

# Gear Weight Optimization Design Model Based on Disturbance and Simulated Annealing Algorithm

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## Abstract

In view of the problems of easily being caught in the local optimum and long operation time of standard simulated annealing algorithm in the gear weight optimization process, this paper put forward an improved genetic and simulated annealing algorithm with the function of disturbance. Firstly, it used Cauchy distribution as the theory of disturbance distance to process the distance between current division and new division so as to reduce the computation complexity of internal circulation. Then genetic algorithm with mixed single point crossover operator is introduced to enlarge the searching space of population. Finally, the advantages of improved genetic algorithm and simulated annealing algorithm are combined to establish the gear weight optimization design model. The simulation experiments show that improved genetic algorithm and simulated annealing algorithm has better result in gear weight optimization and higher working life compared with traditional simulated annealing algorithm.

Key words: SIMULATED ANNEALING ALGORITHM, IGA-SA, GEAR OPTIMIZATION, STRUCTURE OPTIMIZATION, MIXED SINGLE POINT, DISTURBANCE OPTIMIZATION

## Introduction

An optimization design of a product actually is the optimization of design variables, design methods and optimization process so that the optimization results are more reasonable and feasible [1]. Specific to the mechanical design, it is required to put first the strength, deflection, weight, wear, corrosion, etc. In addition, for a complete mechanical transmission combination, it is also needed to optimize the complex objective function with a large number of design variables on the basis of high reliability, which is very difficult in the actual operation [2]. Gear transmission is one of the most important transmission in the mechanical transmission, and the most widely used transmission form among all kinds of transmission. It transfers the motion and power between any two axes in space relying on the direct contact of gear tooth profile. The quality of the gear transmission design not only has effect on its own transmission performance, size and weight, but also may have some

impacts on the performance of the whole machine [3]. Therefore, it is very important to find the optimal design scheme of gear transmission in the design.

With the development of science and technology and the maturity of various theories, the optimization design of the design objects with various design methods is widely used in the design process. G. Gerard wrote an article of the optimization design based on the simultaneous failure theory which was a typical minimum function optimization theory. It can only deal with the simple structure, but the actual mechanical structure is often complicated structure. Therefore, this theory has more limitations [4]. Genetic algorithm based on the simulation of Darwin genetic selection and biological evolution process, is widely used in the mechanical optimization design due to its wide adaptability, parallelism, robustness and other outstanding advantages, and renews the optimization theory [5]. Wang Desheng and Wang Zhankui optimized the gear transmission by ant colony algorithm.

The results showed that this algorithm was an efficient optimization algorithm under the conditions of multiple parameters and complicated constraints [6]. Zhong Bo applied the artificial neural network to optimize the design of gear and proposed a twice training method of the neural network based on genetic algorithm [7]. MATLAB algorithm is also used for solving the optimization design problem. Although the program design is relatively simple, the standard value is processed as a continuous variable [8]. Simulated annealing algorithm is also applied in the optimization design of gear drive [9]. In the current optimization design of gear rotation, genetic algorithm also has more applications. However, in the process of gear rotation optimization, the binary encoding based on genetic algorithm has four problems: first, very large storage capacity; second, low accuracy; third, a large number of redundant continuous integer variables; fourth, the non-continuous standard which cannot guarantee the optimization results even after second optimization. The real number encoding based on genetic algorithm also has similar problems, such as the discrete variables after treatment also need to be processed again that is, the rounding processing [10].

In view of the defects of simulated annealing algorithm, this paper proposes a gear weight optimization model based on optimized simulated annealing algorithm by disturbance and improved genetic operator. Simulation experiments are conducted to verify the effectiveness of this improved strategy.

### **Defect analysis of simulated annealing algorithm**

During the decreasing trend of system energy, simulated annealing algorithm allows the system jumping to a state of higher energy to avoid the local minimum and finally stabilize at the global minimum. Its basic steps are written as follows.

(1) Setting of initial parameters: taking a large enough initial temperature  $T$ ; initial solution  $i$ ; iteration number  $N$  of each  $T$ .

