Double Exponential Prediction Model for Surface Subsidence Distribution Due to Foundation Pit Excavation

Na Li, Wei Wang*, Fang Zhang, Zhang Chen

Department of Building Engineering, Shaoxing University, Shaoxing, China

Abstract

Subsidence of the surrounding surface after foundation pit excavation plays an important role in ensuring the engineering safety. The mathematical model for surface subsidence distribution was studied. The basic rules and mathematical distribution characteristics of surface subsidence were revealed and the traditional mathematical models were analyzed. Double exponential model (DE model) for surface subsidence distribution was proposed. By the fitting contrast of measured data between two project cases, it shows that DE Model is in good agreement with the measured data with the average fitting error of 5% and 13%. Compared with the traditional composite linear-exponential model, the fitting errors of the DE model are reduced by 34% and 16%, respectively.

Keywords: FOUNDATION PIT, SURFACE SUBSIDENCE, MATHEMATICAL MODEL, EXCAVATION

1. Introduction

In recent years, with the rapid economic development, the continuously expanding and sprawling of urban construction, the building is constantly extending to the ground and underground, and the deep foundation pit excavation is becoming more and more common, then the security issues of the surrounding environment are becoming increasingly prominent. It is necessary to further explore the deformation of the foundation pit and the surrounding soil in the process of excavation, and analyze the deformation of the surrounding soil, so as to predict the effects of foundation pit excavation on the surrounding buildings, structures, underground pipelines and roads, etc [1-3]. The soil downward displacement caused by foundation pit excavation within a certain range of enclosure wall is the surface subsidence, which is one of the main hazards of excavation to the surrounding environment. Therefore, in order to effectively control and predict the ground deformation caused by excavation, it is of great urgency to understand the distribution rules of surface subsidence and establish the assessment methods[4-7].

In this paper, a new mathematical model for the surface subsidence curve caused by excavation was proposed and analyzed according to the statistic of engineering measured data. Compared with other models, some meaningful conclusions were obtained which provided the reference for the future prediction and control methods on the surface subsidence due to deep foundation pit excavation [4-9].

2. Subsidence Features

Many studies on the subsidence caused by excavation are carried out by civil engineers in terms of both theoretical analysis and field investigation [8-12]. Based on these measured data and theoretical researches, various forms of surface subsidence curves due to foundation pit excavation are presented, which can be divided into two types: the groove-shaped subsidence and the triangular subsidence which are shown in Figure 1[13-14]. For the foundation pit with interior bracing supporting structure, if there is a greater lateral displacement of the wall in the cantilever excavation stage, the bottom wall deformation is smaller and lateral displacement of the top wall continues to increase during the supporting excavation, under this deformation the triangular subsidence will be generated. If there is a less lateral displacement and its increase is controlled by the supporting and enclosure wall when continuing excavation, the maximum lateral moves to the excavation face, thus the subsidence will become the groove-shaped curve [15-16]. Currently, in terms of the most applied construction technologies, the lateral displacement value of the top wall is small, so the subsidence is generally groove-shaped [12]. In this paper, we mainly focus on the groove-shaped subsidence analysis.

As shown in Figure 1 (a), the subsidence curve can be divided into three characteristic stages as shown in Figure 2. Taking the subsidence as the dependent variable y and the distance to the pit as the independent variable x, we can see that the ① curve is with a increasing and convex shape, the ② curve is with a decreasing and convex shape, while the ③ curve is with a decreasing concave shape. According to the curves characteristics, we can get the mathimatical featurs of subsidence curve:

in stage①,

$$\frac{dy}{dx} > 0,\ \frac{d^2y}{dx^2} < 0 \qquad (1)$$

in stage②,

$$\frac{dy}{dx} < 0,\ \frac{d^2y}{dx^2} < 0 \qquad (2)$$

in stage③,

$$\frac{dy}{dx} < 0,\ \frac{d^2y}{dx^2} > 0 \qquad (3)$$

3. Mathematical Model

3.1. Traditional Models

Chen has studied both the surface subsidence which is similar to triangular surface subsidence with cantilever supporting structure, and the asymmetrical surface subsidence with interior bracing supporting structure, and proposed the prediction model in equation (4) to estimate the surface subsidence [15]:

