

Mobile Sensor Networks for Sampled Position Data Control of an array of Distributed Parameter Systems with Randomly Occurring Sensor Saturations

Xueming Qian^{1,2}, Baotong Cui¹

1. Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi 214122, P.R.China

2. School of Internet of Things and software technology, Wuxi professional college of science and technology, Wuxi 214028, P.R.China

Corresponding author is Xueming Qian

Abstract

This paper develops sampled position data control problem of an array of distributed parameter systems with randomly occurring sensor saturations. A sampled position data decentralized output feedback controller is designed to ensure the distributed parameter systems to be globally asymptotically stable in the mean square using mobile sensor networks, where the measurements are taken in moving sampling points. By making use of the direct Lyapunov method and linear operator inequality technique, sufficient conditions for the stability of such system are given in the form of linear operator inequality and the velocity law of each mobile sensor. It is also shown that static sampled position data control of distributed parameter systems is just a special case of our main results. A numerical simulation illustrates the effectiveness of the proposed control scheme in enhancing system performance. Keywords: SAMPLED POSITION DATA CONTROL, MOBILE SENSORS, DISTRIBUTED PARAMETER SYSTEMS, RANDOMLY OCCURRING SENSOR SATURATIONS

1. Introduction

It is well known that wireless sensor networks have received increasing interest lately due to widely applications in a large amount of field, such as military, environmental, health, home and other commercial applications^[1-4]. With the development of micro-electro-mechanical systems technology, sensor nodes are small in size and lower cost, and many different types of monitoring can be achieved. Traditionally, a great quantity of sensors is deployed densely, which are fixed. However, the sensors are often moving in many applications, for instance, we can know the ecological situation population by placing sensors in wild animals. Such mobile sensor network is composed of

a great number of sensing nodes, which can move in an area employing mobile agents. Recently, arrays of mobile sensor networks have attracted the attentions of researchers as they exhibit some advantages^[5-6]. The use of mobile sensor networks can improve the system performance and reduce power consumption such that sensing nodes can monitor efficiently^[7]. For now, many important results on control problem have been reported for mobile sensor networks. Cooperative control in a class of mobile sensor networks has been studied in [8] to seek out local maxima or minima in a distributed environment. The swarm behavior control of mobile multi-robots with wireless sensor networks has been analyzed in [9]. Since target track-

ing algorithm of mobile sensor networks based on flocking control has been investigated intensively in [10]. Also, the problems of filtering and prediction of mobile sensor networks have been studied in [11] and [12], respectively.

In practice, the use of mobile sensors is plentiful in many industrial process applications, for example, thermal diffusion processes and chemical engineering, etc. These industrial processes are distributed in space inherently which represented by distributed parameter systems. That is their behavior relies on time as well as spatial position. Early work by Khalplov^[13-14] was considered the distributed parameter systems with mobile measurements and controls. Near the work under studied, where mobile actuator-sensor networks for control of parabolic PDEs have been investigated is the work by Uciński^[15] and Chen^[16]. In a similar fashion, solving the control and estimation of distributed parameter systems have been investigated in [17] where the condition of stability and distributed consensus filter have been obtained by the velocity law of each mobile actuator/sensor. The result has been depicted that the performance of system has been enhanced using mobile actuator/sensor compared to the case of fixed-in-space actuator/sensor. The underlying equation that indicates a diffusion process is a class of semi-linear parabolic systems. And, fixed sensors have been used for robust sampled-data control of distributed parameter systems has been investigated in [18]. By utilizing the Lyapunov direct method, the stability conditions have been obtained in the form of linear matrix inequalities through the output feedback control law.

In reality, sensors cannot supply signals of unlimited amplitude on account of physical or technological constraints. Such just mentioned phenomenon is sensor saturation which often occurs in control system. It is well known that saturation may cause oscillatory and instability according to its nonlinear characteristics. Therefore, the control problem of system with sensor saturations has attracted much attention. An output feedback H_∞ controller has been designed for linear systems with sensor saturation in [19]. Set-membership filtering algorithm has been given in [20] for system which subject to sensor saturation. It should be pointed out that sensor saturations may occur in a probabilistic way in mobile sensor networks due to sensor aging or networked environment changes. In [21], such phenomenon in H_∞ filtering problem of a class of nonlinear system has been studied. However, to the best of author's knowledge, the issue of randomly occurring sensor saturations has little considered in distributed parameter systems. Therefore, it is a challenge issue to be solved in this paper.

