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# Airflow angle control of hypersonic flight vehicle with time-delay input

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### Abstract

In order to have an efficient control of its flight, hypersonic flight vehicle needs to have an accurate control of its airflow attitude angle. On the basis of realizing the accurate tracing of airflow attitude angle command, the projected flight path can be realized to accomplish flight missions. Considering the input time-delay caused by the system signal transmission of the aircraft and the actuator dynamic, compensate for the adverse effects caused by it, and design a time-delay compensation controller, which realizes the tracking control of airflow attitude angle for hypersonic flight vehicles with the input time-delay. Simulation result shows that this method has better properties of tracking control and rapid convergence.

Keywords: HYPERSONIC FLIGHT, BACKSTEPPING, AIRFLOW ATTITUDE ANGLE, TIME-DELAY, CONTROLLER

### 1. Introduction

Hypersonic flight vehicle has some characters, such as strong nonlinearity, fast time-varying and strong coupling, etc. Because the hypersonic flight vehicle has a skyscraping flight speed and the engine is sensitive to airflow angles, we have to realize real-time control. In order to realize the control of the aircraft, dynamic inversion, backstep-

ping and some other nonlinear methods are widely used. Through the analyses of hypersonic motion models and some appropriate simplification, it is appropriate to use the design method of backstepping. As a result, this paper takes the model with special structure as the nominal system, and uses backstepping to design the nominal control law for the nominal system[1-3].

**2. Problem Description**

According to the kinematics equation of attitude angle and the kinetics equation of angular rate, abstract the input time-delay effect caused by signal transmission and actuator dynamic as the situation of pure input containing time-delay, and describe the airflow angle attitude control problem of hypersonic flight vehicle as the control problem of input with time-delay for block strict feedback nonlinear systems[4-5]. The block triangle strict feedback nonlinear system has a specific form, i.e. three-dimensional state block and two-layer rough integral chain. The mathematical description is as follows:

$$\begin{cases} \dot{\Omega} = f_s + g_s \omega \\ \dot{\omega} = f_f + g_f g_{f\delta} \delta(t - D(t)) \\ y = \Omega \end{cases} \quad (1)$$

Where nonlinear function vector  $f_s$ ,  $f_f$  and control gain matrices  $g_s$  and  $g_f g_{f\delta}$  are known; delay model  $D(t)$  is also known. For further research and combining practical situation, we can have following hypotheses.

**3. Relevant theory bases**

**3.1. Design procedures of backstepping**

To facilitate the theoretical description and research, the object system is extended to a more general n-layer subsystem form, of which every subsystem has a m-dimensional status. The system function is described as follows:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} & i = 1, 2, \dots, n-1 \\ \dot{x}_n = f_n(x) + g_n(x)u(t) \\ y = x_1 \end{cases} \quad (2)$$

where the dimension of status block vector  $x_i \in R^m$  is consistent with the number of input channels. The status vector  $\bar{x}_i = [x_1^T, x_2^T, \dots, x_i^T]^T$  is composed by  $i$

$$\begin{aligned} \dot{V}_1 &= e_1^T \dot{e}_1 = e_1^T (f_1 + g_1 x_2 - \dot{y}_d) \\ &= e_1^T (f_1 + g_1 x_{2d} - \dot{y}_d) + e_1^T g_1 (x_2 - x_{2d}) \\ &= e_1^T (f_1 - c_1 e_1 - f_1 - \dot{y}_d + \dot{y}_d) + e_1^T g_1 (x_2 - x_{2d}) = -c_1 e_1^T e_1 + e_1^T g_1 e_2 \end{aligned} \quad (6)$$

It can be seen from the above form that the second item  $e_1^T g_1 e_2$  cannot be handled in this step, the dynamic condition of virtual instruction tracking error  $e_2$  needs to be introduced to counteract  $e_1^T g_1 e_2$ , and make sure that the tracking error is asymptotically stable. The rest can be done in the same manner to step  $i$  [8].

