Robust Adaptive Beam-forming Algorithm Using Rotation and Iteration

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Abstract
The introduced the large offset, poor directivity and larger sidelobe for beam forming caused by the pointing deviation of the desired signal are likely to result in signal cancellation in the expected direction. For this, a robust adaptive beam-forming algorithm, called the rotation iterative algorithm (RIA), is proposed in this paper. The proposed algorithm can eliminate the direction deviation by using the rotation adjustment for deviation orientation vector. Furthermore, the presented RIA algorithm can also improve the real-time operability by applying an iterative method to search optimal weight vector on the basis of the steepest descent method. In addition, the proposed method avoids the eigen-decomposition and inversion of the correlation matrix. As a result, computational load can be greatly reduced. Experimental results have shown that the proposed RIA algorithm is able to significantly revise orientation vector deviation, improve signal to interference and noise ratio (SINR) for the output signal, and speed up convergence speed.
Keywords: ADAPTIVE BEAM-FORMING, ROBUST ADAPTIVE, ROTATION ITERATIVE, EIGEN-DECOMPOSITION

1. Introduction

Owing to a wide range of applications, the adaptive beam-forming algorithm has made great and continuous development. In order to suitable for different applications, many criterions of adaptive beam-forming algorithm have been proposed, such as minimum variance, maximum output signal-to-noise ratio and minimum mean square error(MMSE). The constraint criterion of the minimum mean square error takes the mean square error minimization between the output signal and desired signal. The constraint criterion of the minimum variance criterion requires the minimum array output power. Integrating one of these criterions with different constraint conditions, some beam-forming algorithms [1-4] have been proposed.

Under the condition of linear constraint and minimum output power, taking the response vector as the optimal weight vector, linear constrained minimum variance(LCMV) can response timely signal in the desired direction, and that can eliminate noises in the other directions [5]. However, the correlation matrix of the received signal is usually replaced by the correlation matrix of the instantaneous received signal. Thus, LCMV certainly has error with the actual matrix. For reducing the error, some methods are adopted, such as using the expectation estimation of the n times sampling the instantaneous received signal to acquire the correlation matrix of the received signal, or by n times iterative addition with n times sampled data to get the correlation matrix of the received signal. The eigen spatial beam-forming algorithm(ESA) separates the weight vector space into the signal subspace and the noise subspace, and then the weight vector is projected into the subspace. ESA can realize the separation of signal and noise, and limit effectively the noise power output, thus its convergence speed and robustness have a significant improvement. However, ESA cannot reduce the error of correlation matrix. Therefore, the inverse of the correlation matrix still needs to be calculated in spite of the reduction of its computation complexity. Nearby the initial desired direction, the first order Taylor series of steering vector can be applied to modify direction and help look for a new steering vector. When the directions of steering vector and the desired signal are the same, the output power of the optimal beamformer will achieve local optimum [6]. This method is easy to realize, and has nothing to do with the selection of the phase center. But the range of the revised pointing error is wide caused by main lobe broadening. Newton iterative method is essentially the linear part of Taylor series, but it is lower in the accuracy and slower in convergence speed. In order to avoid calculating the inverse of the matrix, the recursive least squares algorithm can be applied to reduce the calculated amount effectively [7]. Nevertheless the pointing error can cause signal cancellation and performance decline. To improve the robustness of the beamforming algorithm, steering vector can be revised by rotating vector [8]. Because the normal LCMV algorithm needs to solve the inverse of the matrix and eigen-decomposition, the computational load is biggish. Using the steepest descent method to search the optimal weight vector and the steering vector recursively can avoid the operation of the inverse matrix and the eigen-decomposition, which can reduce computational complexity and improve the robustness [9]. Suppose that the weight vector is optimal, MMSE can approach the desired steering vector by iteration [3]. In addition, although strong interference can also cause the orientation error, it is proved to be suppressed by broadening the null steering [10]. In order to improve the desired signal cancellation phenomenon, correlation matrix is constructed using orthogonal projection for the mismatch angle of the desired signal, and steering vector can also be estimated under beam-space [11]. When the mismatch of direction-of-arrival(DOA) is large, to make the beamformer stable and the performance optimized, we restructure the Interference plus noise covariance matrix and replace the actual sampling correlation matrix by the desired signal plus noise covariance matrix, and through rotation, the optimal weight vector can be acquired by the orthogonal subspace containing the desired signal [12]. There are many methods to restructure the covariance matrix, like the sparse feature used by Gu [13], which estimates the external product linear combination of the interference vector oriented of the single interference power weighting through compressed sensing(CS), and then restructures the sparse covariance matrix added Interference and noise to replace the diagonal loading factor of Cruise in the sampling correlation matrix. The problem of signal cancellation can be solved because signal components are not contained [14].