In this paper, a novel sampled position data control is presented in distributed parameter systems with randomly occurring sensor saturations using mobile sensor networks. By utilizing a Lyapunov functional which depend on spatial parameter, combine output feedback control scheme, the stability criteria of distributed parameter systems are given which in terms of linear operator inequality and the velocity law of each mobile sensor. The main significance of this paper is that we introduce the stabilization of distributed parameter systems using sampled position data controller via mobile sensors, thus the results improve the earlier work.

Notations. The notations are quite standard. Throughout this paper, let H be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and corresponding induced norm $\|\cdot\|$. Given a linear operator $P: H \rightarrow H$ with dense domain $D(P)$ in H , whereas P^* stands for its adjoint operator. Such an operator $P > 0$ means that P is strictly positive definite, where P is self-adjoint operator in sense i.e. $P = P^*$ and there exists a constant $c > 0$ such that $\langle x, Px \rangle \geq c \langle x, x \rangle$ and for all $x \in D(P)$. $P \geq 0$ means that nonnegative definite operator P is self-adjoint and $\langle x, Px \rangle \geq 0$ for all $x \in D(P)$.

If an operator A generates a strongly continuous semigroup $T(t)$ on the Hilbert space H , the domain of the operator A can be defined as follows: $(x, y)_{D(A)} = \langle x, y \rangle + \langle Ax, Ay \rangle$, $x, y \in D(A)$ ^[22].

2. Problem Formulation

Consider an array of distributed parameter control systems which described the following diffusion process:

$$\begin{aligned} \frac{\partial x(t, \xi)}{\partial t} &= \frac{\partial}{\partial \xi} \left(a(\xi) \frac{\partial x(t, \xi)}{\partial \xi} \right) + u(t, \xi), \\ y_i(t) &= \gamma_i \sigma \left(\int_0^l \varphi(\xi; \xi_i^s(t)) x(t, \xi) d\xi \right) \\ &\quad + (1 - \gamma_i) \int_0^l \varphi(\xi; \xi_i^s(t)) x(t, \xi) d\xi, \\ i &= 1, 2, \dots, N. \end{aligned} \tag{1}$$

where $x(t, \xi)$ denotes the state of system at time t and in space ξ , $\xi \in [0, l]$, $l > 0$, $t \in \mathbf{R}^+$. Diffusion operator $a(\xi) \geq a_0 > 0$. $u(t, \xi)$ denotes the associated control signal. $y_i(t)$ is the measurement of i th mobile sensor. The function $\varphi(\xi; \xi_i^s(t))$ denotes the spatial distribution of the i th mobile sensor. The spatial point $\xi_i^s(t) \in [0, l]$ denotes the time varying centroid of the i th sensor. That is, the moving trajectory of i th sensor.

The initial conditions associated with (1) be of the form

$$x(0, \xi) = x_0(\xi),$$

and have Dilichlet boundary conditions

$$x(t, 0) = x(t, l) = 0.$$

The variables $\gamma_i \in \mathbf{R}$, $i = 1, 2, \dots, N$ is Bernoulli distributed white sequences taking values on 0 and 1 with the following probabilities:

$$\begin{cases} \text{Prob}\{\gamma_i = 1\} = \bar{\gamma}_i \\ \text{Prob}\{\gamma_i = 0\} = 1 - \bar{\gamma}_i \end{cases}$$

where $\bar{\gamma}_i \in [0, 1]$ are known constants.

The saturation function σ is defined as $\sigma: \mathbf{R} \rightarrow \mathbf{R}$,

$$\sigma(v) = \text{sign}(v) \min\{1, |v|\},$$

where $\text{sign}(v)$ is the signum function. Without loss of generality, the saturation level is taken as 1.

Assumption 1 Nonlinear function f satisfies

$$m_i^- \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq m_i^+, \quad i = 1, 2, \dots, N,$$

where fixed constants m_i^-, m_i^+ are allow to be positive, negative or zero. Hence, the nonlinear function may be nonmonotonic and unbounded, and, therefore, more general than the usual Lipschitz condition.