Step  $i$  ( $2 \leq i \leq n-1$ ): Based on (2) and referring to (3), there is

$$\dot{e}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} - \dot{x}_{id} \quad (7)$$

status blocks before and  $x = [x_1^T, x_2^T, \dots, x_n^T]^T$  expresses the overall statuses of the system.  $u \in R^m$  is the control input.  $f_i(\bar{x}_i)$  and  $f_n(x)$  are both the known fully smooth nonlinear function.  $g_i(\bar{x}_i)$  is the gain matrix of controlled quantity  $x_{i-1}$  in the subsystem under the  $x_i$  status.  $g_n$  is the gain matrix of actual controlled quantity  $u$  and  $y \in R^m$  is the system input. For purpose of achieving the design of backstepping for this system, further hypotheses for the system are needed[6-7]:

For the system shown in(2), backstepping method can be used to design the controller. The steps are as follows:

Step 1: Define the output tracking error  $e_1 = x_1 - y_d$ . According to the system function, the dynamic condition of tracking error  $\dot{e}_1$  can be denoted as:

$$\dot{e}_1 = f_1(x_1) + g_1(x_1)x_2 - \dot{y}_d \quad (3)$$

Consider  $x_2$  as the controlled quantity of this error subsystem, namely virtual controlled quantity. To make the tracking error tend to zero, as  $f_1(x_1)$  and  $\dot{y}_d$  are known and  $g_1(x_1)$  is reversible, then take the virtual instruction (ideal virtual control law)  $x_{2d}$  as

$$x_{2d} = g_1^{-1}(-c_1 e_1 - f_1(x_1) + \dot{y}_d) \quad (4)$$

Where  $x_{2d}$  is the design parameter. It can be seen that after the actual status block  $x_2$  tending to the virtual instruction, the tracking error will also tend to zero. In order to study the actual control effect of the system, the following Lyapunov function  $V_1$  can be constructed in allusion to the first subsystem in (2).

$$V_1 = \|e_1\|^2 / 2 \quad (5)$$

Define the virtual instruction tracking error  $e_2 = x_2 - x_{2d}$  and take the derivative:

Like the first step mentioned before, the expected virtual control  $x_{(i+1)d}$  in step  $i$  can be taken as

$$x_{(i+1)d} = g_i^{-1}(-c_i e_i - f_i(\bar{x}_i) + \dot{x}_{id} - g_{i-1}^T e_{i-1}) \quad (8)$$

Where  $c_i > 0$ . The fourth item in the above formula is to counteract  $e_{i-1}^T g_{i-1} e_i$  in step  $i-1$ , and the other items are to counteract the relevant nonlinear items and the known quantities, that generate the expected dynamic status. Similar to the first step described earlier, take the Lyapunov function  $V_i$  after the  $i$ th step as follows:

$$V_i = V_{i-1} + \|e_i\|^2 / 2$$

(9)

Define virtual instruction tracking error  $e_{i+1} = x_{i+1} - x_{(i+1)d}$ . By taking derivate, we can get

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} + e_i^T \dot{e}_i = - \sum_{k=1}^{i-1} c_k \|e_k\|^2 + e_{i-1}^T g_{i-1} e_i + e_i^T (f_i + g_i x_{i+1} - \dot{x}_{id}) \\ &= - \sum_{k=1}^{i-1} c_k \|e_k\|^2 + e_{i-1}^T g_{i-1} e_i + e_i^T (f_i + g_i x_{(i+1)d} - \dot{x}_{id}) + e_i^T g_i (x_{i+1} - x_{(i+1)d}) \\ &= - \sum_{k=1}^i c_k \|e_k\|^2 + e_i^T g_i e_{i+1} \end{aligned} \quad (10)$$

It can be seen from the above form that the second item  $e_i^T g_i e_{i+1}$  cannot be handled in this step, and the dynamic status of virtual instruction tracking error  $e_{i+1}$  needs to be introduced further, which is expected to counteract  $e_i^T g_i e_{i+1}$ , and make sure the tracking error is asymptotically stable. The rest can be done in the same manner to step n.

Step n: Define virtual instruction tracking error  $e_n = x_n - x_{nd}$

$$\dot{e}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)u - \dot{x}_{nd} = f_n(x) + g_n(x)u - \dot{x}_{nd} \quad (11)$$

Also for counteracting the remaining item  $e_{n-1}^T g_{n-1} e_n$  in  $V_{n-1}$  last step and eliminating other known items, thus generate stable linear dynamic. The control quantity  $u$  can be designed as

$$u = g_n^{-1}(-c_n e_n - f_n(\bar{x}_n) + \dot{x}_{nd} - g_{n-1}^T e_{n-1}) \quad (12)$$

Where  $c_n > 0$ . In order to analyze all the dynamic stability in the system, Lyapunov function can be taken as

$$V = \sum_{i=1}^{n-1} V_{i-1} + \|e_n\|^2 / 2 \quad (13)$$

Take the derivative:

