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Research of a Mixed Robust H_2 / H_∞ Controller Design for NCS with Variable Sampling Intervals and Time-delay

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Abstract
Based on Lyapunov stability theory and linear matrix inequalities (LMIs) method, parameter-dependent and parameter-independent states feedback controller, the states feedback controller and dynamic output feedback controller are designed, respectively. The necessary and sufficient condition of existence the above controllers are given by LMIs. Considering the disturbance and noise, the mixed robust H_2 / H_∞ performance of the discrete-time polytopic uncertain system is proposed, which is used to improve the system transient performance and to ensure the robustness of the system simultaneously. Finally, the robust states feedback and dynamic output feedback controllers are designed and an example is given to show its effectiveness.
Key words: LYAPUNOV; LMIS; H_2 / H_∞; ROBUST CONTROL

1. Introduction

NCSs(Networked Control Systems) are some feedback control systems where the feedback loops are closed by some communication networks. NCSs have many advantages such as less wiring, easy installation and maintenance, etc. In recent years, there are many research about NCSs. There are also many special issues for NCSs published by some International Journals[1-6]. In the practical network control system, stability and controller design will be impacted by the external disturbance inevitably. The controller design does not only require that the system should maintain stability, but also should have robust inhibition and satisfy some performance index. Research of the robust controller with variable sampling intervals and time-delay is very less. In the literature [7], NCS with time-varying sampling period and the packet loss is modeled as a discrete time switching system. And it gives the method of H controlled design combined with a cone complementarity linearization method for NCS in the literature [8]. Li Xuan[9] considered fault-tolerant control for NCS with variable sampling intervals and time-delay without noise jamming under the frame of continuous-time system. H_2 and H_∞ control performance are two commonly used in the control systems. Where H_2 can improve the systems' transient performance, H_∞ can ensure the robustness of the control systems with noise jamming and uncertainty.

In this paper, a mixed robust H_2 / H_∞ control is adopted which has performance of both the H_2 and H_∞. It can obtain the balance between dynamic properties and robustness. On the other hand, a system's state can not be measured directly. Therefore, the state feedback controller is designed in the ideal case that all the states can be measured here.

2. A Mixed Robust H_2 / H_∞ State Feedback Control

Assuming the ideal case that all the state can be measured, then we will design a state feedback controller to make the NCS with variable sampling intervals and time-delay maintain asymptotic stable and satisfy corresponding mixed robust H_2 / H_∞ performance.

Considering a continuous-time and linear controlled plant, that's

$$\begin{cases} \bar{x} = A_c x(t) + B_c u(t) + B_w w(t) \\ y_1(t) = C_{1c} x(t) + D_{11c} u(t) + D_{12c} w(t) \\ y_2(t) = C_{2c} x(t) \end{cases} \quad (1)$$

Where $x(t) \in R^{n_1}$ is the state of system, $u(t) \in R^l$ is the system's input, $y_1(t) \in R^p$ is the controlled plant's output of system, $y_2(t) \in R^m$ is the measured output of controlled plant, the outside interference signal $w(t) \in R^d$ is a bounded energy noise, A_c, B_c, B_w, C_{1c}, D_{11c}, D_{12c} and C_{2c} are constant matrix with appropriate dimension.

Sampling intervals h_k and time-delay τ_k in the NCS is unknown, time-variant and bounded, that's, $h_k \in [h_{\min}, h_{\max}]$, $\tau_k \in [\tau_{\min}, \tau_{\max}]$. To convert continuous system Eq.(1) into discrete system, a linear discrete-time system with polytopic uncertainty is obtained as the following,which converts the uncertainty of the sampling intervals and time-delay into the system parameters' uncertainty, that's,

$$\begin{cases} z(k+1) = A(\alpha)z(k) + B(\alpha)u(k) + E(\alpha)w(k) \\ y_1(k) = C_1 z(k) + D_{11}u(k) + D_{12}w(k) \\ y_2(k) = C_2 z(k) \end{cases} \quad (2)$$

Where, $z(k) \in R^n$ is an augmented vector, A_α, B_α and E_α are unknown matrix with appropriate dimension, they all belong to the given convex polyhedron area, that's,

$$\Omega(\alpha) = \{A_\alpha, B_\alpha, C_\alpha\} = \sum_{i=1}^{u} \alpha_i (A_i, B_i, E_i)$$

$C_2 = [C_{c2} \quad 0]$, $D_{11} = D_{11c}$, $D_{12} = D_{12c}$.