For presentation convenience, we denote

$$\begin{aligned} y(t) &= [y_1(t), y_2(t), \dots, y_N(t)]^T, \sigma(\Phi(\xi^s(t))x(t)) = \\ &= [\sigma(\varphi(\xi_1^s(t))x(t)), \sigma(\varphi(\xi_2^s(t))x(t)), \dots, \\ &= \sigma(\varphi(\xi_N^s(t))x(t))]^T, \quad \Lambda_\gamma = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_N), \\ \bar{\Lambda}_\gamma &= \text{diag}(\bar{\gamma}_1, \bar{\gamma}_2, \dots, \bar{\gamma}_N). \end{aligned}$$

System (1) can be rewritten into compact form as:

$$\begin{cases} \dot{x}(t) = Ax(t) + u(t), \\ y(t) = \Lambda_\gamma \sigma(\Phi(\xi^s(t))x(t)) + (I - \Lambda_\gamma) \Phi(\xi^s(t))x(t). \end{cases} \quad (2)$$

State space $L_2(\Omega)$ is a Hilbert space, where $x(t, \cdot) = \{x(t, \xi) : 0 \leq \xi \leq l\}$ denotes the state of system at time t . Let the second order elliptic operator $A = \frac{d}{d\xi}(a(\xi)\frac{d}{d\xi})$, its domain $D(A) = \{\psi \in L_2(\Omega) : \psi, \psi' \text{ absolutely continuous } \psi'' \in L_2(\Omega), \psi(0) = \psi(l) = 0\}$. Operator A bounded and $-A > 0$, due to $a(\xi) > 0$. Linear operator $\Phi(\xi^s(t))$ is output operator related to mobile sensors. The vector of sensor location parameterized by $\xi^s(t) = [\xi_1^s(t), \xi_2^s(t), \dots, \xi_N^s(t)]^T$.

The aim of this paper is to design a sampled position data controller under which globally asymptotically stable in the mean square is guaranteed for system (1). For this, the spatial distribution of i th sensor is given by

$$\varphi(\xi; \xi_i^s) = \delta(\xi - \xi_i^s(t)). \quad (3)$$

That is, every sensor is identical to each other in shape, differing only at the location of their centroid $\xi_i^s(t)$.

Therefore, the output of i th mobile sensor is

$$\begin{aligned} y_i(t) &= \gamma_i \sigma \left(\int_0^l \delta(\xi - \xi_i^s(t)) x(t, \xi) d\xi \right) \\ &+ (1 - \gamma_i) \int_0^l \delta(\xi - \xi_i^s(t)) x(t, \xi) d\xi \\ &= \gamma_i \sigma(x(t, \xi_i^s(t))) + (1 - \gamma_i) x(t, \xi_i^s(t)). \end{aligned} \quad (4)$$

Utilizing the static output feedback control scheme, we can design the sampled position data controller as follows

$$u_i(t) = -k_i y_i(t), \quad k_i > 0, \quad (5)$$

or in matrix form

$$\begin{aligned} u(t) &= -Ky(t) \\ &= -\sum_{i=1}^N k_i [\gamma_i \sigma(x(t, \xi_i^s(t))) \\ &+ (1 - \gamma_i) x(t, \xi_i^s(t))], \end{aligned} \quad (6)$$

with the vector of gain given by $K = [k_1, k_2, \dots, k_N]$.

3. Main Results and Proofs

In order to obtain the main results, lemmas and definition are presented as follows:

Lemma 1^[23] Dirac function $\delta(\xi - \bar{\xi})$, $\bar{\xi} \in [\bar{\xi}_1, \bar{\xi}_2]$ satisfies the following properties:

① If $f(\xi)$ is bounded in \mathbf{R} , and continuous at $\xi = \bar{\xi}$,

$$\int_{\bar{\xi}_1}^{\bar{\xi}_2} \delta(\xi - \bar{\xi}) f(\xi) d\xi = f(\bar{\xi});$$

② If $f(\xi)$ differentiable in \mathbf{R} ,

$$\int_{\bar{\xi}_1}^{\bar{\xi}_2} \delta'(\xi - \bar{\xi}) f(\xi) d\xi = -f'(\bar{\xi}).$$

Lemma 2^[23] Let $f(\xi)$ differentiable in \mathbf{R} , and $\bar{\xi}$ is the only real root of $f(\xi) = 0$ in \mathbf{R} . Then the following result holds:

$$\delta(f(\xi)) = \frac{1}{|f'(\bar{\xi})|} \delta(\xi - \bar{\xi}).$$

Lemma 3^[24] Let $f(t)$ be a non-negative function defined on $[0, +\infty)$. If $f(t)$ is Lebesgue integrable on $[0, +\infty)$ and is uniformly continuous on $[0, +\infty)$, then $\lim_{t \rightarrow +\infty} f(t) = 0$.

