}\left|z_{0l}^{-}-z_{yl}\right|+\xi\max\limits_{y}\max\limits_{l}\left|z_{0l}^{-}-z_{yl}\right|}{\left|z_{0l}^{-}-z_{yl}\right|+\xi\max\limits_{y}\max\limits_{l}\left|z_{0l}^{-}-z_{yl}\right|} \quad (14)$$

where ξ is the distinguishing coefficient and $\xi\in[0,1]$. Generally, ξ= 0.5 is applied[16].

(4) Grey relation matrices S^{+} and S^{-}

For the yth FM_y, risk factors S, O, D and E have different importance. The matrix constituted by grey relation coefficients should be weighted.

Definition 2 Let $\left\{H_{ij}^{0},\eta_{ij}^{x}\left(FM_y,RF_l\right)\right\}$, i=1,2,…5; j=1,2,…5; x=1,2,…X; y=1,2,…Y; l=1,2,…4} be the belief structure provided by TM_x (the xth team member) on FM_y (the yth failure mode) with respect to the weight of RF_l (the lth risk factor). The group belief structure of the risk factor weight is described as:

$$\tilde{\omega}_y(l)=\left\{H_{ij}^{0},\eta_{ij}\left(FM_y,RF_l\right)\right\} \quad (15)$$

where $\eta_{ij}(FM_y,RF_l)$ is a group belief degree of risk factors and $\eta_{ij}\left(FM_y,RF_l\right)=\sum_{x=1}^{x}\mu_x\eta_{ij}^{x}\left(FM_y,RF_l\right)$ i=1,2,…5; j=1,2,…5; x=1,2,…X; y=1,2,…Y; l=1,2,…4. μ_x (μ_x>0) is the weight of TM_x satisfying $\sum_{x=1}^{X}\mu_x=1$.

Eq 15 is defuzzified by using eq 16, namely:

$$\omega_y(l)=\sum_{i=1}^{5}H_{ij}^{0}\frac{\eta_{ij}}{j-i+1} \quad (16)$$

Thus, the average weight of risk factors is determined by:

$$\omega_l=\frac{\sum_{y=1}^{Y}\omega_y(l)}{y} \quad (17)$$

Normalize the average weight of risk factors:

$$\bar{\omega}_l=\frac{\omega_l}{\sum_{l=1}^{L}\omega_l} \quad (18)$$

Weighted grey relation matrix S^{+} and S^{-} are respectively described as:

$$S^{+}=\begin{bmatrix}\bar{\omega}_1\gamma_{11}^{+} & \bar{\omega}_2\gamma_{12}^{+} & \cdots & \bar{\omega}_l\gamma_{1l}^{+}\\ \bar{\omega}_1\gamma_{21}^{+} & \bar{\omega}_2\gamma_{22}^{+} & \cdots & \bar{\omega}_l\gamma_{2l}^{+}\\ \vdots & \vdots & \vdots & \vdots\\ \bar{\omega}_1\gamma_{y1}^{+} & \bar{\omega}_2\gamma_{y2}^{+} & \cdots & \bar{\omega}_l\gamma_{yl}^{+}\end{bmatrix} \quad (19)$$

$$S^{-}=\begin{bmatrix}\bar{\omega}_1\gamma_{11}^{-} & \bar{\omega}_2\gamma_{12}^{-} & \cdots & \bar{\omega}_l\gamma_{1l}^{-}\\ \bar{\omega}_1\gamma_{21}^{-} & \bar{\omega}_2\gamma_{22}^{-} & \cdots & \bar{\omega}_l\gamma_{2l}^{-}\\ \vdots & \vdots & \vdots & \vdots\\ \bar{\omega}_1\gamma_{y1}^{-} & \bar{\omega}_2\gamma_{y2}^{-} & \cdots & \bar{\omega}_l\gamma_{yl}^{-}\end{bmatrix} \quad (20)$$