(2) If the iteration number is in the range of  $k=1, 2, \dots, N$  (starting from  $k=1$ ), then continuing step (3) to (6).

(3) Randomly selecting a solution  $j$ .

(4) Calculating the increment  $\Delta = f(j) - f(i)$  where  $f(x)$  is the evaluation function.

(5) If  $\Delta < 0$  then  $j$  is received as current solution; otherwise,  $j$  is taken as current solution in a probability of  $\exp(-\Delta/T)$ .

(6) If the ending conditions are reached, current solution is output to be the optimum value and the program ends. The ending condition is usually when

some continuous solutions cannot be accepted; otherwise, turn to step (7).

(7) Decreasing  $T$  by  $k$  and  $k = k + 1$ , then turn to step (3).

Simulated annealing algorithm is independent of initial value, namely the solution of algorithm is independent of initial solution state  $S_0$  (the beginning of algorithm iteration); simulated annealing algorithm has the asymptotic convergence which is theoretically proved to be global optimization algorithm converging to global optimal in probability 1.

Simulated annealing algorithm allows the system jumping to a state of higher energy to avoid the local minimum and finally stabilize at the global minimum in a process of decreasing system energy.

### **(1) State generation and acceptance function**

State generation function includes two parts, the process function to get the candidate solution and the state function at different time. State acceptance function represents the process function in a state of acceptation, and usually has following rules: in a stable temperature state, the candidate solution which decreases the objective function is the priority. Then the temperature decreases slowly to make fewer the candidate solution that increases the function value.

Experiments prove that this function doesn't have evident effect on the algorithm performance. Therefore, this function is often process in following way.

$$\min[1, -\Delta E / T] \quad (1)$$

### **(2) Initial temperature**

The higher the initial temperature is set, the larger the probability to obtain the final solution of high quality. But the computation time increases correspondingly, which decreases the feasibility. Theoretically analysis proves that a quasi-equilibrium of algorithm can be obtained at the beginning by setting the initial acceptation probability to 1, namely,

$$x_0 = \frac{\text{Accepted}}{\text{Extract}} \approx 1 \quad (2)$$

According to Metropolis rule,  $\exp(-\Delta f / t_0) \approx 0$  where  $\Delta f$  is the difference value of objective function. If  $x_0 = 0.9$ , when  $\Delta f = 100$ ,  $t_0 > 949$ .

After considering the optimization quality and efficiency, we choose following method to select initial temperature: randomly generating a group of state, determining the maximum objective value error between states and the initial temperature by functions.

### **Temperature updating function**

This function is usually related to the descending process of temperature with exponential form.

$$t_{k+1} = \alpha t_k \quad (3)$$

Here,  $0 < \alpha < 1$ , and the function value can vary in

external circulation.

Ending condition of internal and external circulation

The internal circulation ending condition is used to calculate the candidate set of different temperatures. The external circulation ending condition determines when the algorithm ends.

The simulated annealing algorithm is a heuristic searching algorithm, so it has following defects:

(1) Simulated annealing algorithm can find the global optimum absolutely, but it requires high enough initial temperature, slow enough decreasing speed and the final temperature to zero. This kind of design costs too much time, which is impossible in practical situations.

(2) The selection of parameters has great effect on the quality of solution, which is difficult to handle in different problems.

(3) It costs too much to solve large scale problems.

(4) It is hard to judge whether a temperature reaches to the equilibrium state, namely Markov chain is hard to control.

