A Simplified Model of 3-pump and 4-pump Four Wave Mixing Processes with Same Wavelength Spacing in Optical Fibre

Xu Ruihong*, **Yu Chongxiu, Hsu Dahsiung**

Institute of Information Photonics and Optical Communications, Beijing University of Posts and Telecommunications, Beijing, China

Abstract

Multi-pump constructure in four wave mixing process has been researched both theoretically and experimentally on decade. Traditional propagation equations are complicated to analyse a FWM model which has more than 2 pumps. In this work the propagation equations are simplified and keep the key FWM processes, to illustrate how FWM processes works in multi-pump constructure with the same wavelength spacing, theoretically. Considering the number of total waves in fibre would increase rapidly when pumps are more than 5, the 3-pump and 4-pump FWM process are analysed and simulated in high nonlinear photonic crystal fibre. The result demonstrate the theoretical model, and prove the advantages of multi-pump FWM process.

Keywords: MULTI-PUMP, FOUR WAVE MIXING, PHOTONIC CRYSTAL FIBRE, OPTICAL PARAMETRIC AMPLIFIER.

1. Introduction

Four Wave Mixing (FWM) in optical fibre[1], as its high nonlinear efficiency, simple experimental constructure, etc., has been a more and more remarkable solution in many applications such as photon-pair generation [2], wavelength conversion (WC), optical parametric amplification (OPA), optical quantum-information processing, and optical switching [3]

FWM transfer equations in most thesis referred to only one resonance, which explained the model concerned dual pump and their first order sidebands. However, the higher order sidebands also exist in optical fibre. In some research model sidebands can't be ignored: (1) 2nd order sidebands: Thompson et al. proposed and derived the propagation equations of multiple FWM processes including first order and second-order sidebands, six propagation waves in all[4]; (2) Higher order sidebands: Millot et al. developed a model with high-order sidebands further and explored the interactions between pumps and high-order sidebands[5]; (3) Multiple FWM process: Trillo et al. investigated the nonlinear dynamics of dual-frequency-pump multiple FWM processes[6]; (4) 3-pump and 4-pump FWM process: Liu proposed the propagation equations involved three pumps and four pumps FWM processes, then discovered a self-stability function[7]. Based on Liu's theory, arbitrary number of pumps with same wavelength slot can be developed to set up a FWM solution. Theoretical model of 3-pump and 4-pump wavelength conversion was set up as a model, to demonstrate the simplification and optimization of multi-pump FWM process. The result and the theoretical model are with good agreement.

2. Simplification and Derivation of Multi-Pump FWM Equation

2.1. Nonlinear Polarization Simplification

Governed by Maxwell's Equations, as there is no current or induced magnetization in optical fibre, wave propagation equation can be simplified as,

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \mu_0\left(\frac{\partial^2\mathbf{P_L}}{\partial t^2} + \frac{\partial^2\mathbf{P_{NL}}}{\partial t^2}\right) \quad (1)$$

where linear and nonlinear polarization is given by

$$\mathbf{P_L} = \varepsilon_0\chi^{(1)}\mathbf{E} \quad (2)$$

$$\mathbf{P_{NL}} = \varepsilon_0\chi^{(3)} \vdots \mathbf{EEE} \quad (3)$$

In common fibres, as the absorption loss can be ignored, the refractive index can be

$$n_0^2 = 1 + \chi^{(1)} \quad (4)$$

Eq.(1) can be simplified as

$$\nabla^2\mathbf{E} - \frac{n_0^2}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \mu_0\frac{\partial^2\mathbf{P_{NL}}}{\partial t^2} \quad (5)$$