(5) Grey relation projection

Each row vector of matrix S^{+} or S^{-} respectively means the correlation values between risk of the corresponding failure and the highest risk alternative or the lowest risk alternative. Therefore the grey relation projection of the yth failure mode FM_y on the highest risk sequence and on the lowest risk sequence can be respectively calculated as:

$$P_y^{+}=\left\|S_y^{+}\right\|\cos\left(S_y^{+},S_0^{+}\right)=\sum_{l=1}^{L}\left(\frac{\bar{\omega}_l^{2}}{\sqrt{\sum_{l=1}^{L}\bar{\omega}_l^{2}}}\times\gamma_{yl}^{+}\right) \quad (21)$$

$$P_y^{-}=\left\|S_y^{-}\right\|\cos\left(S_y^{-},S_0^{-}\right)=\sum_{l=1}^{L}\left(\frac{\bar{\omega}_l^{2}}{\sqrt{\sum_{l=1}^{L}\bar{\omega}_l^{2}}}\times\gamma_{yl}^{-}\right) \quad (22)$$

(6) Relative projection

The relative projection of each failure mode to the highest risk sequence is defined as follows:

$$R_y=\frac{P_y^{+}}{P_y^{+}+P_y^{-}} \quad (23)$$

When conducting FMEA, the relative projection value is referred to the relationship between the potential failures and the highest value of the risk factors. The failures with smaller values of relative projection have the larger effect. Thus, all the failures can be prioritized or ranked according to the ascending order of their relative projection values in the FMEA.

4.3. Proposed method

In this paper, a economic FMEA model is developed to overcome the shortcoming that the traditional FMEA ignores cost risks. In the proposed model, a new risk factor E is defined and the ratings are formulated by the proportion of the E_{max}. Then, according to Table 2, the fuzzy belief structure is applied to evaluate risks for potential failures. Last the assessments are dealt and the risk priorities are ranked by using the grey relative projection.

The detail steps of the proposed model are as follows:

Step 1 Determine the objective of the risk evaluation and the potential failures.

Step 2 Recruit a FMEA team and assign weights for the members.

Step 3 Evaluate the risks of failures with fuzzy belief structure according to Table 2.

Step 4 Collect and deal with the risk assessments provided by team members by eq 8. Then defuzzify the assessments by eq 9 to construct the comparative matrix of RFs as eq 10.

Step 5 Determine the reference matrix of *RFs* by eqs 11 and 12 and calculate the grey relation coefficient by eqs 13 and 14.

Step 6 Evaluate the weights of risk factors with fuzzy belief structure according to Table 2 and collect and deal with the risk assessments by eqs 15-18 to obtain the relative weights of risk factors.

Step 7 Receive the weighted grey relation matrix by eqs 19 and 20.

Step 8 Calculate the grey relation project by eqs 21 and 22 and rank the potential failures by eq 23.

5. Example

In this section, a case study verification of a certain type of cylindrical sealer with the proposed FMEA model has been applied to illustrate the effectiveness.

Suppose the FMEA team is made up of five experts who are respectively from technology department, equipment department, technical instructors, scene operators and inspectors. With different professional knowledge, experiences and departments, each of them plays a different role in the team and is assigned a different relative weight, which are assumed respectively to be 0.25, 0.25, 0.2, 0.2 and 0.1. After group discussing, the potential failures of this type cylindrical sealer are described as welding deformation(FM_1), bolt fastening deformation(FM_2), higher roundness of flange rings(FM_3), higher roundness of adjustment rings(FM_4), higher roundness of intermediate fixed rings(FM_5) and higher coaxiality of intermediate fixed ring assemblies(FM_6). The case study verification compares the proposed method in terms of four risk factors *S*, *O*, *D* and *E* with the conventional method in terms of three risk factors *S*, *O* and *D*.

5.1. Analysis by Using the Proposed FMEA Model

According to criterions in Table 1 and Table 2, team members evaluation information of above the six failures, which are described in Table 3.