Design a state feedback controller, that's,

$$u(k) = Kz(k) \quad (3)$$

Eq.(1) is substituted in Eq.(2), we can get Eq.(4), that's,

$$\begin{cases} z(k+1) = \bar{A}_{cl}(\alpha)z(k) + \bar{E}(\alpha)w(k) \\ y_1(k) = \bar{C}_1 z(k) + D_{12}w(k) \\ y_2(k) = C_2 z(k) \end{cases} \quad (4)$$

Where, $\bar{A}_{cl}(\alpha) = A(\alpha) + B(\alpha)K$, $\bar{E}(\alpha) = E(\alpha)$, $\bar{C}_1 = C_1 + D_{11}K$.

The task of H_2 / H_∞ robust control is that the state feedback controller designed by Eq.(3) to make the closed-loop system (4) asymptotically stable under all parameter uncertainty allowed for the convex polytopic uncertain systems Eq.(2), and meanwhile it must satisfy the following performance index, that's,

(i) Robust H_2 performance. Minimizing $\eta > 0$ to make the norm of H_2 minimize, that's, $\|T_{y_2 w}(\zeta)\|_\infty < \eta$ for the closed-loop system $T_{y_2 w}(\zeta)$;

(ii)Robust H_∞ performance. Given $\gamma > 0$, the norm of H_∞, that's, $\|T_{y_1 w}(\zeta)\|_\infty < \gamma, \forall w(k) \neq 0$ for the closed-loop system $T_{y_1 w}(\zeta)$;

3. Main Conclusion

Lemma 1[10] For the given $\eta > 0$, to make the closed-loop system Eq.(4) asymptotically stable, and $\|T_{y_2 w}(\zeta)\|_\infty < \eta$, the necessary and sufficient conditions is that there are real symmetric positive definite matrices P_{2i} and $W_i, i = 1, 2, \ldots, u$, and for all time-varying parameters $\alpha \in L$, there are $P_2(\alpha) = \sum_{i=1}^{\mu} \alpha_i P_{2i}$ and $W(\alpha) = \sum_{i=1}^{\mu} \alpha_i W_i$, meanwhile, the following inequalities must be satisfied, that's,

$$trace(W(\alpha)) < \eta^{1/2} \quad (5)$$

$$C_2 P_2(\alpha) C_2^T - W(\alpha) < 0 \quad (6)$$

$$\bar{A}_{cl}(\alpha) P_2(\alpha) \bar{A}_{cl}^T(\alpha) - P_2(\alpha) + \bar{E}(\alpha) \bar{E}^T(\alpha) < 0 \quad (7)$$

Theorem 1 For the given $\eta > 0$, to make the closed-loop system Eq.(4) asymptotically stable, and $\left\| T_{y_2 w}(\zeta) \right\|_\infty < \eta$, the necessary and sufficient conditions is that there are real symmetric positive definite matrices P_{2i}, $W_i, i = 1,2,...,u$ and matrix G, for all time-varying parameters $\alpha \in L$, the following inequalities must be satisfied, that's,

$$trace(W_i) < \eta^{1/2} \quad (8)$$

$$\begin{bmatrix} -W_i & C_2 G \\ * & -G - G^T + P_{2i} \end{bmatrix} < 0 \quad (9)$$

$$\begin{bmatrix} -P_{2i} & \bar{A}_{cl} G & E_i \\ * & -G - G^T + P_{2i} & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (10)$$

Prove.

To make a sum for inequalities (8) after the left is multiplied by α_i, then we can get the inequalities (5), meanwhile, to make a sum for inequalities (9) and (10) after the left is multiplied by α_i, then the theorem 1 can be proved, q.e.d.

Before analyzing the H_∞ performance, first of all, we give the bounded real theorem of the discrete time system. Considering a linear discrete-time system, that's,

$$\begin{cases} \tilde{x}(k+1) = A\tilde{x}(k) + B\tilde{w}(t) \\ \tilde{y}(k) = C\tilde{x}(k) + D\tilde{w}(t) \end{cases} \quad (11)$$

Where $\tilde{x}(k) \in R^n$ is the state of system, $\tilde{y}(k) \in R^m$ is the controlled plant's output of system, the outside interference signal $\tilde{w}(k) \in R^q$ is a bounded energy noise. If for one kind of external disturbance $\tilde{w}(k)$, the the controlled plant's output of system $\tilde{y}(k)$ is always small, then the system is considered to have a good inhibition performance for external disturbance.