According to most of the FWM experiments, the optical waves are assumed to be polarized. The electric field of the guided modes inside the fibre can be written as [8]

$$\mathbf{E} = \frac{1}{2}\hat{x}\sum_{m=1}^{M} E_m \exp[i(\beta_m z - \omega_m t)] + c.c. \quad (6)$$

where m=1, 2, …, M represents the number of wave existing in fibre; $\hat{x}$ is the unit vector; $\beta_m = n_m\omega_m / 2$ is the propagation constant including material and wave guiding effects in refractive index n_m. Polarization $\mathbf{P_{NL}}$ can be modified as a triple sum as

$$\mathbf{P_{NL}} = \frac{1}{8}\varepsilon_0\chi^{(3)}\hat{x}\sum_{p,q,r} C_{p,q,r}E_pE_qE_r \exp[i(\xi_p + \xi_q + \xi_r)] \quad (7)$$

where $\xi_m = \beta_m z - \omega_m t$, $p,q,r = 1,2,...,M$

2.2. The Number of Waves in Optical Fibre

In this work, when the 1st order sideband taken into account only, incident pump waves are labeled as a, b, c, …, n, sideband waves as 1, 2, 3, …, ascending according to frequency. N-pump (N≥2) FWM processes with same wavelength spacing would induce 3N-2 wave in all, including both incident pump waves and sideband waves (M=3N-2). N=2 is the usually the basic model of FWM. There would be 2 pump waves and 2 sideband waves in fibre.

In classical theory, FWM only occurs when the mismatch of phase is nearly zero. In numerical equation it is:

$$\Delta\beta z = (\beta_j + \beta_k) - (\beta_h + \beta_l)z \approx 0 \quad (8)$$

here j and k represent inner frequencies, while h and l the both side. When j = k, it is called degenerated process; while j ≠ k, it is non-degenerated process. Therefore, according to Eq.(5), there would be 7 waves in 3-pump process (3 pumps and 4 1st sidebands), which induce 22 FWM processes (9 degenerated FWM and 13 non-degenerated FWM). If $\omega_h\omega_j\omega_l$ represents the degenerated FWM process (j = k), $\omega_h\omega_j\omega_k\omega_l$ represents the non-degenerated FWM process (j ≠ k) that phase matches as $\omega_j + \omega_k = \omega_h + \omega_l$, the degenerated FWM processes are $\omega_1\omega_2\omega_a$, $\omega_1\omega_a\omega_c$, $\omega_1\omega_b\omega_4$, $\omega_2\omega_a\omega_b$, $\omega_2\omega_b\omega_3$, $\omega_a\omega_b\omega_c$, $\omega_a\omega_c\omega_4$, $\omega_b\omega_c\omega_3$ and $\omega_c\omega_3\omega_4$; non-degenerated FWM processes are $\omega_1\omega_2\omega_a\omega_b$, $\omega_1\omega_2\omega_b\omega_c$, $\omega_1\omega_2\omega_c\omega_3$, $\omega_1\omega_2\omega_3\omega_4$, $\omega_1\omega_a\omega_b\omega_3$, $\omega_1\omega_a\omega_c\omega_4$, $\omega_2\omega_a\omega_b\omega_c$, $\omega_2\omega_a\omega_c\omega_3$, $\omega_2\omega_a\omega_3\omega_4$, $\omega_2\omega_b\omega_c\omega_4$, $\omega_a\omega_b\omega_c\omega_3$, $\omega_a\omega_b\omega_3\omega_4$ and $\omega_b\omega_c\omega_3\omega_4$. In the applications like wavelength conversion and optical parametric amplification, the efficiency of FWM determines the overall performance of the system. Considering the power difference, in all the FWM processes, only the processes which one or no sideband wave involved are taken as the "key FWM". In 3-pump FWM system, they are $\omega_1\omega_a\omega_c$, $\omega_2\omega_a\omega_b$, $\omega_a\omega_b\omega_c$, $\omega_a\omega_c\omega_4$ and $\omega_b\omega_c\omega_3$ of degenerated FWM; $\omega_2\omega_a\omega_b\omega_c$ and $\omega_a\omega_b\omega_c\omega_3$ of non-degenerated FWM. Liu et al figured out the quantity of the FWM waves in fibre, both degenerated FWM and non-degenerated FWM waves, then organised into Table.1.