$$y = \frac{Ax+u}{e^{\frac{x^2}{m}}} \qquad (4)$$

Wherein, A, u and m are three undetermined parameters. Taking the supporting pile top as the original point of the coordinate system and the x-axis as the distance to the edge of the foundation pit, the y-axis is vertical downward. When x=0, y=u, it represents the subsidence at the edge of the foundation pit; when $x \to \infty$, y=0, it represents the surface subsidence is 0 at infinity distance, which meets the boundary conditions of the surface subsidence curves during excavation. When A, u, and m are all greater than 0, it can better simulate the groove-shaped subsidence. When A=0, the function model is shown as the following equation:

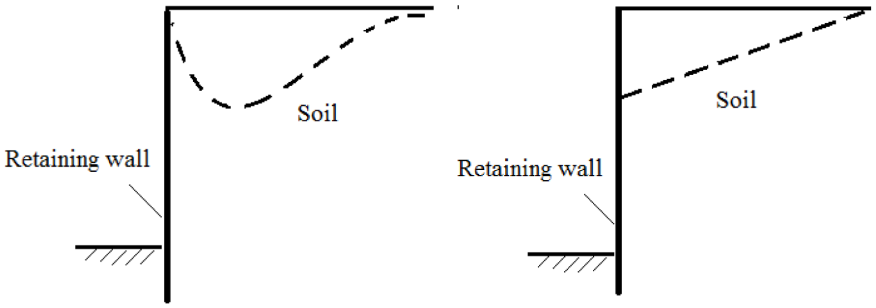
$$y = \frac{u}{e^{\frac{x^2}{m}}} \qquad (5)$$

The function model still satisfies the boundary conditions which can better simulate the triangular subsidence. In this paper, the model proposed by Chen is called as the composite linear-exponential model (CLE Model).

Wang has also proposed the subsidence model expression based on the maximum sedimentation and range, which can be simplified to equation (6) [12]. This model can better simulate the surface subsidence curve, but the shortcoming is that when x=0, then y=0, that is when the distance to the pit is 0, the subsidence is 0. This is inconsistent with the actual.

$$y = axe^{-bx^2} \qquad (6)$$

Wherein, y represents the subsidence, x represents the distance to the pit, a is the parameter related to the largest subsidence and subsidence influence range, b



(a) groove-shaped subsidence (b) triangular subsidence

Figure 1. Typical surface subsidence of the foundation pit

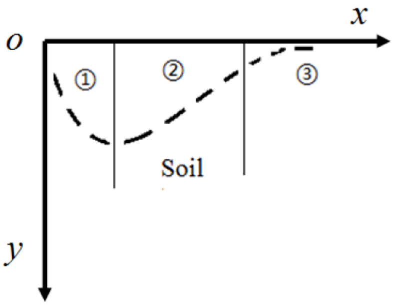


Figure 2. Groove-shaped curves characteristic

is the parameter related only to the subsidence influence range.

3.2. New Model

According to the measured data of surface subsidence in actual project cases, the traditional models are analyzed and studied. In consideration of the surface subsidence curve characteristics, we propose the following function model:

$$y = a\left(e^{-bx^2} - e^{-cx}\right) + k \tag{7}$$

Wherein, y represents the subsidence, x represents the distance to the pit; a, b, c, and k are undetermined parameters, which are all greater than 0, and $c>b$. Because it contains two kinds of exponential model, the equation (7) is called as the double exponential model (DE Model).

The first-order derivative and second-order derivative can be obtained from equation (7):

$$\frac{dy}{dx} = ace^{-cx} - 2abxe^{-bx^2} \tag{8}$$

$$\frac{d^2y}{dx^2} = 4ab^2x^2e^{-bx^2} - ac^2e^{-cx} - 2abe^{-bx^2} \tag{9}$$

It is easy to show that, with the parameter values change, the equation (8) and (9) can meet the concavity and convexity requirements of the subsidence curves described in the previous section from equation (1) to (3) and present the curve shape in Figure 2. The typical model curves and parameters are shown in Figure 3.