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{n-1} \dot{V}_{i-1} + e_n^T \dot{e}_n = - \sum_{i=1}^{n-1} c_i \|e_i\|^2 + e_{n-1}^T g_{n-1} e_n + e_n^T (f_n + g_n u - \dot{x}_{nd}) \\ &= - \sum_{i=1}^{n-1} c_i \|e_i\|^2 + e_{n-1}^T g_{n-1} e_n + e_n^T (f_n - c_n e_n - f_n + \dot{x}_{nd} - g_{n-1}^T e_{n-1} - \dot{x}_{id}) \\ &= - \sum_{i=1}^n c_i \|e_i\|^2 \leq 0 \end{aligned} \quad (14)$$

As  $u$  can be directly equal to the design value, which is unlike the virtual instruction causing the instruction tracking error. It can be seen from (14), besides the origins  $\{e_i = 0, n=1, 2, \dots, n\}$ , the derivative of its Lyapunov function is negative. As a result, the error system is asymptotically stable, i.e., the original system output gradually tracks a given output[9].

### 3.2. Time-delay compensator

Time-delay systems have many process modes and the Lyapunov mode is commonly used. For nonlinear systems, especially input time-delay nonlinear systems, the available results are hard to acquire. Here a recently developed time-delay compensation control is introduced. This method has better universality. As it is a compensation control, it is appropriate for the case of actuation with time-delay for the hypersonic flight vehicle under study[10].

Considering a common nonlinear system with time-delay in the input, its time-delay is time-varying and the time-domain model of time-delay is known. The system can be described as follows:

$$\dot{x}(t) = f(x(t), u(t - D(t))) \quad (15)$$

where  $x \in R^n, u \in R^m$  and  $t \in R_+$   $f$  is first-order derivable and continuous through origin.

If the controller of an asymptotically stabilized nominal system  $\dot{x} = f(x, u)$  without time-delay can be taken as  $u(t) = k(t, x)$  then the time-delay compensation controller used to control system (15) has the following forms:

$$u(t) = k(\phi^{-1}(t), P(t)) \quad (16)$$

where  $P(t)$  has the following form:

$$P(\theta) = (\phi^{-1}(\theta) - t) \int_0^{\phi^{-1}(\theta) - t} f(P(\phi(t + y(\phi^{-1}(\theta) - t))), u(\phi(t + y(\phi^{-1}(\theta) - t)))) dy + x(t) \quad (17)$$

$$= \int_{\phi(t)}^{\theta} f(P(\sigma), u(\sigma)) \frac{d\sigma}{\phi'(\phi^{-1}(\sigma))} + x(t), \quad \phi(t) \leq \theta \leq t$$

Firstly, convert them to a system described by partial differential equations, i.e. take time and time-delay as the independent variables of the system. Then, a reversible transformation of the original system can be acquired, and it can be proved that this transformation can make the corresponding properties of the system unchanged. Through the stability demonstration of the target system after conversion, the system stability theorem can be achieved[11].

$$\begin{aligned} \dot{x}(t) &= f(x(t), U(0, t)) \\ U_t(x, t) &= \pi(x, t) U_x(x, t), \quad x \in [0, 1] \\ U(1, t) &= u(t) \end{aligned} \quad (18)$$

$$\|x(t)\| + \sup_{\phi(t) \leq \theta \leq t} \|u(\theta)\| \leq \alpha \left( \|x(0)\| + \sup_{\phi(0) \leq \theta \leq 0} \|u(\theta)\|, t \right), \quad t \geq 0 \quad (20)$$

According to lemma, it is obtained that

$$\begin{aligned} \sup_{x \in [0, 1]} \|w(x, t)\| &\leq \sup_{x \in [0, 1]} \|u(x, t)\| + \hat{\rho} \cdot \rho_1 \left( \|x(t)\| + \sup_{x \in [0, 1]} \|u(x, t)\| \right) \\ \sup_{x \in [0, 1]} \|u(x, t)\| &\leq \sup_{x \in [0, 1]} \|w(x, t)\| + \hat{\rho} \cdot \rho_2 \left( \|x(t)\| + \sup_{x \in [0, 1]} \|w(x, t)\| \right) \end{aligned} \quad (21)$$

#### 4. Controller design and the stability analysis

In order to handle the hypersonic flight vehicle inner signal transformation and the time-delay problems caused by actuation surface response by using time-delay compensation, it needs to design controller according to the nominal situation of airflow angle attitude system (namely no time-delay). Firstly, the attitude controller is designed by standard backstepping method to get the relevant time-delay compensation controller. Then, aiming at the calculate inflation problem of the method and the emerging problem of unknown system accurate derivation, use dynamic surface to design controller, and the relevant time-delay compensation controller can be acquired[12].

By using the above backstepping methods, we can switch the attitude tracking problems to the stabilization problems of the error system.