Table 1. Numbers of FWM processes with multi pumps

N	All FWM		Key FWM (idler=0 or 1)	
	D	ND	D	ND
2	2	1	2	0
3	9	13	5	2
4	20	50	10	7
5	36	125	16	17
6	56	252	24	33
7	81	144	33	57
8	110	715	44	90
9	144	1078	56	134
10	182	1547	70	190

* D=degenerated, ND=non-degenerated [7]

It is obviously that as the amount of pumps increases, the number of all waves grows in a much faster way. When N≥5 even, there is hardly significance of research.

2.3. Simplification of Wave Propagation Equation

To solve Eq.(5), $\mathbf{P_{NL}}$ and $\mathbf{E}$ are figured out respectively. In quasi-continuous wave assumption, we take ω_1 in 3-pump FWM process as an example, 2nd order derivation of Eq.(7) would be,

$$\mu_0 \frac{\partial^2 \mathbf{P_{NL}}(\omega_1)}{\partial t^2}$$
$$= -\frac{3\chi^{(3)}\omega_1^2}{8c^2}\hat{x}\Bigg[\Big(|E_1|^2 + 2\sum_{h\neq 1}|E_h|^2\Big)E_1$$
$$+E_2^2 E_a^* e_{122a} + E_a^2 E_c^* e_{1aac} + E_b^2 E_4^* e_{1bb4}$$
$$+2E_2 E_a E_b^* e_{12ab} + 2E_2 E_b E_c^* e_{12bc}$$
$$+2E_2 E_c E_3^* e_{12c3} + 2E_2 E_3 E_4^* e_{1234}$$
$$+2E_a E_b E_3^* e_{1ab3} + 2E_a E_c E_4^* e_{1ac4}\Bigg] e^{i(\beta_1 z - \omega_1 t)} \tag{9}$$

in which according to Eq.(8), the phase match is in the form as

$$e_{hjkl} = e^{iz[\beta(\omega_j)+\beta(\omega_k)-\beta(\omega_h)-\beta(\omega_l)]} \tag{10}$$

According to [4], a simplification of $\mathbf{E}$ could be,

$$\nabla^2 \mathbf{E}(\omega_h) - \frac{n_o^2}{c^2}\frac{\partial^2 \mathbf{E}(\omega_h)}{\partial t^2}$$
$$\approx i\beta_h \frac{\psi_h}{N_h}\frac{\partial A_h}{\partial z}\hat{x}\, e^{i(\beta_h z - \omega_h t)} \tag{11}$$

Substituting Eq.(9) and Eq.(11) into Eq.(5), in [7], the rigorous propagation equation of ω_1 should be,

$$\frac{\partial A_1}{\partial z} = i\gamma_1 (M_1$$
$$+A_2^2 A_a^* e_{122a} + A_a^2 A_c^* e_{1aac} + A_b^2 A_4^* e_{1bb4}$$
$$+2A_2 A_a A_b^* e_{12ab} + 2A_2 A_b A_c^* e_{12bc}$$
$$+2A_2 A_c A_3^* e_{12c3} + 2A_2 A_3 A_4^* e_{1234}$$
$$+2A_a A_b A_3^* e_{1ab3} + 2A_a A_c A_4^* e_{1ac4}) \tag{12}$$

in which M represents the SPM and XPM part as

$$M_1 = (|A_1|^2 + 2\sum_{h\neq 1}|A_h|^2)A_1 \tag{13}$$

and $\gamma = n_2\omega_h/(cA_{eff})$ is the nonlinearity coefficient for all the 7 wavelengths.

As in Sec2.2, the efficiency of OPA and WC depending on the power of wave involving in FWM, the equations can be simplified into the model that only “key FWM” make effort. Hence, Eq.(12) can be,

$$\frac{\partial A_1}{\partial z} = i\gamma_1 (M_1 + A_a^2 A_c^* e_{1aac}) \tag{14}$$