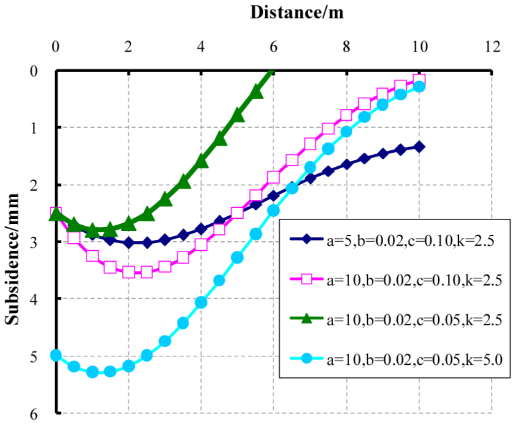


Figure 3. Typical curve and parameter

4. Model Comparison

4.1. Project Case 1

The project is located on the eastern side of Fengtan Road, Hangzhou city, Zhejiang Province, China. Foundation soils from top to bottom are: miscellaneous fill, silty clay, mucky soil and silty clay. The excavation depth of this project is about 12m with the use of internal shoring row piles supporting structure. The row piles are the composite structure of bored piles and cement mixing piles. Two support structures are set up with the center height of -4.25m and -9.1m, and with three steps of excavation. The field measured surface subsidence is shown in Table 1 [17]. Applying the function model to fit the actual measured subsidence curve, the measured surface subsidence and model fitting curve are shown in Figure 4.

Table 1. The field measurement subsidence of case1

Order	Distance/m	Subsidence/mm
1	0.00	1.35
2	3.06	6.33
3	6.12	12.79
4	7.21	13.73
5	9.17	13.19
6	12.01	11.58
7	14.85	9.69
8	17.91	7.81
9	20.97	6.33
10	24.03	4.98
11	26.87	4.31
12	29.71	3.77
13	32.77	3.50
14	35.83	3.37
15	38.88	2.96
16	41.94	2.83

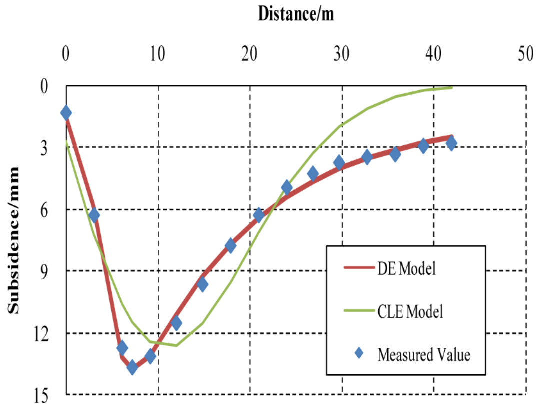


Figure 4. Predictive and measured subsidence curves of case 1

4.2. Project Case 2

This project is the New Shanghai International Building project, Shanghai, China. Soil formations of this foundation from top to bottom are: miscellaneous fill, silty clay, silty clay, mucky soil, and silty soil containing silty clay, and the average internal friction angle is 11°. Using the concrete diaphragm wall to block soils, the wall thickness is 800mm and the depth is 26m. Three support structures are set up with the center height of -0.5m, -4.5m and -8.5m. With three steps of excavation, the total excavation depth is 13.5m [15]. Applying the function model to fit the actual measured subsidence curve, the measured surface subsidence and model fitting curve are shown in Figure 5.

