

## A Measurement Method of Geometric Parameters of Spatial Circle Based on Reference Points

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**Abstract**

For the binocular vision measurement, there are a lot of mismatching points because feature points of the spatial circle's edge are not obvious. The method of pasting reference points on the edge of spatial circle is proposed to seek the feature points. First of all, the pixel points of the spatial circle's edge are extracted and elliptic equation is fitted by the least square method. Secondly, the coordinates of reference points' centers are extracted, every two reference points are connected as one line, the intersections of the line and the spatial circle's edge are taken as the feature points, and the three-dimensional coordinates of spatial circles' edge points are obtained by binocular stereo measurement. Finally, the geometric parameters of spatial circle are fitted based on its geometrical properties. The measurement data based on this paper's method and coordinate measuring machine are compared, furthermore the error sources are analyzed. Experimental results show that the method is easy to implement and has lower cost and higher measurement accuracy.

**Key words:** REFERENCE POINTS, FEATURE POINTS, SPATIAL CIRCLE

**1. Introduction**

In the process of machining, it is very significant for parts assembly that the precise localization of workpiece circular holes center and radius extraction. General speaking, the geometrical parameters of circular holes can be achieved by three coordinate measuring machine(CMM) [1], however the workpieces can't be measured on the production site due to CMM location and larger volume.

With of non-contact, higher speed, flexibility, and higher automation advantages etc [2], visual detection technology is more and more widely applied in the industrial measurement field. The algorithm of hole center corresponding points based on ellipse fitting was proposed by the national key precision instrument laboratory in Tianjin university[3]. The spatial coordinates of the center of the circular holes were measured by this method, but the measurement error was large when there is a certain intersection angle between the spatial circle plane and the image plane. The measurement method of the spatial circle based on the line structured light was presented by reference [4]. Even though this method has higher measurement accuracy, the special light source and strict environmental condition are necessary. Meanwhile, a great amount of calculation for this method is needed due to computing every kind of images. Based on the parallax basis of the corresponding pixel points [5], the matching relationship of every point in left and right images is one of the most important issues for binocular stereo vision measurement. However, matching is the most difficult and critical step for the actual stereo images. Because of the influence of noise, shelter, illumination change and perspective distortion, there was some deviation for the same point in the left and right image. Although many constraints to reduce the mismatching of corresponding points have been established, effects were not effective for seeking feature points of the circular holes edge.

In this paper, a method is proposed by pasting reference points around circular holes to find the feature points and calculate the geometry parameters of the circular holes. This method has higher flexibility and higher precision, and avoids the above-mentioned shortcomings effectively.

**2. Circular hole edge detection and elliptic equation fitting**

Due to workpiece hole depth and illumination environment, there are shadow on the edge of the hole during photographing. Thereby measurement results for the radius of circular hole would be affected. Therefore, on the one hand, the workpiece should be lighted uniformly during the image acquisition, and the strategy is that a back light source is placed at the bottom of the workpiece. On the other hand, the image is pre-processed by the median filter to reduce most of the salt-and-pepper noise.

During the edge detection, the precise edge localization and noise interference restrain can't be satisfied simultaneously. Some edge detection operators are chosen to filter noise but to increase edge localization difficulty. In this paper, Sobel operator, Prewitt operator, Roberts operator, LoG operator and Canny operator are chosen to detect the edge of the workpiece respectively. The detected effects are shown as follows.

See from figure 1, relative to other operators, Canny operator reasonably balances the edge precise positioning and the noise interference restrain. In this paper the Canny edge detection operator is chosen to extract the edge of circular hole.

Because of the influence of the camera perspective projection distortion, the projection of the spatial circles are appeared as an ellipse in the image plane. If there is a certain angle between the spatial circle plane and the image plane, then there will be a deviation between the projection of the spatial circles center and fitted center of ellipse. The angle is bigger, the deviation is bigger.