In order to eliminate estimation error and pointing error as well as improve real-time performance of algorithm and simplify calculations, taking advantages of the above methods, this paper proposes RIA for the robust adaptive beam-forming. RIA can eliminate orientation error by rotate operation, and avoid inverse operation of covariance matrix by using the steepest decent operation [15]. However, the weighted vector has been iterative searched for optimum.

2. Theoretical basis

Suppose that isotropic homogeneous M element linear array with interval of half wavelength has received a desired signal and p (p+1 < M) interference signals from the directions of θ_0, θ_1, …, θ_p, , and the noise is white noise. Here, all signals and noise are mutually non-correlative, and then the received signal x(t) at t moment can be described in equation (1).

$$x(t)=\sum_{i=0}^{p}a(\theta_i)s_i(t)+n(t)=\mathbf{A}(\theta)\mathbf{S}(t)+n(t) \qquad (1)$$

where $\mathbf{A}(\theta) = [a(\theta_0), a(\theta_1),\dots, a(\theta_p)]$ is a steering vector, $\mathbf{S}(t) = [s_0(t), s_1(t),\dots, s_p(t)]^T$ is source signal vector, $s_0(t)$ and $s_i(t)$ ($i = 1, 2, \dots, p$) are the complex envelopes of desired and interference signals and $n(t)$ is a noise.

The correlation matrix $\mathbf{R}_x$ of the received signal $x(t)$ can be described in equation (2).

$$\mathbf{R}_x=E\left\{x(t)x^H(t)\right\}=\sigma_0^2a(\theta_0)a^H(\theta_0) + \sigma_i^2\sum_{i=1}^{p}a(\theta_i)a^H(\theta_i)+\sigma_n^2\mathbf{I} \qquad (2)$$

Where, σ^2 denotes the power, the subscript 0, i and n indicate the desired signal, interference signal and noise, and **I** is an M order unit matrix. Normally, we cannot get the correlation matrix $\mathbf{R}_x$. Usually, it can be estimated by using the average of finite snapshots. Suppose using n times snapshot sampling, and then we can get an estimation matrix $\tilde{\mathbf{R}}_x$ shown in equation (3). When $N\to\infty$, $\tilde{\mathbf{R}}_x\approx\mathbf{R}_x$.

$$\tilde{\mathbf{R}}_x=\frac{1}{N}\sum_{n=1}^{N}x(n)x^H(n) \qquad (3)$$

In order to get optimal weight vector ω, the beamforming idea of LCMV is adding restrains to some directions to result the minimal output power. Considering that it is hard to get the practical steering vector, it can be replaced by the estimation matrix $\tilde{\mathbf{R}}_x$. It can be expressed as shown in equation (4).

$$\min_{\omega} \omega^H \tilde{\mathbf{R}}_x \omega \quad s.t. \quad \omega^H a(\theta_0) = 1 \tag{4}$$

The optimal weight vector ω_{opt} can be got by solving equation (5).

$$\omega_{opt} = \frac{\tilde{\mathbf{R}}_x^{-1} a(\theta_0)}{a^H(\theta_0)\tilde{\mathbf{R}}_x^{-1} a(\theta_0)} \tag{5}$$

Actually, the pointing error contained in steering vector will cause the performance decline of the LCMV algorithm.

3. Rotating iterative beam-forming

When the error direction is on the edge of beam main lobe, based on the direction vector $a(\tilde{\theta}_0)$ of supposing expected signal direction $\tilde{\theta}_0$, we use first order Taylor series to search and correct the deflection error by iteration and approximation. During the iteration, the correction value makes the assumed direction approach the expected direction, which means correction value and the pointing error are consistent, and then the research can converge to the expected angle [6]. However, when the condition can't be satisfied, the research failed and the performance will worsen badly. It will result some circumstances such as signal-to-noise ratio of the output signal attenuating and beam robustness weakening. For that, a RIA based on the error analysis of direction vector has been proposed.