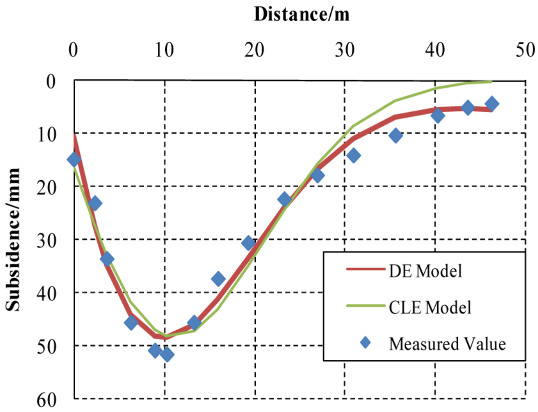


Figure 5. Predictive and measured subsidence curves of case 2

4.3. Relative Error

According to the fitting data of the two cases, the percentage error can be calculated and shown in Table 2 and Table 3, and the absolute percentage error are shown in Figure 6 and Figure 7.

Table 2. Percentage Error of DE Model and CEL Model of case1

Order	DE (%)	CLE (%)
1	13.1	100.81
2	-6.8	13.9
3	3.4	-16.6
4	0.6	-16.1
5	-0.2	-5.2
6	-3.7	9.5
7	-3.9	19.7
8	-1.1	22.5
9	1.9	13.6
10	9.2	-0.5
11	8.8	-24.3
12	8.3	-46.8
13	1.6	-68.5
14	-6.7	-83.3
15	-5.2	-91.0
16	-10.0	-95.9

The average value of the absolute percentage error of CEL model in Case 1 and Case 2 are 39% and 29% respectively; while the average value of the absolute percentage error of DE model are 5% and 13%, compared with CEL model fitting errors, it is reduced by 34% and 16%. From the aspects of the model fitting curve, the percentage error and the absolute percentage error, we can get that the most error rate of DE model is less than that of CEL model, only at a closer distance to the pit, the error rate is occasionally greater than that of CEL model; at a long distance to pit, the convergence of data of DE model is significantly better than that of CEL model. Therefore, DE model is relatively better in fitting.

Table 3. Percentage Error of DE Model and CEL Model of case2

Order	DE (%)	CLE (%)
1	-29.7	8.2
2	17.9	17.1
3	2.6	-2.6
4	-3.5	-8.5
5	-5.5	-7.5
6	-6.3	-6.7
7	0.6	3.5
8	10.0	15.2
9	9.4	14.7
10	7.6	9.2
11	-6.5	-12.0
12	-23.0	-38.4
13	-33.0	-63.5
14	-18.4	-78.6
15	2.3	-87.5
16	24.4	-92.6

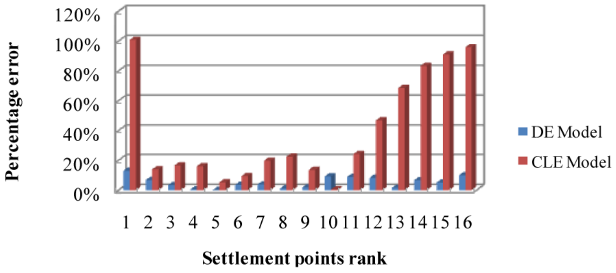


Figure 6. Comparison chart of the absolute percentage error of Case 1

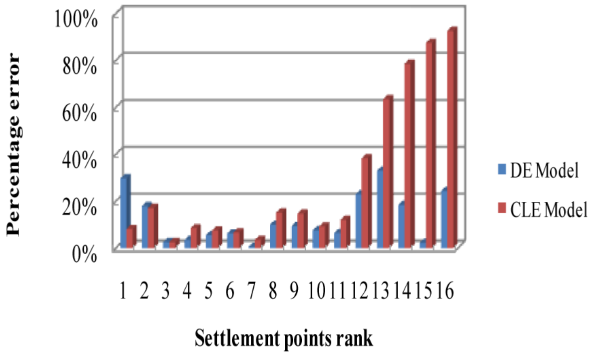


Figure 7. Comparison chart of the absolute percentage error of Case 2

Conclusions

In this paper, a new mathematical model, DE model, was proposed to describe the surface subsidence distribution caused by foundation pit excavation. Based on the two project cases, a comparison with the traditional model is conducted, which shows that the new proposed DE model has better fitting effect than the traditional CEL Model. The parameters of the DE model can be calculated by counter-analysis method which is easy to use and can predict the surface subsidence due to foundation pit excavation, so as to provide reference for controlling subsidence.