Definition 1 The distributed parameter system (2) is said to be globally asymptotically stable in the mean square if $\lim_{t \rightarrow +\infty} \mathbf{E} |x(t)|^2 = 0$.

For notation simplicity, we let

$$M_1 = \text{diag} \left(m_1^- m_1^+ \varphi^* (\xi; \xi_1^S(t)) \varphi (\xi; \xi_1^S(t)), \right. \\ \left. m_2^- m_2^+ \varphi^* (\xi; \xi_2^S(t)) \varphi (\xi; \xi_2^S(t)), \dots, \right. \\ \left. m_N^- m_N^+ \varphi^* (\xi; \xi_N^S(t)) \varphi (\xi; \xi_N^S(t)) \right), \\ M_2 = \text{diag} \left(\frac{m_1^- \varphi (\xi; \xi_1^S(t)) + m_1^+ \varphi^* (\xi; \xi_1^S(t))}{2}, \right. \\ \frac{m_2^- \varphi (\xi; \xi_2^S(t)) + m_2^+ \varphi^* (\xi; \xi_2^S(t))}{2}, \dots, \\ \left. \frac{m_N^- \varphi (\xi; \xi_N^S(t)) + m_N^+ \varphi^* (\xi; \xi_N^S(t))}{2} \right).$$

Together with decentralized output feedback control (6), we deduce the closed-loop control system

$$\dot{x}(t) = [A - K(I - \Lambda_\gamma)\Phi(\xi^S(t))]x(t) \\ - K\Lambda_\gamma\sigma(\Phi(\xi^S(t))x(t)) \\ = A_c(\xi^S(t))x(t) - K\Lambda_\gamma\sigma(\Phi(\xi^S(t))x(t)). \quad (7)$$

Theorem 1 Under the decentralized output feedback control scheme (6) and the spatial distribution of sensor is given by (3), the distributed parameter system (2) is globally asymptotically stable in the mean square, if the following parameter-dependent linear operator inequality:

$$\Psi = \begin{bmatrix} \Upsilon(\xi^S(t)) & A_c(\xi^S(t))K\bar{\Lambda}_\gamma + \Omega M_2 \\ * & -\Omega \end{bmatrix} < 0, \quad (8)$$

where

$$\Upsilon(\xi^S(t)) = -A_c^*(\xi^S(t))A_c(\xi^S(t)) \\ - A_c(\xi^S(t))A_c(\xi^S(t)) - \Omega M_1, \quad (9)$$

hold in the Hilbert space $D(A)$, and the velocity law of each mobile sensor is given by

$$\dot{\xi}_i^S(t) = \mu_i k_i (1 - \bar{\gamma}_i) \frac{\partial x(t, \xi)}{\partial \xi} \Big|_{\xi = \xi_i^S(t)} x(t, \xi_i^S(t)), \quad (10)$$

with $\mu_i, i = 1, 2, \dots, N$, are velocity gain of each mobile sensor.

Proof. It is not difficult to see that closed loop operator $-A_c(\xi^S(t)) > 0$ due to $-A > 0$. In order to establish the stability conditions, we introduce the following parameter-dependent Lyapunov functional candidate

$$V(t) = - \left\langle x(t), A_c(\xi^S(t))x(t) \right\rangle. \quad (11)$$

The weak infinitesimal operator $LV(t)$ along the trajectory of the system (2) is calculated as follows:

$$LV(t) = \lim_{h \rightarrow 0^+} \frac{1}{h} \left\{ \mathbf{E} \{ V(x(t+h), t+h) | x(t) \} - V(x(t), t) \right\} \\ = - \mathbf{E} \left\langle \dot{x}(t), A_c(\xi^S(t))x(t) \right\rangle - \mathbf{E} \left\langle x(t), A_c(\xi^S(t))\dot{x}(t) \right\rangle \\ - \mathbf{E} \left\langle x(t), \frac{dA_c(\xi^S(t))}{dt} x(t) \right\rangle. \quad (12)$$