### **The simulated annealing algorithm based on disturbance and genetic algorithm**

#### **Annealing algorithm based on disturbance**

The disturbance model of annealing algorithm has an important effect on the convergence speed, but large number of algorithms only randomly give disturbance to a node. The fixed disturbance model is a constant function and independent of the temperature change, so that it will decrease the rate of convergence. Therefore, this paper choose a new disturbance model using Cauchy distribution as the theory of disturbance distance, expressed as follows.

$$p(\Delta x) = \frac{1}{\pi} \frac{T}{T^2 + (\Delta x)^2} \quad (4)$$

It can be derived that,

$$\Delta x = T \tan(\pi \cdot z) \quad (5)$$

Here  $z$  is the random number, in a range of  $[0, 0.5]$ . In the task graph,  $\Delta x$  is node change number from current node distribution to new distribution, namely the distance between current division and new division. Because the node number and annealing temperature varies in value and range.  $\Delta x$  requires to be normalized. But  $\Delta x \in [0, N]$ , the upper limit of internal circulation complexity is  $o(K \cdot N)$  where  $N$  is the task number and  $K$  is the annealing number of each temperature. Obviously, the complexity of internal circulation calculation is only related to  $N$ . When  $N$  is larger, the calculation time will increase greatly.

Suppose the external cooling time of simulated annealing algorithm in the random disturbance mo-

del is  $M$ , then the complexity of algorithm using this disturbance model is  $o(M \cdot K \cdot N)$ . Under the extreme condition, the computation time of improved disturbance model is larger than that of random disturbance model. Therefore, this paper processes the  $\Delta x$  to decrease the computational complexity of internal circulation and make  $\Delta x$  still change with  $T$ , shown as follows.

$$\Delta x = \varepsilon \Delta x \quad (6)$$

$\varepsilon$  is small value, and the complexity of internal circulation after improvement is  $o(K)$ . The computational complexity decreases greatly.  $\Delta x$  is still the function of  $T$ . There is larger disturbance distance in higher temperature meaning searching in larger area. When the temperature is lower, the searching only continues in current division, which can accelerate the convergence speed.

### **Simulated annealing algorithm optimized by improved genetic algorithm**

Genetic algorithm is introduced to optimize the simulated annealing algorithm. The mutation operator of standard genetic algorithm is improved.

The crossover operation is main operation of genetic algorithm. Only continuous crossover operation can new individual be generated so as to obtain better individuals. In order to increase the searching area of population, this paper proposed the crossover operator based on mixed single point.

Suppose the sample is evaluated into the  $t$  generation, the two male parents  $X_a^t = [x_{a,1}^t, \dots, x_{a,k}^t, \dots, x_{a,n}^t]$  and  $X_b^t = [x_{b,1}^t, \dots, x_{b,k}^t, \dots, x_{b,n}^t]$  do crossover operation to get the  $t+1$  generation individual  $X_a^{t+1} = [x_{a,1}^{t+1}, \dots, x_{a,k}^{t+1}, \dots, x_{a,n}^{t+1}]$  and  $X_b^{t+1} = [x_{b,1}^{t+1}, \dots, x_{b,k}^{t+1}, \dots, x_{b,n}^{t+1}]$  with the crossover point  $x_{a,k}^t$  and  $x_{b,k}^t$ . Here, the crossover way is to calculate the  $k$  gene of individual  $X_a^t$  and  $X_b^t$  through equation (7) to generate two genes  $x_{a,s}^{t+1}$  and  $x_{b,s}^{t+1}$ . These two genes are put back to original space, then the genes from  $k+1$  to  $n$  of  $X_a^t$  and  $X_b^t$  are interchanged to generate two individuals finally. The calculation way of  $k$  gene is,

$$\begin{cases} x_{a,s}^{t+1} = \alpha x_{a,k}^t + (1-\alpha)x_{b,k}^t \\ x_{b,s}^{t+1} = (1-\alpha)x_{a,k}^t + \alpha x_{b,k}^t \end{cases} \quad (7)$$

$$\text{where } \alpha = \frac{\text{rank}(X_a^t)}{\text{rank}(X_a^t) + \text{rank}(X_b^t)},$$

$\text{rank}(X)$  meaning the rank of individual  $X$ . With evolution generation increasing,  $\alpha$  is more and more approaching to 0.5. In  $t$  generation, the main steps of mixed single point crossover are listed below.