It should be noted that, only two kinds of foundation pit supporting as bored pile and concrete diaphragm wall are selected in the project cases. As for the other forms of supporting structures which are not considered, the size and depth of the pit which are not taken into account in this paper, it is necessary to further to be explored in future studies.

Acknowledgements

This work is supported in part by the National Natural Science Foundation of China (41202222), the Pandeng Foundation for the academic leader of the subjects of Zhejiang Province University (pd2013387).

References

1. Shi Cheng-hua, Peng Li-min. Ground surface settlement caused by foundation pit excavation and dewatering. China Civil Engineering Journal, 2006, 39(5), p.p.117-121.
2. Nie Zong-quan, Zhang Shang-gen, Meng Shao-ping. Surface settlement of deep foundation pits by excavation. Chinese Journal of Geotechnical Engineering, 2008, 30(8), p.p.1218-1223.
3. Liu Yun-si. Analysis of the pile anchor bracing affect adjacent super deep excavation with earth-rock combination. Electronic Journal of Geotechnical Engineering, 2011, 16(T), p.p. 1563-1579
4. Yang Xiao-li, Qin Chang-bing. Limit analysis of supporting pressure in tunnels with regard to surface settlement. Journal of Central South University, 2015, 22(1), p.p.303-3097.
5. Ludmila Strokova. Effect of the over-consolidation ratio of soils in surface settlements due to tunneling. Sciences in Cold and Arid Regions, 2013, 5(5), p.p. 637-643
6. Takashi Sasaoka, Hiroshi Takamoto, Hideki Shimada. Surface subsidence due to underground mining operation under weak geological condition in Indonesia. Journal of Rock Mechanics and Geotechnical Engineering, 2015, 7(3), p.p. 337-344.
7. Liu Xin-rong, Wang Jun-bao, Guo Jian-qiang. Time function of surface subsidence based on Harris model in mined-out area. International Journal of Mining Science and Technology, 2013, 23(2), p.p. 245-248.
8. Marino Gennaro, Gautam Bira. Mine subsidence damage during construction of medical center and remedial measures taken. Electronic Journal of Geotechnical Engineering, 2011, 16(J), p.p. 1099-1117
9. Ding Zhi, Wang Da, Wang Jin-yan. Deformation characteristics of Zhejiang soft soil deep foundation pits and their predictive analysis. Rock and Soil Mechanics, 2015, 36(11), p.p. 506-512
10. Wang Li-zhong, Long Fan. Base stability analysis of braced deep excavation in undrained anisotropic clay with upper bound theory. Science China-Technological Sciences, 2006, 57(9), p.p. 1865-1876
11. Andrea Lisjak, Daniel Figi, Giovanni Grasselli. Fracture development around deep underground excavations: Insights from FDEM modeling. Journal of Rock Mechanics and Geotechnical Engineering, 2014, 4(6), p.p.493-505
12. Wang Qiang, Chen Chun-shu. Study on applicability of a new model for describing ground settlement induced by foundation pit excavation. Geotechnical Engineering Technique, 2014, 28(4), p.p.168-171.
13. Liu Nian-wu, Gong Xiao-nan, Lou Chun-hui. Deformation behavior of nearby facilities analysis induced by excavation in soft clay. Journal of Zhejiang University, 2014, 48(7), p.p.1141-1147.
14. Yin Sheng-bin, Ding Hong-yan. Numerical analysis of estimation of ground surface settlement outside pit induced by soft-soil excavation. Rock and Soil Mechanics, 2012, 33(4), p.p.1210-1216.
15. Chen Wan-peng. (*Ph.D. thesis*, Nanjing University of Technology, 2006).
16. Zhang Shang-gen, Yuan Zheng-ru. Analysis of ground settlement induced by excavation of deep foundation pit in soft soil. Chinese Journal of Underground Space and Engineering, 2013, 9(1), p.p.1753-1757.
17. Zhang Jie. (*Ph.D. thesis*, Zhejiang University of Technology, 2013)