Taking into account the operator $A_c(\xi^S(t))$ is self-adjoint, we have

$$- 2 \mathbf{E} \left\langle \dot{x}(t), A_c(\xi^S(t))x(t) \right\rangle \\ = - 2 \left\langle A_c(\xi^S(t))x(t), A_c(\xi^S(t))x(t) \right\rangle \\ + 2 \left\langle K\bar{\Lambda}_\gamma\sigma(\Phi(\xi^S(t))x(t)), A_c(\xi^S(t))x(t) \right\rangle \\ = \left\langle \zeta(t), \Psi_0 \zeta(t) \right\rangle$$

where $\zeta(t) = [x^T(t), \sigma^T(\Phi(\xi^S(t))x(t))]^T$,

$$\Psi_0 = \begin{bmatrix} \Upsilon_0(\xi^S(t)) & A_c(\xi^S(t))K\bar{\Lambda}_\gamma \\ * & 0 \end{bmatrix},$$

$$\Upsilon_0(\xi^S(t)) = -A_c^*(\xi^S(t))A_c(\xi^S(t)) - A_c(\xi^S(t))A_c(\xi^S(t)).$$

With Assumption 1 and $N \times N$ positive definite diagonal matrix Ω , it follows that

$$- 2 \mathbf{E} \left\langle \dot{x}(t), A_c(\xi^S(t))x(t) \right\rangle \\ \leq \left\langle \zeta(t), \Psi_0 \zeta(t) \right\rangle + \left\langle \zeta(t), M(\xi^S(t))\zeta(t) \right\rangle \\ = \left\langle \zeta(t), \Psi \zeta(t) \right\rangle$$

$$\text{where } M(\xi^S(t)) = \begin{bmatrix} -\Omega M_1 & \Omega M_2 \\ \Omega M_2 & -\Omega \end{bmatrix}.$$

It is easy to see, first two terms in the right hand side of (12) are negative if $\Psi < 0$.

And the third term in the right hand side of (12) is

$$- \mathbf{E} \left\langle x(t), \frac{dA_c(\xi^S(t))}{dt} x(t) \right\rangle \\ = \left\langle x(t), \frac{d(K(I - \bar{\Lambda}_\gamma)\Phi(\xi^S(t)))}{dt} x(t) \right\rangle \\ = \int_0^l \dot{\xi}_i^S(t) K(I - \bar{\Lambda}_\gamma) \frac{\partial \varphi(\xi; \xi_i^S(t))}{\partial \xi} x^2(t, \xi) d\xi \\ = \sum_{i=1}^N k_i (1 - \bar{\gamma}_i) \dot{\xi}_i^S(t) \int_0^l \frac{\partial \delta(\xi - \xi_i^S(t))}{\partial \xi} x^2(t, \xi) d\xi \\ = - \sum_{i=1}^N 2k_i (1 - \bar{\gamma}_i) \dot{\xi}_i^S(t) \frac{\partial x(t, \xi)}{\partial \xi} \Big|_{\xi = \xi_i^S(t)} x(t, \xi_i^S(t)).$$

The choice

$$\dot{\xi}_i^s(t) = \mu_i k_i (1 - \bar{\gamma}_i) \frac{\partial x(t, \xi)}{\partial \xi} \Big|_{\xi = \xi_i^s(t)} x(t, \xi_i^s(t)),$$

with $\mu_i > 0$ such that the result above is negative. Then, it follows readily from (8)-(10) that

$$LV(t) \leq -c |x(t)|^2, \tag{13}$$

where $c > 0$.

Furthermore, $-A_c(\xi^s(t))$ satisfy the following inequality in $D(A)$:

$$\begin{aligned} & b_1 \langle x(t), x(t) \rangle \leq \\ & \langle x(t), -A_c(\xi^s(t))x(t) \rangle \\ & \leq b_2 [\langle x(t), x(t) \rangle + \langle Ax(t), Ax(t) \rangle] \end{aligned} \tag{14}$$

where $b_1 > 0, b_2 > 0$. Letting $\alpha = b_2/c$, we have

$$\int_0^t \mathbf{E} |x(s)|^2 ds \leq \alpha (|x_0|^2 + |Ax_0|^2).$$

This implies that

$$\lim_{t \rightarrow +\infty} \mathbf{E} |x(t)|^2 = 0,$$

and therefore the system (2) is globally asymptotically stable in the mean square.