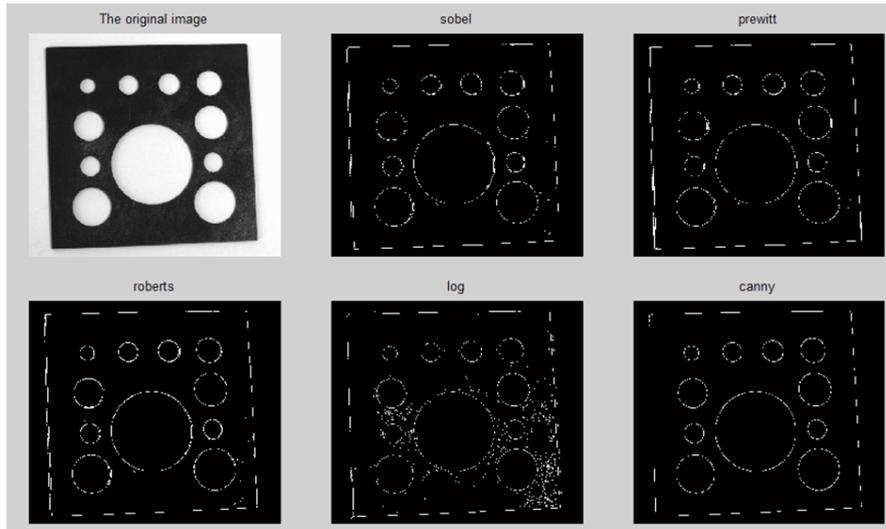


Figure 1. Extraction of ellipse Edge

The Least squares ellipse simulation is one of common methods. Assuming the general form of the ellipse is shown as in formula (1).

$$ax^2 + bxy + cy^2 + dx + ey + g = 0 \quad (1)$$

The sum of mean square error is shown in formula 2.

$$f(a, b, c, d, e, g) = \sum_{i=1}^n (ax_i^2 + bx_i y_i + cy_i^2 + dx_i + ey_i + g_i)^2 \quad (2)$$

In order to avoid zero solution, parameters should be given some restrictions. constraint condition is set as  $a + c = 1$ . In order to get the minimum value of formula (2), formula (3) will be obtained the partial derivatives for b, c, e, g respectively according to the extremum principle, and every derivatives value is setting to 0, that is:

$$\frac{\partial f}{\partial b} = \frac{\partial f}{\partial c} = \frac{\partial f}{\partial d} = \frac{\partial f}{\partial e} = \frac{\partial f}{\partial g} = 0 \quad (3)$$

A linear system of equations can be obtained, combined with the constraint condition  $a + c = 1$ , the coefficient of equations a, b, c, d, e and g are obtained.

### 3. Extraction of feature points for the edge of circular hole

For less texture information of workpiece surface, there is no any regularity for extracting the feature points, which are often useless. It is inevitable to be very difficult to match. Therefore, in this paper, method of sticking reference points is adopted to obtain feature points. Intersection is used to be a matching point between the reference points connected line and the edge of circular hole. In this way, the false matching problem will be avoided effectively in the stereo matching.

As shown in figure 2, the accurate positioning circular non-coding logo design is selected in this pa-

per. Reference points are stuck on around two circular holes of workpiece surface randomly. The number of reference points should not be too much, because every two reference points can be combined randomly. So when the number of reference points is n, the number of intersection will be  $n(n-1)/2$  between the reference points connecting line and the edge of circular hole. So the geometric parameters of fitted spatial circle have been able to meet the precision requirements. Considering there may be no intersection between line of the adjacent two points and ellipse edge when circular hole is smaller, 8 to 10 reference points are able to meet the requirements.