(1) Selecting randomly two male parents from the population  $p1^t = [x_{a,1}^t, \dots, x_{a,k}^t, \dots, x_{a,n}^t]$  and  $p2^t = [x_{b,1}^t, \dots, x_{b,k}^t, \dots, x_{b,n}^t]$ , and choosing the  $k$  variable from them,  $k = \text{round}(\text{rand} \cdot (n-1)+1)$  where  $n$  is the

number of decision variables.

(2) If  $rand < p_c$ ,  $p_c$  the probability of crossover, then

$$\alpha = \frac{rank(p1)}{rank(p1) + rank(p2)}$$

and turn to step (3); otherwise,  $rand > p_c$ , turn to step (5).

$$(3) \begin{cases} ch1'(k) = \alpha x_{a,k}' + (1-\alpha)x_{b,k}' \\ ch2'(k) = (1-\alpha)x_{a,k}' + \alpha x_{b,k}' \end{cases}$$

is the transformation form of Equation (7).

(4) Generating two individuals of new generation

$$ch1'^{t+1} = [p1'(1:k-1), ch1'(k), p2'(k+1:n)] \quad (8)$$

$$ch2'^{t+1} = [p2'(1:k-1), ch2'(k), p1'(k+1:n)] \quad (9)$$

(5) Taking these two parent individuals as the offspring individuals, namely  $ch1'^{t+1} = p1'$  and  $ch2'^{t+1} = p2'$ .

Repeating the process above until the number of new generation individuals equals to the population number and ending the crossover operation. Then the advantages of improved genetic algorithm and simulated annealing are combined to form a new optimized IGA-SA algorithm.

There are four chromosomes of a group of individuals after crossover and mutation operation: two parent individuals and two offspring individuals. In the traditional genetic algorithm, parent individuals will be replaced by their offspring individuals. But in the improved simulated annealing algorithm, two chromosomes of next generation is selected from these four chromosomes with the criterion of fitness value. The chromosomes with higher fitness are more probable to be retained; the chromosomes with lower fitness are not always abandoned. The probability selection is done by simulated annealing algorithm based on local selection strategy. This probability is related to current control temperature  $T_i$  and other three parameters, namely  $f_{best}$ ,  $f_{worst}$  and  $f_i$ .  $f_{best}$  is the individual with best fitness in father parent individuals ;  $f_{worst}$  is the individual with worst fitness;  $f_i$  is the fitness of offspring fitness;  $T_i$  is the control temperature. In each temperature  $T_i$ , the fitness function value  $f_i(i=1,2)$  of test chromosome is compared with  $f_{worst}$ . If the following condition is met, the chromosome  $i$  is accepted.

$$\min[1, \exp(f_i - f_{worst}) / T_i] > r \quad (10)$$

$r$  is a random number between 0 and 1. If chromosome  $i$  is accepted, the best and worst chromosomes will be updated.

At the beginning, the mutation probability of GASA is given to a larger value and adjusted by SA. After every ten generations,  $p_m$  is updated to  $\alpha * p_m$  until  $p_m$  reaches a certain value. Here  $\alpha$  is the cooling rate of SA. Therefore, in the initial stage, if the cooling mechanism of SA is operated exactly, the

initialized high temperature can ensure the parent individuals being replaced by son individuals whatever the son individuals are more appropriate. More importantly, initially high mutation rate can improve the variety of population so as to avoid the premature problem of traditional genetic algorithm. In addition, at a later stage, the mutation probability and the temperature decreases, so the probability of replacing father generation with son generation greatly reduces.

This way can add the best individuals into the next generation. Because of the decrease of mutation probability, the probability of removing the useful individuals in the final generation is small.

The solving process of improved genetic algorithm and simulated annealing algorithm (IGA-SA) algorithm is written as follows.

(1) Initializing the population, setting the population number  $N$ , maximum evolution generation  $m$ , crossover probability  $P_c$ , mutation probability  $P_m$  and cooling control temperature  $T_i = T_0$ .

(2) Calculating the fitness of all the individuals in the population.

(3) Generating new generation individuals after selection, crossover and mutation operation.

(4) Calculating the fitness of individuals in the new population.