Remark 1: In fact, if choose the parameter-dependent Lyapunov functional as $V(t) = -e^{\alpha t} \langle x(t), A_c(\xi^s(t))x(t) \rangle$, we can verify the system (2) is exponentially stable in the mean square along the similar line of proof of Theorem 1, and the proof is omitted.

Remark 2: It is shown in Theorem 1 that one of the stability conditions of abstract equation (2) is in terms of linear operator inequality which using sampled position data by mobile sensors. When sampled position data by fixed sensors, such stability condition of system (2) is degraded to linear matrix inequality. So the results given in this paper are extending the previous works.

In what follows, a more general spatial distribution of i th sensor is presented.

$$\varphi(\xi; \xi_i^s) = \delta(f(\xi)), \tag{15}$$

where $f(\xi)$ differentiable in $[0, I]$, and $\xi_i^s(t)$ is the only real root of $f(\xi) = 0$ in $[0, I]$.

Theorem 1 can be generalized in the following.

Theorem 2 Under the decentralized output feedback control scheme (6) and the spatial distribution of sensor is given by (15), the distributed parameter system (2) is globally asymptotically stable in the mean

square, if the linear operator inequality (8) hold in the Hilbert space $D(A)$, and the velocity law of each mobile sensor is given by:

$$\dot{\xi}_i^s(t) = \frac{\mu_i k_i (1 - \bar{\gamma}_i) \partial x(t, \xi)}{|f'(\xi_i^s(t))| \partial \xi} \Big|_{\xi = \xi_i^s(t)} x(t, \xi_i^s(t)). \tag{16}$$

Proof. Along the similar line of proof of Theorem 1, we can obtain the results of Theorem 2. Hence, the proof is omitted here.

From Theorem 2, we have the following result easily. By (15), we choose

$$f(\xi) = (\xi - \xi_i^s(t))g(\xi), g(\xi) \neq 0. \tag{17}$$

Then the following corollary obtained directly.

Corollary 1 Under the control scheme (6), and the spatial distribution of sensor is given by (15) with $f(\xi)$ satisfy (17), the distributed parameter system (2) is globally asymptotically stable in the mean square, if the linear operator inequality (8) hold in the Hilbert space $D(A)$, and the velocity law of each mobile sensor is given by:

$$\dot{\xi}_i^s(t) = \frac{\mu_i k_i (1 - \bar{\gamma}_i) \partial x(t, \xi)}{|g(\xi_i^s(t))| \partial \xi} \Big|_{\xi = \xi_i^s(t)} x(t, \xi_i^s(t)). \tag{18}$$

If $f(\xi)$ in (15) satisfy

$$\lim_{\xi \rightarrow \xi_i^s(t)} \frac{f(\xi)}{\xi - \xi_i^s(t)} = 1, \tag{19}$$

we have the following corollary.

Corollary 2 Under the control scheme (6), and the spatial distribution of sensor is given by (15) with $f(\xi)$ satisfy (19), the distributed parameter system (2) is globally asymptotically stable in the mean square, if the linear operator inequality (8) hold in the Hilbert space $D(A)$, and the velocity law of each mobile sensor is given by:

$$\dot{\xi}_i^s(t) = \mu_i k_i (1 - \bar{\gamma}_i) \frac{\partial x(t, \xi)}{\partial \xi} \Big|_{\xi = \xi_i^s(t)} x(t, \xi_i^s(t)). \tag{20}$$

It is not difficult to find that the result of Corollary 2 can become the form of (10) in Theorem 1 when we choose $f(\xi) = \xi - \xi_i^s(t)$.

4. A Numerical Example

In this section, a simulation example is provided to illustrate the effectiveness of the results obtained. Consider a one-dimensional distributed parameter system (1) with three mobile sensors in $\Omega = [0, 1]$. The initial condition associated with (1)

is $x(0, \xi) = \sin(\pi\xi)e^{-6\xi^2}$ and boundary condition is $x(t, 0) = x(t, 1) = 0$. The coefficient of transmission diffusion operator is $a = 0.005$. The gain of controller are $k_1 = 8, k_2 = 10, k_3 = 12$ and probabilities are taken as $\bar{y}_1 = 0.7, \bar{y}_2 = 0.75, \bar{y}_3 = 0.6$.