1) According to the attitude angle loop, design virtual control law. Define  $x_1 \square \Omega$ ,  $f_1 \square f_s$ ,  $g_1 \square g_s$ ,  $c_1 = c_s$  and  $e_1 \square e_s = \Omega - \Omega_d$ . Design attitude angle loop control law as

$$\omega_d = g_s^{-1}(-c_s e_s - f_s + \dot{\Omega}_d) \quad (22)$$

2) According to the angular rate loop, design angular rate loop control law. Define  $x_2 \square \omega$ ,  $f_2 \square f_f$ ,

where

$$\pi(x, t) = \frac{1 + x \left( \frac{d(\phi^{-1}(t))}{dt} - 1 \right)}{\phi^{-1}(t) - t} \quad (19)$$

The reason that choose such a transmission rate function is to seek infinite-dimension actuator status, so as to meet  $U(0, t) = u(\phi(t))$  and  $U(1, t) = u(t)$

Theorem 1: If the control system meets the above conditions, there is a KL generic function  $\alpha$  to make

$g_2 \square g_f g_{f, \delta}$ ,  $c_f \square c_2$  and  $e_2 \square e_f = \omega - \omega_d$ . Design angular rate loop control law as

$$\delta_c = g_2^{-1}(-c_f e_f - f_f - g_s^T e_s + \dot{\omega}_d) \quad (23)$$

where  $\dot{\omega}_d$  is the output of the nonlinear differential tracker, in which  $\omega_d$  is the input.

Theorem2: According to the attitude motion system equation (1) of hypersonic flight vehicles, when  $D(t) = 0$ , and in the case of using backstepping to design tracking controller, the system is ISS for the additional inputs  $d_1$  and  $d_2$

**Prove:** Through the backstepping transformation, attitude tracking control system is converted to the following error system.

$$\begin{cases} \dot{e}_1 = f_s + g_s \omega_d(e_1) - \dot{\Omega}_d + g_s e_2 + d_1 \\ \dot{e}_2 = f_f + g_f g_{f, \delta} \delta(t) - \dot{\omega}_d(e_1) + d_2 \end{cases} \quad (24)$$

As for the above subsystem  $\dot{e}_1 = f_s + g_s \omega_d(e_1) - \dot{\Omega}_d + g_s e_2 + d_1$ , take  $e_2, d_1$  as inputs and  $e_1$  as output, and its open loop CLF is:

$$V_1 = \|e_1\|^2 \quad (25)$$

$$\dot{V}_1 = 2e_1^T \dot{e}_1 = 2e_1^T (f_s + g_s \omega_d(e_1) - \dot{\Omega}_d + g_s e_2 + d_1) \quad (26)$$

Take  $g_s \omega_d(e_1) = \dot{\Omega}_d - f_s + v$

$$\dot{V}_1 = 2e_1^T (v + g_s e_2 + d_1)$$

$$\dot{V}_1 \leq 2e_1^T v + \frac{(\xi_1^2 \|g_s e_2\|^2 + \|e_1\|^2)}{\xi_1} + \frac{(\xi_2^2 \|d_1\|^2 + \|e_1\|^2)}{\xi_1} \quad (27)$$

$$\dot{V}_1 \leq 2e_1^T v + \frac{(\xi_1 + \xi_2) \|e_1\|^2}{\xi_1 \xi_2} + \xi_1 \|g_s\|_F^2 \|e_2\|^2 + \xi_2 \|d_1\|^2$$

Take  $v = -[\frac{(\xi_1 + \xi_2)}{2\xi_1\xi_2} + k_1]e_1$

$$\dot{V}_1 \leq -2k_1 \|e_1\|^2 + \xi_1 \|g_s\|_F^2 \|e_2\|^2 + \xi_2 \|d_1\|^2 \quad (28)$$

then:

$$\omega_d(e_1) = g_s^{-1} (\dot{\Omega}_d - f_s - [\frac{(\xi_1 + \xi_2)}{2\xi_1\xi_2} + k_1]e_1) \quad (29)$$

In actual applications, it is necessary to find out the analytical expression. However, in simulink simulations, it is acquired directly through the differential of  $\omega_d$ . In order to avoid this differential operation, the instruction filter is used.  $P(e_f)$  and  $P(e_s)$  are respectively the corresponding state components of error state components  $e_f$  and  $e_s$  after compensation.