(5) Judging whether the population is converged, then jump to step (3), otherwise resetting the counter  $t = 0$  and cooling generation, and doing cooling operation on new generation individuals. According to Metropolis rule, current population individual is chosen as the new genetic population, if the counter is lower than the preset cooling generation, then turn to step (1); otherwise, the individual in this cooling operation is used to replace the individuals of worse fitness value.

(6) If the evolution generation reaches to the maximum, cooling process is activated following some way, and the evolution generation  $r = r + 1$  and jump to step (2); otherwise, the whole optimization process ends and outputs the optimal solution.

## **Gear weight design optimization model based on IGA-SA algorithm**

In the structural design of gear, people always wish to design a promising product with small volume, light weight and high performance. Therefore, this paper takes the light weight as the objective.

(1) Design variable

According to the ideal performance index of electromagnetic gear, iron core, flange section and gear teeth section must be passed by enough flux to ensure the electromagnetic gear with enough field density and satisfied magnetic moment. Under this condition,

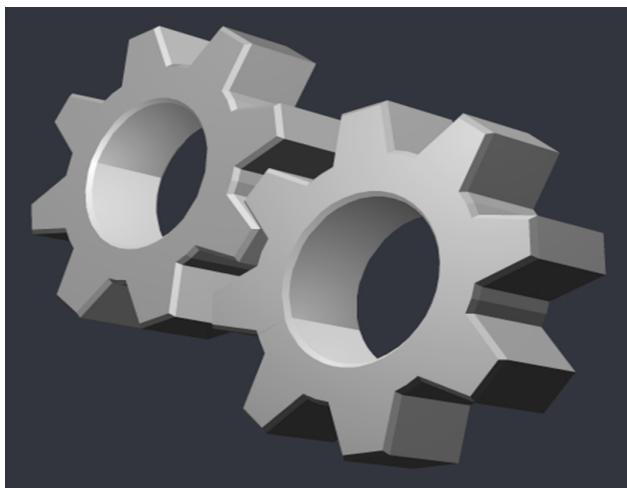
it is meaningful to minimize the electromagnetic gear volume. Due to the performance requirement, that gear shaft cannot conduct magnetic will not influence the magnetic performance of gear. The transmission torque is small so that a common gear shaft can meet the strength. The diameter of shaft  $D$  is a constant given by structure. The optimization variable of electromagnetic gear is determined by equation (11).

$$\begin{aligned} X &= [x_1, x_2, \dots, x_6]^T \\ &= [D_1, D_2, h, L, Z, A]^T \end{aligned} \quad (11)$$

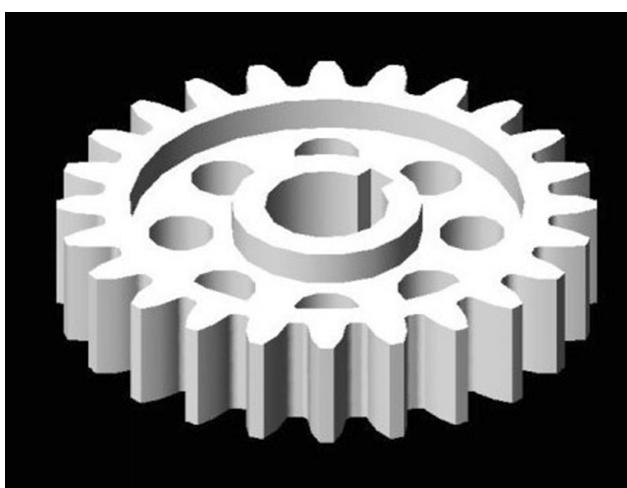
Here,  $D_1$  is flange outer diameter of pole teeth,  $D_2$  the outer diameter of conductive iron core,  $h$  teeth height,  $L$  effective length of pole teeth,  $Z$  number of pole pairs equal to 1/2 number of teeth, and  $A$  thickness of pole teeth flange.