Determine the risk factors comparison matrix by eqs 7, 8 and 9:

$$Z_6(4)=\begin{bmatrix} FM_1 \\ FM_2 \\ FM_3 \\ FM_4 \\ FM_5 \\ FM_6 \end{bmatrix}=\begin{bmatrix} 5.00 & 4.10 & 4.52 & 3.20 \\ 2.55 & 3.10 & 2.90 & 2.26 \\ 2.65 & 2.00 & 2.80 & 3.15 \\ 2.10 & 2.05 & 2.65 & 2.50 \\ 2.65 & 2.05 & 2.40 & 2.33 \\ 3.15 & 2.95 & 2.20 & 2.23 \end{bmatrix}$$

Table 3. Assessments of failures for S, O, D and E

	S					O				
	TM_1	TM_2	TM_3	TM_4	TM_5	TM_1	TM_2	TM_3	TM_4	TM_5
FM_1	H_{55}	H_{55}	H_{55}	H_{55}	H_{55}	H_{44}	H_{44}	H_{55}	H_{33}	H_{55}
FM_2	H_{33}	H_{22}	H_{22}	H_{33}	H_{33}	H_{33}	H_{33}	H_{33}	H_{33}	H_{44}
FM_3	H_{22}	H_{33}	H_{44}	H_{22}	H_{22}	H_{22}	H_{22}	H_{11}	H_{33}	H_{22}
FM_4	H_{22}	H_{22}	H_{22}	H_{22}	H_{33}	H_{33}	H_{22}	H_{11}	H_{22}	H_{22}
FM_5	H_{33}	H_{22}	H_{33}	H_{33}	H_{22}	H_{22}	H_{22}	H_{11}	H_{33}	$(H_{22},0.5)$ $(H_{33},0.5)$
FM_6	H_{33}	H_{22}	H_{55}	H_{33}	H_{33}	H_{33}	H_{44}	H_{22}	H_{33}	H_{22}

Continued Table 3

	D					E				
	TM_1	TM_2	TM_3	TM_4	TM_5	TM_1	TM_2	TM_3	TM_4	TM_5
FM_1	H_{44}	H_{55}	H_{55}	H_{44}	$(H_{44},0.3)$ $(H_{55},0.7)$	H_{44}	H_{22}	H_{33}	H_{44}	H_{33}
FM_2	H_{22}	H_{22}	H_{44}	H_{44}	H_{33}	H_{22}	H_{22}	H_{22}	H_{33}	$(H_{22},0.4)$ $(H_{33},0.6)$
FM_3	H_{44}	H_{22}	H_{33}	H_{22}	H_{33}	H_{55}	H_{22}	H_{22}	H_{44}	H_{22}
FM_4	H_{33}	H_{22}	H_{33}	H_{33}	H_{33}	H_{44}	H_{22}	H_{22}	H_{22}	H_{22}
FM_5	H_{22}	H_{22}	H_{44}	H_{22}	H_{22}	H_{33}	H_{22}	$(H_{11},0.6)$ $(H_{22},0.4)$	H_{33}	H_{22}
FM_6	H_{33}	H_{11}	H_{33}	H_{22}	H_{22}	H_{33}	H_{22}	$(H_{11},0.6)$ $(H_{22},0.4)$	H_{33}	H_{11}

Determine the risk factors reference matrix by the highest levels and the lowest levels of all the risk factors:

$$Z_0^+ = \begin{bmatrix} 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 \end{bmatrix} \quad Z_0^- = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Collect the assessments of the six failures for the weights of S, O, D and E given by the FMEA team members. As shown in Table 4.