The closed loop system is simulated in the time interval $[0, 10]$ with three mobile sensors. As a comparison, we also consider three fixed-in-space sensors. The static sensors are fixed at $\xi_1^s = 0.15, \xi_2^s = 0.50, \xi_3^s = 0.85$, where also are the initial locations of mobile sensors. In the simulation, the state L_2 norm for the closed loop system and the case of mobile networks are shown in Figure 1. It is clear that the performance of system which contained mobile sensors is better than the static case. Figure 2 describes the state distribution of static and mobile networks at four different time instants. Figure 3 depicts the trajectory of three sensors for the fixed and mobile cases.

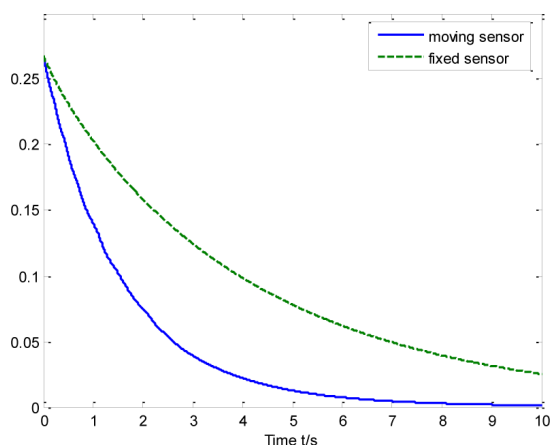


Figure 1. Evolution of spatial L_2 norm

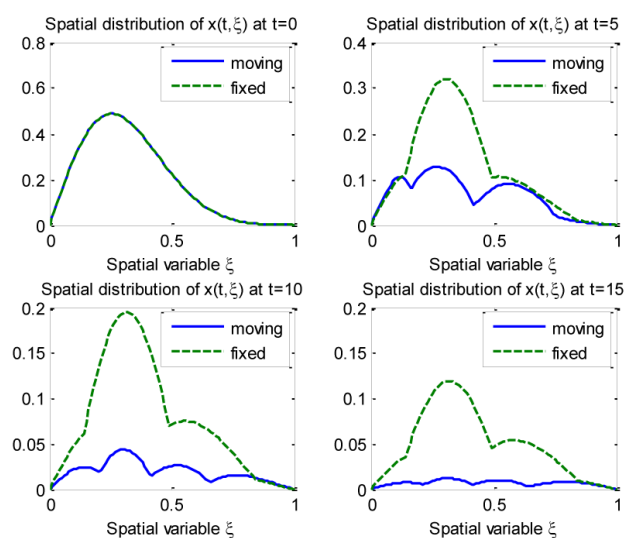


Figure 2. Comparison of closed loop state and spatial variable at different times

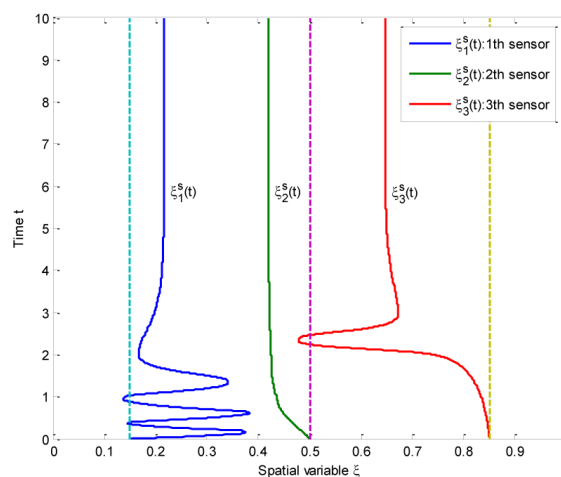


Figure 3. The trajectory of three sensors

Conclusions

In this paper, a novel sampled position data control approach has been proposed for stabilization of an array of distributed parameter systems based on the point measurement of mobile sensors combine with decentralized output feedback controller. Also, the influence of randomly occurring sensor saturations has been considered in distributed parameter control system. By means Lyapunov functional method and linear operator inequality technique, sufficient conditions have been derived for globally asymptotically stable in the mean square in terms of linear operator inequality and the velocity law of each mobile sensor. Finally, a simulation example is given to illustrate the usefulness of the obtained results.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (No.61174021, 61473136, and 61104155).

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