### (2) Objective function

Taking the minimum volume of electromagnetic



**Figure 1.** Three-dimensional image of the sample 1 gear



**Figure 3.** Three-dimensional image of the sample 2 gear

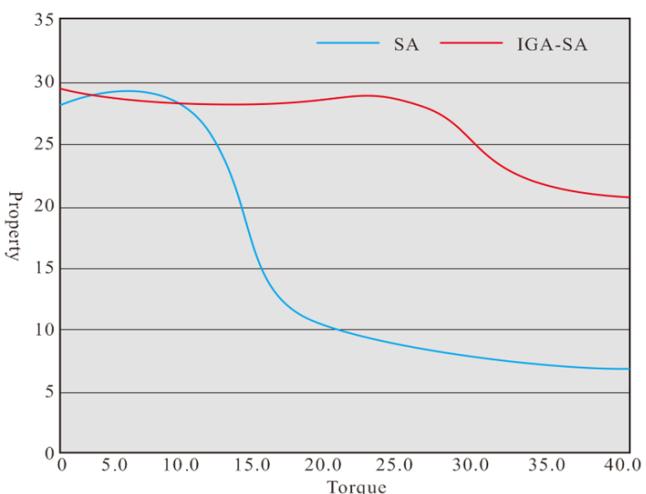
gear as the objective function, the volume sum of left pole teeth and conductive iron core is calculated. Once the structural parameters of left pole teeth is determined, the structural parameters of right pole teeth can be determined accordingly. Thus,

$$\begin{aligned} \min f(x) = & \pi[x_6(x_1 + 2x_3)^2 + x_2^2x_4 - (x_4 + x_6)D^2]/4 \\ & + \pi(180^\circ/x_5 - \theta)x_1x_3x_4x_5/180 \end{aligned} \quad (12)$$

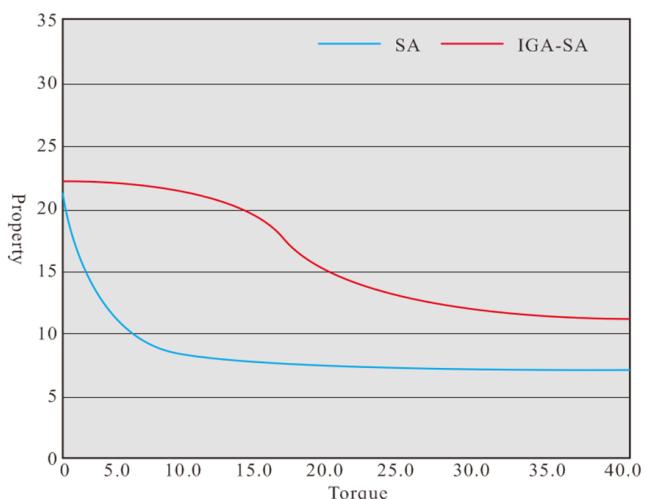
Here,  $D$  is the diameter of electromagnetic gear and  $\theta$  is central angle of pole teeth gap.

### Simulation experiment

To verify the effectiveness of the proposed algorithm, this paper takes two kinds of gears for example, and adopts standard annealing algorithm and the proposed IGA-SA algorithm to optimize the weight and test their working life. The comparison results are shown below.



**Figure 2.** The comparative optimization results of the life test



**Figure 4.** The comparative optimization results of the life test

From the results above, it is seen that the proposed improved genetic algorithm and simulated annealing algorithm has better result in gear weight optimization and higher working life compared with traditional simulated annealing algorithm.

### Conclusions

Gear optimization design plays an important role in the modern machine design. With the advancement in optimization techniques, intelligent optimization methods, like genetic algorithm and simulated annealing algorithm with high optimization performance, no special information and global searching ability, have achieved wide attention in various region and provide some methods for the gear optimization. In view of the defects of traditional simulated annealing algorithm, this paper proposes a gear weight optimization model based on simulated annealing algorithm optimized by disturbance and improved genetic operator. The simulation experiments show that the improved IGA-SA algorithm has better result in gear weight optimization and higher working life compared with traditional simulated annealing algorithm.

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