If the direction of desired signal is θ_0 and the direction of the steering vector exists deflection error $\triangle\theta$, then the normalization of error space frequency is $u = \sin(\theta_0+\triangle\theta)/2$. When $\triangle\theta \to 0$, the trigonometric function $\sin(\theta_0+\triangle\theta)$ can be expanded as shown in equation (6).

$$\begin{aligned} \sin(\theta_0+\Delta\theta) &= \sin\theta_0\cos\Delta\theta + \cos\theta_0\sin\Delta\theta \\ &\approx \sin\theta_0 + \Delta\theta\cos\theta_0 \end{aligned} \tag{6}$$

Supposing $\varphi = \sin\theta_0$, $\triangle\varphi = \triangle\theta\cos\theta_0$, then the error steering vector can be expressed as shown in equation (7).

$$\begin{aligned} a(\theta_0+\Delta\theta) &= \left[1, e^{-j\pi(\varphi+\Delta\varphi)}, \cdots, e^{-j\pi(\mathrm{M}-1)(\varphi+\Delta\varphi)}\right]^T \\ &= \left[1, e^{-j\pi\varphi}\cdot e^{-j\pi\Delta\varphi}, \cdots, e^{-j\pi(\mathrm{M}-1)\varphi}\cdot e^{-j\pi(\mathrm{M}-1)\Delta\varphi}\right]^T \end{aligned} \tag{7}$$

Supposing $a(\Delta\theta) = \left[1, e^{-j\pi\Delta\varphi}, \cdots, e^{-j\pi(\mathrm{M}-1)\Delta\varphi}\right]^T$, then the error steering vector can be expressed as shown in equation (8).

$$a(\theta_0+\Delta\theta) = a(\theta_0)\cdot * a(\Delta\theta) \tag{8}$$

Therefore, the steering vector of desired signal getting from the error steering vector can be expressed as shown in equation (9).

$$a(\theta_0) = a(\theta_0+\Delta\theta)\cdot / a(\Delta\theta) \tag{9}$$

Where "$\cdot *$" and "$\cdot /$" express Hadamard multiplication and division of Corresponding element among column vectors. When the deflection error is in the half power main lobe which means $|\triangle\theta|<(0.886/\mathrm{M})\sec\theta_0\,(rad)$, normally a good direction adjustment can be got. But, the actual error is uncertain. Therefore, the angle error needs to be corrected gradually.

In order to improve the calculating speed and the real-time operability, the steepest descent method should be combined. The calculating speed can be improved by an order of magnitude compared with the normal LCMV formation algorithm through this way. Getting the estimate of the correlation matrix of received data by iteration can operate in real time and don't need enough amounts of snapshot data, and that can Iterative optimization for the weight vector. The estimate of the correlation matrix can be expressed as as shown in equation (10).

$$\tilde{\mathbf{R}}_{n+1} = \left(1-\frac{1}{n}\right)\tilde{\mathbf{R}}_n + \frac{1}{n}\left[x(n)x^H(n)\right] \tag{10}$$

where $\tilde{\mathbf{R}}_n$ is the correlation matrix estimated from the n times snapshot data. When the quantity of snapshots is large enough, $\tilde{\mathbf{R}}_{n+1} \approx \tilde{\mathbf{R}}_n$. The function of linear constraint can be formed by using Lagrange multiplier factor, and the steepest descent method can make the weight vector decline along the direction of maximal gradient which can be expressed as shown in equation (11).

$$\omega(n+1) = \omega(n) - \mu\nabla_\omega \mathrm{J} \tag{11}$$

Where μ is iteration step length, J is the linear constraint function of equation (5) formed by using Lagrange multiplier factor. $\nabla_\omega \mathrm{J}$ is defined as shown in equation (12).

$$\nabla_\omega \mathrm{J} = 2\tilde{\mathbf{R}}_x\omega(n) + \lambda_n a \tag{12}$$

Where ω(n+1) satisfies with $\omega^H(n+1)a = 1$, and a is error steering vector. Therefore, λ_n can be obtained according to equation (13).

$$\lambda_n = \frac{1}{\mu}\left(a^H\omega(n+1) - 2\mu a^H\tilde{\mathbf{R}}_n\omega(n) - 1\right) \tag{13}$$

Thus, the weight vector can be derived from equation (14).

$$\omega(n+1) = \omega(n) - 2\mu\left(\tilde{\mathbf{R}}_n\omega(n) - a^H\tilde{\mathbf{R}}_n\omega(n)a\right) \tag{14}$$

Supposing that $\varepsilon(n)x(n) = -\tilde{\mathbf{R}}_n\omega(n) + a^H\tilde{\mathbf{R}}_n\omega(n)a$ where $x(n)$ and $\varepsilon(n)$ are received signal and corrected error, the equation (13) becomes the weighted vector generation algorithm of MMSE.