Lemma 2[11] (Bounded Real Lemma of the Discrete Time System) For the above discrete time system (11), the following conditions under the given constant $\gamma > 0$ is equivalent , that's,

(i) System is asymptotically stable and $\left\| T_{\tilde{y}w}(z) \right\|_\infty < \gamma$;

(ii) There is a real symmetric positive definite matrix $P > 0$, to make

$$\begin{bmatrix} -P & AP & B & 0 \\ * & -P & 0 & PC^T \\ * & * & -I & D^T \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (12)$$

Theorem 2 For the given $\gamma > 0$, to make the closed-loop system Eq.(4) asymptotically stable, and $\left\| T_{y_1 w}(\zeta) \right\|_\infty < \gamma$, the necessary and sufficient conditions is that the real symmetric positive definite matrix $P_{\infty i} (i = 1,2,...,\mu)$ and the general matrix Γ satisfy the following inequality, that's,

$$\begin{bmatrix} -P_{\infty i} & \bar{A}_{cl,i}\Gamma & E_i & 0 \\ * & -\Gamma - \Gamma^T + P_{\infty i} & 0 & \Gamma^T \bar{C}_1^T \\ * & * & -I & D_{12}^T \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, i = 1,2,...\mu \quad (13)$$

Prove.

By the lemma 2, to make the closed-loop system (4) asymptotically stable and satisfies $\left\| T_{y_1 w}(\zeta) \right\|_\infty < \gamma$, the necessary and sufficient conditions is that there are real symmetric positive definite matrices $P_{\infty i}$, for all time-varying parameters $\alpha \in L$, there is $P_\infty(\alpha) = \sum_{i=1}^{\mu} \alpha_i P_{\infty i}$, and the following inequalities must be satisfied, that's,

$$\begin{bmatrix} -P_\infty(\alpha) & \bar{A}_{cl}(\alpha) P_\infty(\alpha) & \bar{E}(\alpha) & 0 \\ * & -P_\infty(\alpha) & 0 & P_\infty(\alpha) \bar{C}_1^T \\ * & * & -I & D_{12}^T \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (14)$$

q.e.d.

Theorem 3 For a given scalar $\gamma > 0$, there exists a state feedback matrix $u(k) = Kz(k)$ in the uncertain discrete-time system (2) to make the closed-loop system (4) asymptotically stable. And minimize η to make $\left\| T_{y_2 w}(\zeta) \right\|_2 < \eta$ and satisfy $\left\| T_{y_1 w}(\zeta) \right\|_\infty < \gamma$, the necessary and sufficient conditions is that there are real symmetric positive definite matrices P_{2i}, $P_{\infty i}, i = 1,2,...,u$ and general matrix G, Z to make the following optimization problems

$$\eta = \min_{G,Z,P_{2i},P_{\infty i}} trace(W_i), i = 1,2,...,\mu \quad (15)$$

satisfy

$$\begin{bmatrix} -W_i & C_2 G \\ * & -G - G^T + P_{2i} \end{bmatrix} < 0 \quad (16)$$

$$\begin{bmatrix} -P_{2i} & A_iG+B_iZ & E_i \\ * & -G-G^T+P_{2i} & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (17)$$

$$\begin{bmatrix} -P_{\infty i} & A_iG+B_iZ & E_i & 0 \\ * & -G-G^T+P_{\infty i} & 0 & G^TC_1^T+Z^TD_{11}^T \\ * & * & -I & D_{12}^T \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (18)$$

and have a optimal solution G and Z. Then the corresponding parameter of mixed robust H_2 / H_∞ state feedback control is $K = ZG^{-1}$.

Prove.

Expanding the inequalities (9) and (10), and assuming $\Gamma = G$, $Z = KG$, then the inequalities (17) and (18) can be obtained. Meanwhile, as $-G-G^T+P_{2i}<0$, $-G-G^T+P_{\infty i}<0$, then there is

$G+G^T>0 \Rightarrow G$ *is* nonsingular matrix

$\Rightarrow G^{-1}$ *existes*

where P_{2i}, $P_{\infty i}$ are real symmetric positive definite matrix.

Q.e.d.