### 5. Simulated experiment

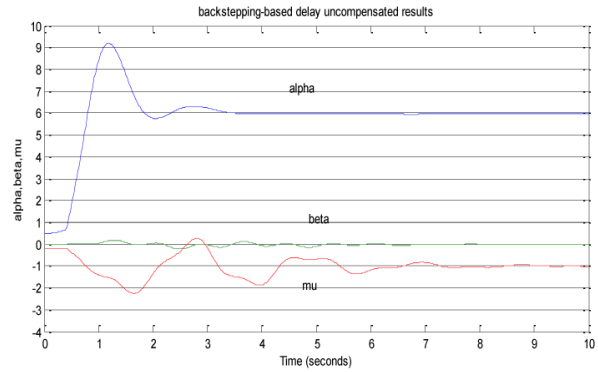
In order to verify the designed control law and analyze the influences of control performance and design parameters. Simulink is adopted and the corresponding simulation results are given under different simulation conditions.

It assumes that some parameters of the aircraft are constants, such as mass, moment of inertia, etc.  $M=136080\text{tKg}$ . The initial flight speed is  $V_0=2200\text{m/s}$ . The thrust is a constant, namely  $T=400\text{KN}$ . The initial value of the flight height is  $H_0=27\text{km}$ . The initial value of the attack angle is  $\alpha_0 = 1.0^\circ$ . The initial value of the sideslip angle is  $\beta_0 = 1.0^\circ$ .  $\mu_0 = 0.5^\circ$  denotes the initial value of the roll angle around speed.  $p_0 = q_0 = r_0 = 0 \text{ rad/s}$  is the initial value of the component of the body angular rate, and the simulation time is 10s. Input time-delay function is  $D(t) = \frac{1+t}{2(1+2t)}$ .

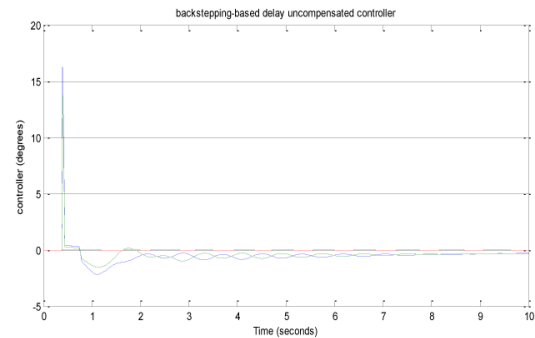
The expected attack angle instruction is that the 6-degree constant outputs through a first-order link, and  $\alpha = \frac{6}{s+2}$ ; sideslip angle instructions are  $\beta = 0^\circ$ ,  $\mu = -1^\circ$ .

(1) When there is no time-delay compensation, the tracking results and control signals are as follows:

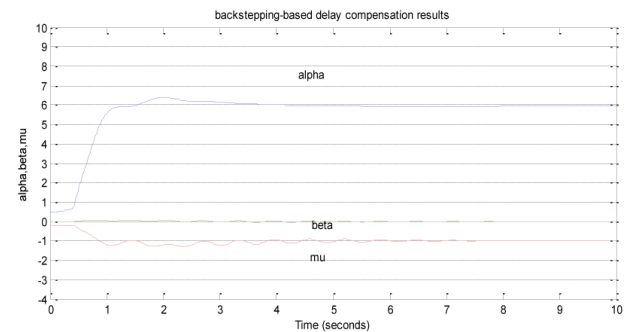
(2) For the same control gain, when using time-delay compensation, the tracking results and control signals are as follows:



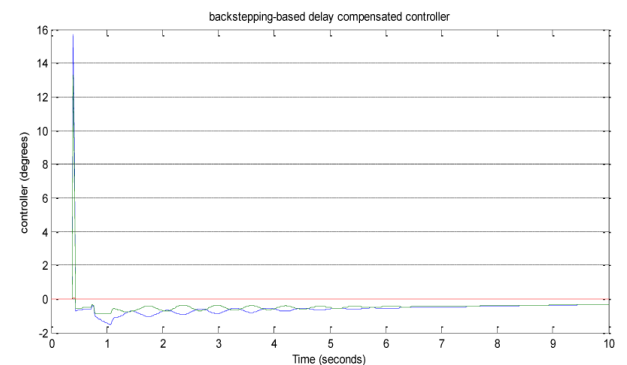
**Figure 1.** Tracking results without compensation



**Figure 2.** Controlled quantity without compensation (the deflection angle of the rudder surface)



**Fig 3.** Tracking results with time-delay compensation



**Fig 4.** Controlled quantity with time-delay compensation (the deflection angle of the rudder surface)

## 6. Conclusion

For the hypersonic flight vehicle existing disturbances, the paper has considered the flight control problem, and designed a time-delay compensation controller. Firstly, for the hypersonic flight vehicle, it realizes the tracking control of airflow attitude angle for hypersonic flight vehicles with the input time-delay. The designed methods are applied to the hypersonic flight vehicle, that realize the tracking control, and achieve the good control performance and effectiveness.

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