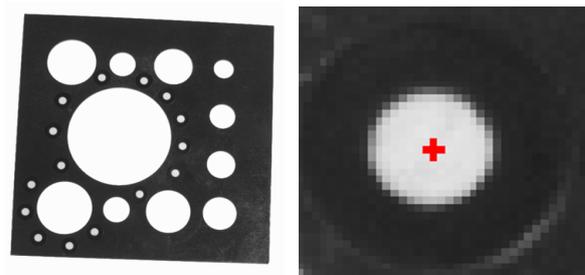


Figure 2. Extraction of center of reference points

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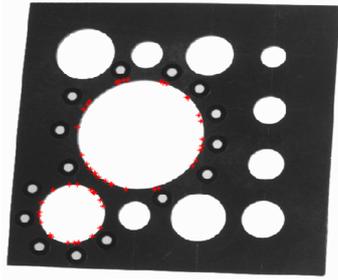


Figure 3. Extracted feature points of the edge of circular holes

#### 4. Fitting of Spatial circle

According to the calibrated camera inside and outside parameters, the pixel coordinates of the circular hole edge points are transformed into the spatial three-dimensional coordinates. Because all three-dimensional coordinates are on one spatial plane, the spatial points are on plane fitting. Set spatial arbitrary plane equation represented as equation (4).

$$ax + by + cz - 1 = 0 \quad (4)$$

Take the coordinates of n points into equation (4), so:

$$A \cdot C - W = 0 \quad (5)$$

Where  $A = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \\ z_1 & z_2 & \cdots & z_n \end{pmatrix}^T$ ,  $C = (a, b, c)^T$ ,

$$W = (1, 1, \dots, 1), \hat{C} = (A^T A)^{-1} A^T W$$

can be calculated by the least square method

As shown in figure 4, the coordinates of two points of the spatial circle are  $P_1(x_1, y_1, z_1), P_2(x_2, y_2, z_2)$ . A chord of the spatial circle is consisted by these two connecting points. The direction vector of this chord is expressed as  $\overline{P_1 P_2}(x_2 - x_1, y_2 - y_1, z_2 - z_1)$ . Suppose coordinates of the spatial circles' center is  $P_0(x_0, y_0, z_0)$ , and the midpoint of  $P_1$  and  $P_2$  is  $P_{12}$ , the direction vector of  $P_{12}$  and  $P_0$  is

$$\overline{P_0 P_{12}} \left( \frac{x_2 + x_1}{2} - x_0, \frac{y_2 + y_1}{2} - y_0, \frac{z_2 + z_1}{2} - z_0 \right)$$

Based on the geometric characteristics of spatial circle, two direction vectors are vertical. So  $\overline{P_1 P_2} \cdot \overline{P_0 P_{12}} = 0$ ; namely:.

$$(x_2 - x_1, y_2 - y_1, z_2 - z_1) \cdot \left( \frac{x_2 + x_1}{2} - x_0, \frac{y_2 + y_1}{2} - y_0, \frac{z_2 + z_1}{2} - z_0 \right) = 0$$

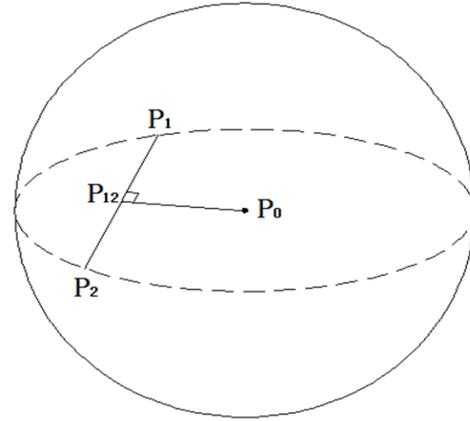


Figure 4. Structural drawing of the spatial circle

The above equation can be simplified as equation (6).

$$\Delta x_{12} \cdot x_0 + \Delta y_{12} \cdot y_0 + \Delta z_{12} \cdot z_0 - l_1 = 0 \quad (6)$$

Where  $\Delta x_{12} = x_2 - x_1, \Delta y_{12} = y_2 - y_1, \Delta z_{12} = z_2 - z_1, l_1 = (x_2^2 + y_2^2 + z_2^2 - x_1^2 - y_1^2 - z_1^2)$ .

n points of the spatial circle can be obtained n - 1 equation like equation (6), and error equation (7) will be gotten by the least square method.