4. Numerical Simulatioms

In this paper, the example is borrowed from literature [6], and continuous-time linear system is given as follows

$$\begin{cases} \bar{x}(t) = \begin{bmatrix} -2 & -6 \\ -2.1 & -5.8 \end{bmatrix} x(t) + \begin{bmatrix} 1.5 \\ -8.5 \end{bmatrix} u(t) + \begin{bmatrix} 6.5 \\ 3 \end{bmatrix} w(t) \\ y_1(t) = \begin{bmatrix} -1.5 & 0.1 \end{bmatrix} x(t) - 0.5u(t) \\ y_2(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(t) \end{cases} \quad (19)$$

Where the sampling interval h_k is switched in the set of $\{0.1, 0.11, 0.12\}$, the corresponding upper limit of time-delay is $\tau_{k\max} = 0.3h_k$. Here we assume sampling interval and time-delay to be continuously variable, that's, $h_k \in [0.1\ 0.12]$ s, $\tau_k \in [0, 0.1]$ s, the delay is expanded its scope here. The sampling period is unknown, bounded and time-varying.

The characteristic roots of the system matrix A_c are $\lambda_1 = 0.2$, $\lambda_1 = -7.5$, and the continuous time system is unstable. The value of the the external interference signal $w(t)$ is as follows, that's,

$$w(t) = \begin{cases} \sin(t) & 2.0s \le t \le 8.0s \\ 0 & others \end{cases} \quad (20)$$

(i) Only considering the H_2 performance to solve optimization problem $\min_{G,Z,P_{2i}} trac(W_i)$, which is crestrained by the inequality (16) and (17). Using the parameter $P_{2i}, i = 1,2,\ldots,\mu$ depended on Lyapunov function and the parameter $P_2 = P_{2i}, i = 1,2,\ldots,\mu$ does not depend on Lyapunov function, respectively, to get the optimal H_2 performance index, that's, $\eta_{\min} = 0.8721$ and $\eta_{\min} = 0.8806$.

(ii) Only considering the H_∞ performance to solve optimization problem $\min_{G,Z,P_{2i}} \gamma$, which is crestrained by the inequality (18). Using the parameter $P_{\infty i}, i = 1,2,\ldots,\mu$ depended on Lyapunov function and the parameter $P_\infty = P_{\infty i}, i = 1,2,\ldots,\mu$ does not depend on Lyapunov function, respectively, to get the optimal H_2 performance index $\gamma_{\min}$ as the table 1 below.

Table 1. Robust Performance H_∞ of NCS with Variable Sampling Period

Lyapunov function	H_2 performance index $\gamma_{\min}$
Parameter dependent of $P_{\infty i}$	2.5542
Parameter independent of $P_\infty = P_{\infty i}$	2.6781
The result in literature [6]	4.0073

(iii) Considering the mixed H_2 / H_∞ performance to solve optimization problem inequality (15). Using the parameter $P_{2i}, P_{\infty i}, i = 1,2,\ldots,\mu$ depended on Lyapunov function and the parameter $P_\infty = P_{\infty i}, P_2 = P_{2i}, i = 1,2,\ldots,\mu$ does not depend on Lyapunov function, respectively, to get the optimal H_2 performance index $\eta_{\min}$ as the table 2 below.

Table 2. Robust Performance H_2 / H_∞ of NCS with Variable Sampling Period

γ	Parameter dependent	Parameter independent ($P_2 = P_{2i}, P_\infty = P_{\infty i}$)	Parameter independent ($P_{2i} = P_{2\infty}$)
2.5542	1.0411	No result	No result
2.6781	0.9276	0.9371	No result
3	0.8914	0.8861	1.0412
3.5	0.8756	0.8861	0.9869
4	0.8721	0.8861	0.9578
4.5	0.8720	0.8861	0.9411
5	0.8720	0.8861	0.9295
6	0.8720	0.8861	0.9152
7	0.8720	0.8861	0.9072
8	0.8720	0.8861	0.8985
9	0.8720	0.8861	0.8985

The figure 1 is the relationship between the actual H_2 robust performance and H_∞ robust performance in the case of $\gamma > 0$ given. The results above the dotted line is that it adopts public Lyapunov function, the result below solid line is that parameter is used which is dependent of Lyapunov function. The curve in figure 1 shows that H_2 performance index $\eta_{\min}$ decreases with that the disturbances rejection gamma γ increases. It suggests that the system's robustness has a conflict between the other performance of the system. Improving the robustness of the system is on the understanding that reducing the dynamic, and vice versa. At the same time it also shows that by using common Lyapunov function the results' conservative is larger.