Defuzzify and normalize the relative importance weights by eqs 15-18:

$$\bar{\omega}_1 = 0.298\,,\ \bar{\omega}_2 = 0.254\,,\ \bar{\omega}_3 = 0.232\,,\ \bar{\omega}_4 = 0.216$$

Calculate and assign weights to the grey relation coefficients by eqs 13, 14, 19 and 20. Then the weighted grey relation matrix S^+ and S^- are respectively described as:

$$S^+ = \begin{bmatrix} 0.069 & 0.068 & 0.058 & 0.069 \\ 0.110 & 0.083 & 0.079 & 0.086 \\ 0.108 & 0.109 & 0.081 & 0.070 \\ 0.124 & 0.107 & 0.084 & 0.081 \\ 0.108 & 0.107 & 0.089 & 0.085 \\ 0.096 & 0.086 & 0.094 & 0.087 \end{bmatrix} \quad S^- = \begin{bmatrix} 0.128 & 0.125 & 0.107 & 0.125 \\ 0.196 & 0.149 & 0.142 & 0.152 \\ 0.192 & 0.191 & 0.145 & 0.126 \\ 0.218 & 0.188 & 0.150 & 0.155 \\ 0.192 & 0.188 & 0.158 & 0.150 \\ 0.174 & 0.154 & 0.166 & 0.153 \end{bmatrix}$$

Calculate the grey relation projection P_y^+ and P_y^- by eqs 21 and 22, then the relative projections are obtained by eq 23. Results are shown in Table 5.

As known from Table 5, the value of R_1 is the lowest, which means that FM_1 is of the highest risk and should be given more attention to take measures.

5.2. Comparison and Discussion

To compare with conventional method in terms of three risk factors S, O and D, the same assessment information without E in Tables 3 and 4 are dealt in the same way. Results are shown in Table 6:

In the proposed method in terms of four risk factors and the conventional method in terms of three risk factors, all the assessment information of risk factors S, O, D and E and their relative weights were given by FMEA team members according to the same evaluation ratings. In the two method, the same approaches were applied to deal with evaluations and to rank failures. As seen from Table 6, risk priority rankings were $FM_1 > FM_2 = FM_6 > FM_3 > FM_5 > FM_4$. While in the production, FM_3 had fewer processing steps, fewer consumables, shorter working hours and lower cost than FM_6. Similarly, FM_4 had smaller size, fewer consumables, shorter working hours and lower cost than FM_5. The cost risks of FM_3 and FM_4 were respectively lower than those of

Table 4. Assessments of failures for the weights of S, O, D an d E

	S					O				
	TM_1	TM_2	TM_3	TM_4	TM_5	TM_1	TM_2	TM_3	TM_4	TM_5
FM_1	H^0_{33}	H^0_{55}	H^0_{55}	H^0_{33}	H^0_{55}	H^0_{44}	H^0_{44}	H^0_{55}	H^0_{33}	H^0_{44}
FM_2	H^0_{22}	H^0_{55}	$(H^0_{44},0.5)$ $(H^0_{55},0.5)$	H^0_{22}	H^0_{55}	H^0_{33}	H^0_{33}	H^0_{44}	H^0_{33}	H^0_{33}
FM_3	H^0_{44}	H^0_{55}	H^0_{44}	H^0_{44}	H^0_{44}	H^0_{33}	H^0_{33}	$(H^0_{33},0.8)$ $(H^0_{44},0.2)$	H^0_{33}	H^0_{22}
FM_4	H^0_{33}	H^0_{55}	H^0_{44}	H^0_{33}	H^0_{44}	H^0_{33}	H^0_{33}	H^0_{33}	H^0_{33}	H^0_{22}
FM_5	H^0_{22}	H^0_{55}	H^0_{44}	H^0_{22}	H^0_{44}	H^0_{33}	H^0_{44}	H^0_{22}	H^0_{33}	H^0_{22}
FM_6	H^0_{22}	H^0_{55}	H^0_{44}	H^0_{22}	H^0_{44}	H^0_{44}	H^0_{33}	H^0_{33}	H^0_{33}	H^0_{11}