Then the simplified propagation equations of other waves in 3-pump FWM are as followed,

$$\frac{\partial A_2}{\partial z} = i\gamma_2 (M_2 + A_a^2 A_b^* e_{2aab} + 2A_a A_b A_c^* e_{2abc}) \tag{15}$$

$$\frac{\partial A_a}{\partial z} = i\gamma_a (M_a + A_b^2 A_c^* e_{abbc}) \tag{16}$$

$$\frac{\partial A_b}{\partial z} = i\gamma_b (M_b + 2A_a A_c A_b^* e_{bacb}) \tag{17}$$

$$\frac{\partial A_c}{\partial z} = i\gamma_c (M_c + A_b^2 A_a^* e_{cbba}) \tag{18}$$

$$\frac{\partial A_3}{\partial z} = i\gamma_3 (M_3 + A_c^2 A_b^* e_{3ccb} + 2A_c A_b A_a^* e_{3cba}) \tag{19}$$

$$\frac{\partial A_4}{\partial z} = i\gamma_4 (M_4 + A_c^2 A_a^* e_{4cca}) \tag{20}$$

With the similar analysis and simplification, 4-pump propagation equations can be

$$\frac{\partial A_1}{\partial z} = i\gamma_1 (M_1 + A_a^2 A_d^* e_{1aad}) \tag{21}$$

$$\frac{\partial A_2}{\partial z} = i\gamma_2 (M_2 + A_a^2 A_c^* e_{2aac} + 2A_a A_b A_d^* e_{2abd}) \tag{22}$$

$$\frac{\partial A_3}{\partial z} = i\gamma_3 (M_3 + A_a^2 A_b^* e_{3aab} + A_b^2 A_d^* e_{3bbd}$$
$$+2A_a A_b A_c^* e_{3abc} + 2A_a A_c A_d^* e_{3acd}) \tag{23}$$

$$\frac{\partial A_a}{\partial z} = i\gamma_a (M_a + A_b^2 A_c^* e_{abbc} + 2A_b A_c A_d^* e_{abcd}) \tag{24}$$

$$\frac{\partial A_b}{\partial z} = i\gamma_b (M_b$$
$$+A_c^2 A_d^* e_{bccd} + 2A_a A_c A_b^* e_{bacb} + 2A_a A_d A_c^* e_{badc}) \tag{25}$$

$$\frac{\partial A_c}{\partial z} = i\gamma_c (M_c$$
$$+A_b^2 A_a^* e_{cbba} + 2A_b A_d A_c^* e_{cbdc} + 2A_a A_d A_b^* e_{cadb}) \tag{26}$$

$$\frac{\partial A_d}{\partial z} = i\gamma_d (M_d + A_c^2 A_b^* e_{dccb} + 2A_b A_c A_a^* e_{dbca}) \tag{27}$$

$$\frac{\partial A_4}{\partial z} = i\gamma_4 (M_4 + A_c^2 A_a^* e_{4cca} + A_d^2 A_c^* e_{4ddc}$$
$$+2A_b A_d A_a^* e_{4bda} + 2A_c A_d A_b^* e_{4cdb}) \tag{28}$$

$$\frac{\partial A_5}{\partial z} = i\gamma_5 (M_5 + A_d^2 A_b^* e_{5ddb} + 2A_c A_d A_a^* e_{5cda}) \tag{29}$$

$$\frac{\partial A_6}{\partial z} = i\gamma_6 (M_6 + A_d^2 A_a^* e_{6dda}) \tag{30}$$

They include degenerated FWM processes $\omega_1\omega_a\omega_d$, $\omega_2\omega_a\omega_c$, $\omega_3\omega_a\omega_b$, $\omega_3\omega_b\omega_d$, $\omega_a\omega_b\omega_c$, $\omega_a\omega_c\omega_4$, $\omega_a\omega_d\omega_6$, $\omega_b\omega_c\omega_d$, $\omega_b\omega_d\omega_5$, $\omega_c\omega_d\omega_4$; and non-degenerated FWM processes $\omega_2\omega_a\omega_b\omega_d$, $\omega_3\omega_a\omega_b\omega_c$, $\omega_3\omega_a\omega_c\omega_d$, $\omega_a\omega_b\omega_c\omega_d$, $\omega_a\omega_b\omega_d\omega_4$, $\omega_a\omega_c\omega_d\omega_5$, $\omega_b\omega_c\omega_d\omega_4$.