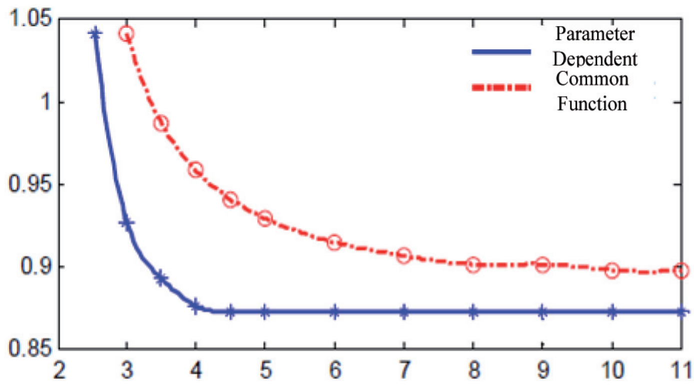

Figure 1. The Relationship between H_2 Robust Performance Index $\eta_{\min}$ and rejection gamma γ

Initial state $z[0] = [1 \quad -1 \quad 0]^T$, select $\gamma = 9$, we can get the state feedback gain in the case of parameter dependent is that,

$$K = [-0.4226 \quad 0.3154 \quad -0.2997]$$

Figure 2 is the state response of the closed-loop system and the system's controlled output curve. It shows that in the case of the external interference, the state feedback controller is obtained under the situation that the parameter selected is dependent on Lyapunov function. It can be seen that for the unstable continuous time systems, the state feedback controller designed can keep the system asymptotic stable in this paper.

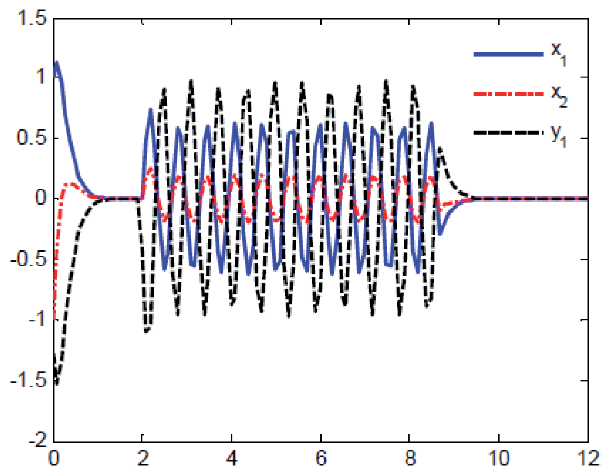

Figure 2. The State Response of the Closed-loop System and the System's Controlled Output Curve

In the case of the external interference, figure 3 is the state response of the closed-loop system and the system's controlled output curve in literature [7].

Comparing figure 2 with figure 3, it can be seen that the proposed mixed robust H_2 / H_∞ state feedback controller in this paper, under the action of no interference it only took 1.2 s to make the system quickly reach a stable state. While in literature [7], it need take 2.0s to enter into a stable state. Therefore it can be seen that the state feedback controller designed in this paper is better than the results in literature.

Acknowledgements

National Undergraduate Training Programs for Innovation and Entrepreneurship National Training Programs of Innovation and Entrepreneurship for Undergraduate, China (NO.201411396009), Research Department of education in Shaanxi Province, China (NO.2013jk1160).

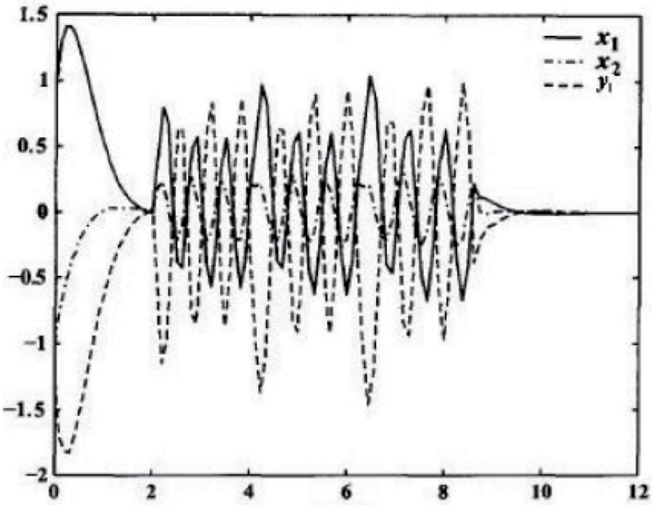

Figure 3. The State Response of the Closed-loop System and the System's Controlled Output Curve in literature [7]

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