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_{n-1} \end{pmatrix} = \begin{pmatrix} \Delta x_{12} & \Delta x_{23} & \cdots & \Delta x_{(n-1)n} \\ \Delta y_{12} & \Delta y_{23} & \cdots & \Delta y_{(n-1)n} \\ \Delta z_{12} & \Delta z_{23} & \cdots & \Delta z_{(n-1)n} \end{pmatrix} \cdot \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} - \begin{pmatrix} l_1 \\ l_2 \\ \vdots \\ l_{n-1} \end{pmatrix} \quad (7)$$

Equation (7) can be simplified as  $V = B \cdot X - L$ . Where P is unit matrix, and which adds a limitation for the equation (7): the center of the circle must be in the fitted spatial plane, namely  $C \cdot X - W = 0$ . Then calculate according to the indirect adjustment principle of attached conditions, deduction equation is:

$$\begin{pmatrix} B^T P B & C^T \\ C & 0 \end{pmatrix} \cdot \begin{pmatrix} X \\ K_s \end{pmatrix} - \begin{pmatrix} B^T P L \\ W \end{pmatrix} = 0 \quad (8)$$

Where  $K_s$  is relative vector of constraints condition.

Solution of the least square method is shown in equation (9).

$$\begin{pmatrix} X \\ K_s \end{pmatrix} = \begin{pmatrix} B^T P B & C^T \\ C & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} B^T P L \\ W \end{pmatrix} \quad (9)$$

By this method the center coordinate of spatial circle is calculated, and the distance of the edge points to the center of the circle is obtained.

$$r_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2}, \quad (i=1, 2, \dots, n) \quad (10)$$

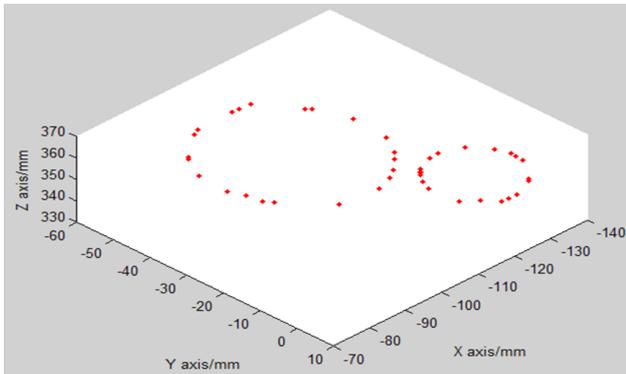
After system errors are excluded by Laiyte criterion, average value is calculated as the radius of spatial circle.

$$r = \sum_{i=1}^n \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} / n \quad (11)$$

**5. Experimental verification and error analysis**

Two panasonic anpvc1210 industrial cameras are used to collect the image. Camera resolution is 1200 \* 1600 pixels; lens is Japanese Computar which focal length is 8 mm. First of all, two cameras are calibrated. Secondly, two camera's internal and external parameters are work out.

Extract center coordinates of the reference points, set every combined two points to form several lines, and intersect with the edge of two circular hole on the workpiece. Combine the linear equations with two elliptic equations to get two solutions, which is the



**Figure 5.** Distribution of edge points for Spatial circular

See from figure 6, the measuring values fluctuate around 20 mm. Due to the influence of the measuring environment, material and angles, the choice of threshold is interfered in the edge detection. If there is a certain angle between the light ray and workpiece surface, shadow is left on the edge of the hole, which affects the precise localization of edge pixels points. So there would be deviation for fitting elliptic equations. Combined the linear equation, calculated feature points sometimes are not on the edge of holes.