The inverse of correlation matrix doesn't need to be calculated in this algorithm, therefore the calculations $O(M^3)$ in normal LCMV declines to $O(M^2)$, and that can operate in real time and don't need enough amounts of snapshot data, which can improve the calculating speed observably.

Based on above all, the proposed RIA algorithm is described as follows:

Step 1: when $n = 0$, initialize $\omega(0)$, step length μ and $\tilde{\mathbf{R}}_0 = \sigma^2\mathbf{I}$ (σ^2 is a very small constant), the range of rotation angle $\triangle\theta$ and rotation angle step.

Step 2: $n = 1,2,\ldots$, updating the estimation matrix $\tilde{\mathbf{R}}_n$ and the weight vector $\omega(n+1)$ based on the equation (10) and equation (14).

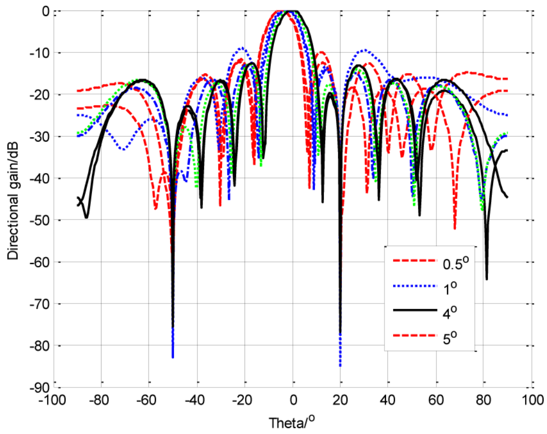
Step 3: according to the equation (9), rotating and correcting the deflection error and updating steering vector, and then go to step 2.

4. Experimental results and Analysis

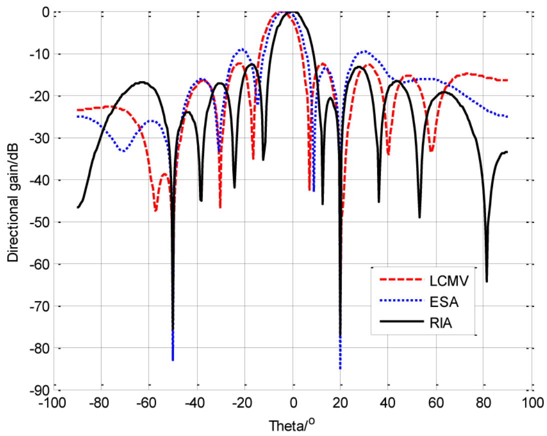
Experimental simulation based on MatLab has been conducted to verify the operability and the effectiveness of the proposed RIA algorithm. Simulation experiments are based on an isotropic homogeneous 10 elements linear array with interval of half wavelength. The incident signals are far field signals. Supposing that the incident angles of a desired signal and two interference signals are 0°, -50° and 20°, the angle of pointing error is 5°, and comparing the algorithm RIA with the eigen spatial algorithm(ESA) and the LCMV algorithm. Every result of simulation experiment gains from the average of 100 experiment results of Monte Carlo.

Experiment 1: Comparison of beam diagram of error correction is shown in Figure 1.

Experiment 1 has offered the simulation beam diagram which pointing error of the number of snapshots is 300, while SNR and interference and noise ratio (INR) are 20dB and 40dB. Figure 1 is the beam diagram which corresponds of error angle of different correct direction. The correction angle is 0° referred from the Figure 1, which means when there is no correction error angle, and there is direction that deviated from desired signal existing in the beam diagram. When the angle of correction error makes the pointing error approach the direction which the main lobe half power corresponds, the point of beam diagram is near the direction of desired signal. From the result of simulation of LCMV, ESA and RIA, it can be seen that when the pointing error is 5°, RIA can point to the direction of actual signal adequately, but



(a)



(b)

Figure 1. Beam diagram of error correction. (a) Beam diagram of the error correction angle. (b) Beam diagram of different algorithms

the LCMV and ESA both have deviation of the main lobe, and the angle of deviation of LCMV is smaller than ESA. In addition, the side lobes of beam diagram of RIA are lower than other both algorithms, and that has relatively small width, but the shortage is that the tail lobe is a little higher. Generally, RIA can correct the error of steering vector to some extent, so RIA can improve the performance of beam diagram.