3. Simulation and Results

Fig.(1) shows the basic theory of multi-pump FWM system structure. Actually, to generate the multi-pump model with same frequency spacing, a fibre ring structure is the typical solution, shown in Fig.(2). By adjusting the FBGs, 3-pump and 4-pump source can be obtained.

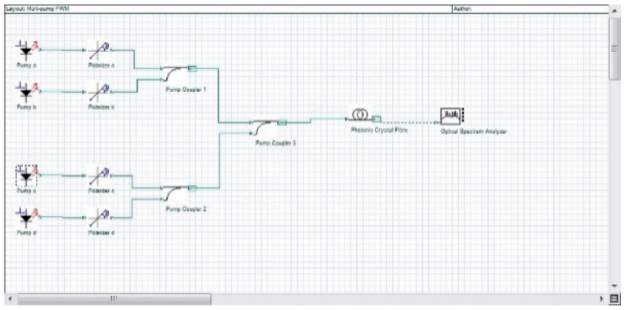

Figure 1. Theory of Multi-pump FWM

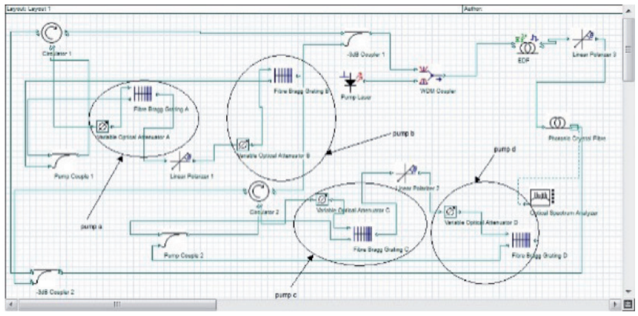

Figure 2. Simulation System Setup

It is worth noting that photonic crystal fibre is used for its high linearity which contribute the most in FWM. Its nonlinear coefficient often reaches up to 20/W/km, even more than 200/W/km. In simulation here we assume that γ=20/W/km over the range of 1480–1620 nm, instead of 11/W/km of traditional SMF[9] [10] [11]; while D=1ps/nm/km, α=1.2dB/km. The initial phases of the pump waves are set to be 0, ideally. All the simulation are in the circumstance of OptiSystem 7.0.

In Fig.3 and Fig.4, these results are shown as followed:

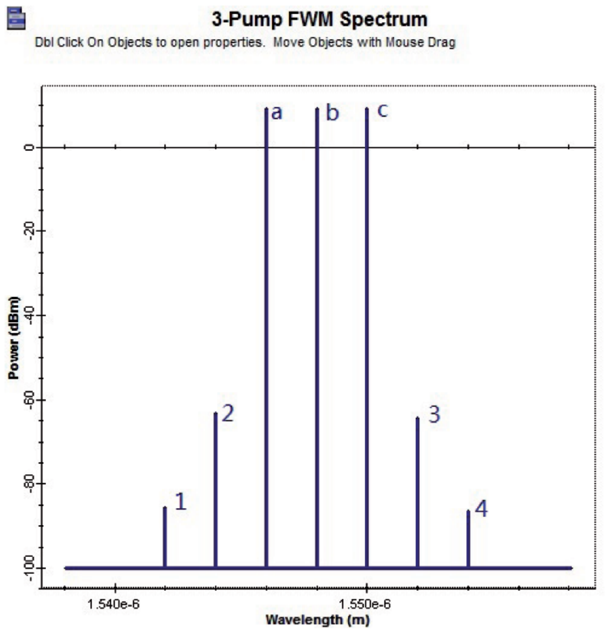

Figure 3. 3-pump FWM output

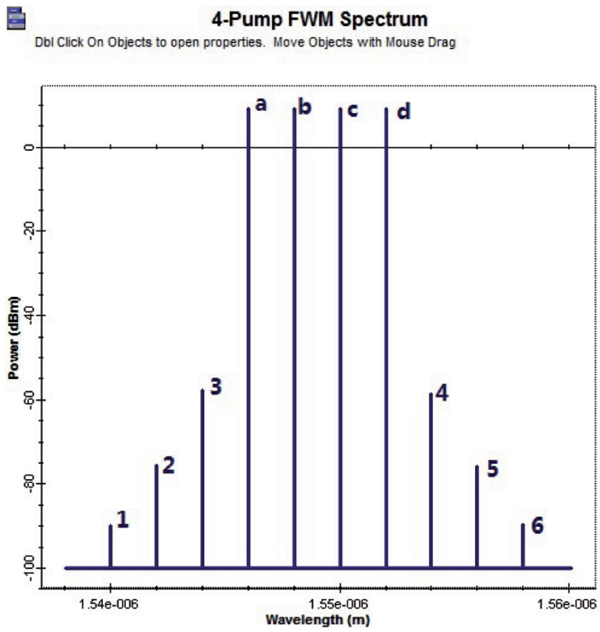

Figure 4. 4-pump FWM output

(1) With the number of pumps increases, the symmetric induced wave pairs, for instance, wave 2 and wave 3 in 3-pump, are approximately of the same power. (2) The power difference between pump and induced wave nearest to pumps are slightly different. In 4-pump situation it's smaller than that in 3-pump. In another word, with same pump power, the power of idler nearest to the pumps in 4-pump FWM is slightly larger than that in 3-pump. (3) The power of waves fluctuate as the initial phase. Assumed the perturbation of pump power along the fibre is linear, which is expressed as P(L)=P(0)+δP. The stability of the whole FWM process is decided by δP. After the calculation in 3-pump,

$$\delta P_b(L) \propto -\sin[\varphi_a(0)+\varphi_c(0)-2\varphi_b(0)] \tag{30}$$

$$\delta P_{a,c}(L) \propto \sin[\varphi_a(0)+\varphi_c(0)-2\varphi_b(0)] \tag{31}$$

while in 4-pump,

$$\begin{aligned}\partial P_a(L)/\partial z \propto \{&\sin[\varphi_a(0)+\varphi_c(0)-2\varphi_b(0)]\\ &+2\sin[\varphi_a(0)+\varphi_d(0)-\varphi_b(0)-\varphi_c(0)]\}\end{aligned} \tag{32}$$

$$\begin{aligned}\partial P_d(L)/\partial z \propto \{&\sin[\varphi_b(0)+\varphi_d(0)-2\varphi_c(0)]\\ &+2\sin[\varphi_a(0)+\varphi_d(0)-\varphi_b(0)-\varphi_c(0)]\}\end{aligned} \tag{33}$$

$$\begin{aligned}\partial P_b(L)/\partial z \propto \{&\sin[\varphi_b(0)+\varphi_d(0)-2\varphi_c(0)]\\ &-2\sin[\varphi_a(0)+\varphi_c(0)-2\varphi_b(0)]\\ &-2\sin[\varphi_a(0)+\varphi_d(0)-\varphi_b(0)-\varphi_c(0)]\}\end{aligned} \tag{34}$$

$$\begin{aligned}\partial P_c(L)/\partial z \propto \{&\sin[\varphi_a(0)+\varphi_c(0)-2\varphi_b(0)]\\ &-2\sin[\varphi_b(0)+\varphi_d(0)-2\varphi_c(0)]\\ &-2\sin[\varphi_a(0)+\varphi_d(0)-\varphi_b(0)-\varphi_c(0)]\}\end{aligned} \tag{35}$$

where φ(0) is the initial phase of pumps.