**Table 1. The error analysis**

	This method/mm	CMM/mm	Absolute error/mm	Relative error/%
Radius of circle 1	20.155	20.013	0.142	0.709
Radius of circle 2	10.636	10.512	0.124	1.179
Distance of centers of circles	36.123	36.165	0.042	0.116

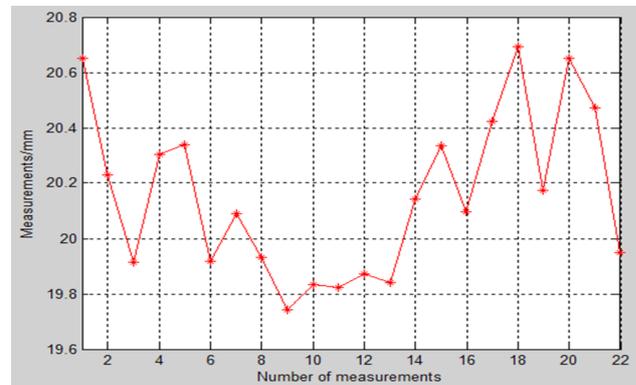
That can be seen from table 1, measurement error of distance of centers of circles is smaller than radius of circle, which is because that the radius measurement is related to the extraction precision of

pixel coordinates of the two matching points. Combine with the calibrated inside and outside parameters of two cameras, the spatial three-dimensional coordinates of the edge points can be calculated. 22 edge points are calculated on the bigger circle, and 20 edge points are calculated on the smaller circle. The spatial distribution of edge points is shown as figure 5.

These measuring values are fitted to the spatial circles. According to formula (9), two coordinates of center are obtained:

$$\begin{cases} (-102.3, -33.3, 356.1) \\ (-123.3, -4, 353.6) \end{cases}$$

After center coordinates are obtained, the distance from the edge points of the bigger circular hole to circle center are calculated according to the formula (10). The calculated data graph is shown as figure 6.



**Figure 6.** Distance of the edge points and center

As shown in table 1, radius and center distance for two groups circle holes are measured 10 times, because the reference points can be arbitrary combined when the different reference points and different combinations ways are selected. The average value of these measurement values is compared with the measured data of CMM. The spatial detection error of CMM is  $E \leq 3\mu m$ , so the measured data of CMM can be used to verify the accuracy of this method.

edge pixels points. It is hard to guarantee that extracted edge points for the circular holes are just on the edge. So the method of sticking reference points on around the hole is helpful to improve positioning

accuracy of center.

According to the calculated value, the fitted effect diagram of two circular holes is shown in figure 7.

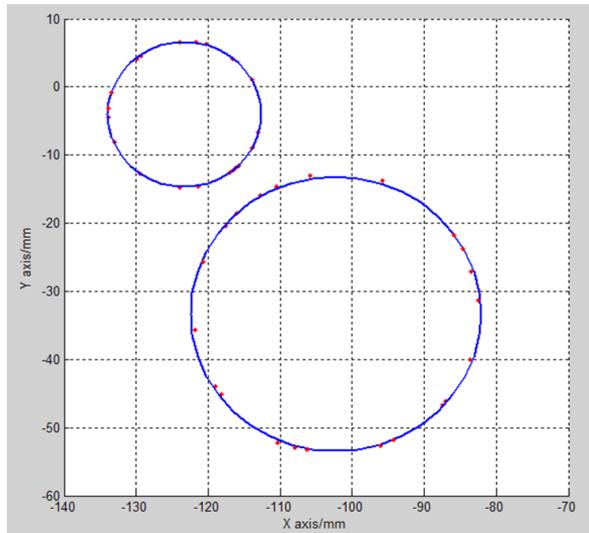


Figure 7. Fitting of spatial circle

## 6. Conclusions

The feature points can be obtained through the method of sticking reference points on the edge of circular holes. The problem of workpiece mismatching has been solved, which is caused by less texture information. Firstly, extract the pixels points of edge for the circular holes and execute ellipse fitting by the least squares method; secondly, take the intersection of the among reference points connecting lines and the circular hole edge as a matching point, according to the geometric characteristics of space circular, the geometric parameters of the spatial circle has been calculated; at least, the error has been analyzed. Experiments has proved that this method is easy to implement, with higher accuracy for spatial circular hole location. It provides reference significance for the circular hole in the process of machining assembly.

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