Experiment 2: The diagram of the output SINR along with the change of the input SNR is shown in Figure 2.

When the quantity of snapshots is 300 and the INR is 50dB, along with the change of the input signal SNR, the simulation results of the output SINR of the 3 algorithms are shown as the Figure 2. Along with the increase of the output SNR, the Figure 2 shows that the output SINR of LCMV has certain advantages when SNR is under 0dB, but it declines when SNR is greater than 0dB. The output SINR of ESA is declines as well when SNR is greater than 30dB. But the output SINR of RIA has a sustained increasing;

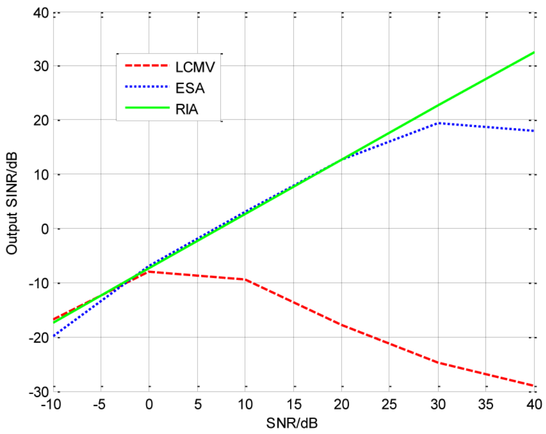


Figure 2. The output SINR along with the change of the input SNR

when the SNR is under 0dB, the result is the same as LCMV when the SNR is between 0dB and 20dB, and the result is the same as ESA when the SNR is greater than 20dB. It shows that RIA has obvious advantages. When the pointing error exists, the simulation results illustrate that RIA can form a robuster beam and has an obvious Inhibition ability for the pointing error, and that has an obvious superiority for the output SINR, especially when SINR is larger.

Experiment 3: The analysis of SINR affected by the quantity of snapshots is shown in Figure 3.

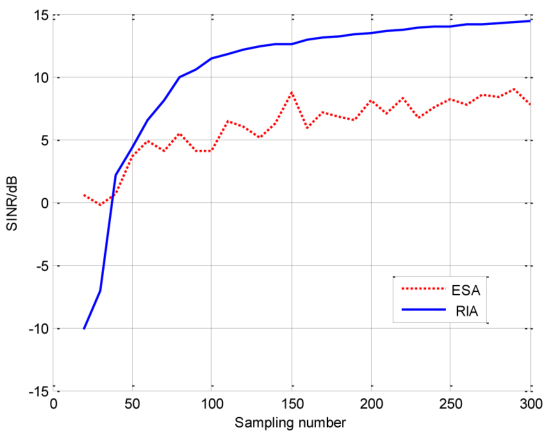


Figure 3. SINR affected by the quantity of snapshots

When SNR and INR are 20dB and 50dB, and when the pointing error is 5°, Figure 3 shows that the output SINR of ESA is increases along with the increase of the hits, but it is unstable and hard to converge. For RIA, the output SINR increases along with the increase of hits, and that stabilizes gradually and converges eventually. Although the output SINR of RIA is worse than ESA when the hits is small, but the convergence rate of RIA is faster when the quantity of snapshots is small. Less than 50 snapshots, RIA has surpassed ESA, and when the quantity of snapshots is greater than 100, RIA can stabilize basically. It can be illustrated that RIA can form robust beam when the quantity of snapshots is small.

Conclusions

For practical phased array radar, the quantity of array elements of is huge, and a little error can cause large offset of beam for desired signal. The directive property is not good and has bigger side lobe, which results in the decline of radar capacity. For the pointing error, RIA has been proposed, which combines the advantages of the steepest descent method and rotating vector method, and use iteration to search the best weight vector. It can avoid the eigen-decomposition and the inverse operation of correlation matrix. Therefore, RIA can simplify calculation. All experiments show that RIA can correct the error existing in the steering vector obviously to some extent and improve the output SINR. In addition, the convergence speed of RIA is fast, and RIA stabilizes basically when the quantity of snapshots is larger than 100.

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The Research on the Model of Image Denoising Based on the Fusion of Anisotropic Diffusion and Total Variation Models

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Abstract
In this paper, the author researched on the model of image denoising based on the fusion of anisotropic diffusion and total variation models. The noise is present at almost all data. The noise can degrade image quality, as a result the interpretations and analysis of the image will be much harder. Denoising is the process of reducing the noise.