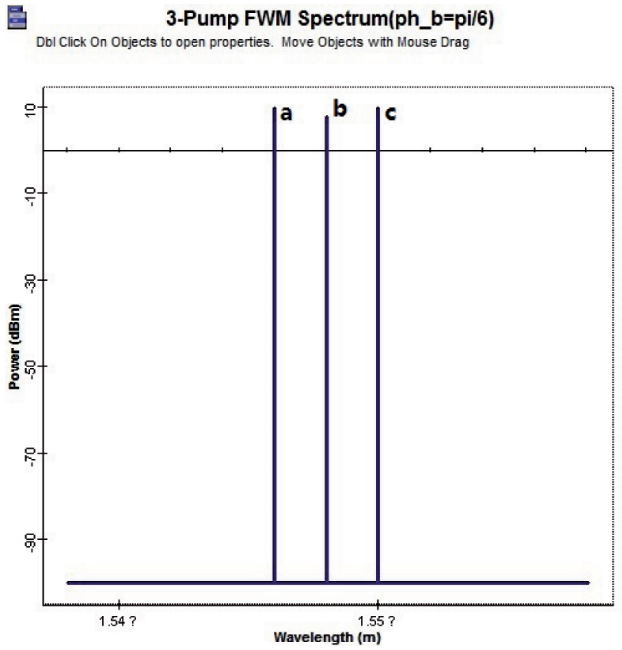

Figure 5. 3-pump FWM output, with sidebands filtered

In Eq.(30)-(35), it is obviously that (1) as the pump added, the power fluctuation is less affected by phase perturbation; (2) by adjusting the initial phase, the energy flow governed by energy conservation can be controlled. Take 3-pump as an example. If $\varphi_a(0)+\varphi_c(0)-2\varphi_b(0)<\pi/2$, the power of pump B is getting smaller while A and C larger along the fibre. Shown in Fig.5, with the sidebands filtered, phase of ω_b is $\pi/6$ away from $\omega_a=\omega_c$, the curve of ω_b is slightly downward, while two others upward for energy conservation. In wavelength conversion experiments, if the incident signal input ω_b were to transfer to higher frequency ω_a, an extra pump with lower frequency ω_c could be added to enhance power of ω_a, by adjusting their initial phase.

4. Conclusions

The propagation equations of FWM process are solved; with the most affected factors, they are simplified to illustrate the multi-pump FWM processes. Considering the number of wave increases rapidly when pumps are more than 5, 3-pump and 4-pump FWM process are simulated to prove their advantages in stability and power increasing, to single or dual pump FWM process.

5. Acknowledgements

This work is supported by National High-tech R&D Program (863 Program) (Grant No. 2013AA031501).

References

1. Takesue H, Inoue K. Generation of polarization entangled photon pairs and violation of Bell's inequality using spontaneous four-wave mixing in fiber loop[J]. Phys.rev.a, 2004, 70(10):031802.
2. Lin Q, Yaman F, Agrawal G P. Photon-pair generation in optical fibers through four-wave mixing: Role of Raman scattering and pump polarization[J]. Physical Review A, 2007, 75(2):441-445.
3. J. Fulconis, O. Alibart, W.J. Wadsworth, et al. Quantum interference with photon pairs using two micro-structured fibres[J]. New Journal of Physics, 2007, 9(8).
4. Thompson J R, Roy R. Nonlinear dynamics of multiple four-wave mixing processes in a single-mode fiber.[J]. Physical Review A, 1991, 43(9):4987-4996.
5. Millot G ,. Multiple four-wave mixing-induced modulational instability in highly birefringent fibers.[J]. Optics Letters, 2001, 26(18):1391-1393.
6. Trillo S, Wabnitz S, Kennedy T A. Nonlinear dynamics of dual-frequency-pumped multiwave mixing in optical fibers.[J]. Physical Review A, 1994, 50(2):1732-1747.
7. Liu X M. Theory and experiments for multiple four-wave-mixing processes with multifrequency pumps in optical fibers[J]. Physical Review A, 2008, 77(4):1912-1917.
8. Agrawal G.P. Nonlinear Fiber Optics, Third Edition and Applications of Nonlinear Fiber Optics.2001:244-272
9. Hansryd J, Andrekson P A, Westlund M, et al. Fiber-based optical parametric amplifiers and their applications[C]// IEEE J. Select. Topics Quantum Electron2002:506--520.
10. Biancalana F, Skryabin D V, Russell P S. Four-wave mixing instabilities in photonic-crystal and tapered fibers.[J]. Physical Review E, 2003, 68(4):352-375.
11. Monro T M, Ebendorff-Heidepriem H. PROGRESS IN MICROSTRUCTURED OPTICAL FIBERS[J]. Annual Review of Materials Research, 2006, 36(36):467-495.