Continued Table 4

	D					E				
	TM_1	TM_2	TM_3	TM_4	TM_5	TM_1	TM_2	TM_3	TM_4	TM_5
FM_1	H^0_{22}	H^0_{22}	H^0_{44}	H^0_{44}	H^0_{44}	H^0_{55}	H^0_{22}	H^0_{44}	H^0_{55}	H^0_{33}
FM_2	H^0_{44}	H^0_{22}	H^0_{33}	H^0_{44}	H^0_{33}	H^0_{33}	H^0_{11}	H^0_{33}	H^0_{33}	H^0_{33}
FM_3	H^0_{33}	H^0_{22}	H^0_{22}	H^0_{33}	H^0_{22}	H^0_{44}	H^0_{22}	H^0_{22}	H^0_{44}	H^0_{22}
FM_4	H^0_{44}	H^0_{22}	H^0_{22}	H^0_{44}	H^0_{22}	H^0_{33}	H^0_{22}	H^0_{22}	H^0_{22}	H^0_{22}
FM_5	H^0_{44}	H^0_{22}	H^0_{33}	H^0_{44}	H^0_{22}	H^0_{33}	H^0_{22}	H^0_{22}	H^0_{33}	H^0_{22}
FM_6	H^0_{33}	H^0_{22}	H^0_{33}	H^0_{44}	H^0_{11}	H^0_{33}	H^0_{11}	H^0_{22}	H^0_{33}	H^0_{33}

Table 5. Risk priority rankings of the six failures

Failure modes	P_y^+	P_y^-	R_y	ranking
FM_1	0.132	0.242	0.353	1
FM_2	0.181	0.323	0.359	2
FM_3	0.186	0.332	0.359	2
FM_4	0.201	0.356	0.361	3
FM_5	0.196	0.347	0.361	3
FM_6	0.181	0.323	0.359	2

Table 6. Risk priority rankings of six failures without E

Failure modes	P_y^+	P_y^-	R_y	ranking
FM_1	0.143	0.265	0.350	1
FM_2	0.202	0.362	0.358	2
FM_3	0.220	0.390	0.360	3
FM_4	0.235	0.413	0.363	5
FM_5	0.224	0.397	0.361	4
FM_6	0.202	0.362	0.358	2

FM_6 and FM_5. The risk priority rankings in Table 5, $FM_1 > FM_2 = FM_3 = FM_6 > FM_4 = FM_5$, was thought that FM_3 and FM_6 had the same risk and FM_4 and FM_5 had the same risk considering both quality and cost, which was more practical and more rational. Differences between Table 5 and Table 6 indicated that risk factor E played a very important role in risk priorities ranking. Therefore when conducting FMEA, both quality risk and cost risk should be taken into consideration to obtain more reasonable rankings.

Conclusions

Under the consideration of risk in terms of quality and cost in the process, this paper proposed the 4th risk factor economy (E) after the three risk factors S, O and D in traditional FMEA. Thus a new economy FMEA method is proposed which consists of four risk factors. E is defined as the ratio of total outputs and the inputs of all production elements in the life cycle of the project and the ratings of E is formulated by the proportion of the E_{max} in the corresponding field. Compared with traditional FMEA, the proposed method has several advantages. First, Fuzzy belief structure is used to well reflect and model the assessments which are uncertain, incomplete and diverse. Second, for different failures ,the relative importance weight of each risk factor is different, which are full considered in this paper. Third, the projections of failures respectively on the highest risk alternative and the lowest alternative are used to obtain a more accurate risk priority rankings. Last, cost risk is taken into consideration and failures are ranked on the basis of evaluations for S, O, D and E.

In the future, the research of FMEA will focus on the following directions. First, As the FMEA team members are from different departments and have different knowledge and experience, assessments of failures for the weights of S, O, D and E have strong subjectivity. So other approaches should be developed to make assessments more objective. Second, cost risk should be considered and the relative risk indicators and ratings can be improved by other reasonable methods. Last, the calculation of risk prioritization method at present is too complex, future research should simplify the data processing. With the improvement of FMEA method, the FMEA will be more convenient and reliable in quality improvement field.

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