# Ant Colony Algorithm Based on Solving Continuous Space Optimization Problem

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Abstract
Ant colony algorithm which is mainly used to solve discrete combinatorial optimization problems through human biological evolution and limited its application in continuous space optimization problems. In the paper, the author puts forward a new algorithm--- Improved Continuous Ant Colony Optimization (ICACO), which is initialized through chaotic sequence. This Logistic map creates an ergodicity method of chaotic sequence and then updates pheromone from all the ant on the location. Through common test functions to the algorithm performance testing, computer simulation experimental results show that the algorithm optimization is of high efficiency and convergence speed can be increased significantly.

Key words: ANT COLONY ALGORITHM, CONTINUOUS SPACE OPTIMIZATION, IMPROVING CONTINUOUS ANT COLONY OPTIMIZATION, CHAOTIC SEQUENCE

## 1. Introduction

Ant colony algorithm is firstly putted forward by M. Dorigo in 1992 in his doctoral thesis. This system is a new optimization algorithm which belongs to volutionary computation [1][2]. The algorithm does not depend on the specific problem and has the global optimization ability, the essence of parallelism. So compared with genetic algorithm and simulated annealing method, it has the advantages of good reliability, fast resolution, and better computer implementation. Thus it has gained more and more attention among researchers in the field of system optimization, solving more complex combinatorial optimization problems, network routing problems, multi-robot task allocation problems, data mining, pattern formation, and divided problems. Ant colony algorithm of the global optimization performance is only applicable to the combination & discrete space optimization problems. Therefore it is very meaningful to explore and explore expand its application field, mining potential of the algorithm to solve the continuous space optimization problems.

In recent years, domestic and foreign scholars have got some research findings in these aspects, which mainly include the following aspects[3]-[18]:

(1) The continuous spatial discretization, namely to the solve continuous optimization problems into the discrete optimization problem, then solved by directly using ant colony optimization algorithm [3]-[5]. This method is only applicable to solving simple and less variable continuous optimization problems, but as for large-scale continuous optimization problems, this method shows poor adaptability .

(2) The combination of basic ant colony algorithm and evolutionary algorithm is able to get a hybrid ant colony optimization algorithm of solving the continuous space optimization problems[6]-[13]. This continuous ant colony optimization algorithm has a disadvantage of the ants in one area often repeatedly searching. Thereby it reduces the optimization efficiency of the algorithm.

(3) Breaking away from traditional ant colony algorithm model framework, it puts forward a new ant colony algorithm and uses it to solve the continuous space optimization problems[16]-[18].

So for continuous optimization problems, how to define the behavior of ants, the existence and distribution of the ants, pheromones are the problems that need to be solved. In view of the fact that the ant colony algorithm is only applicable to the optimization problems of discrete space limitations, the author puts forward a kind of ant colony algorithm in this paper to solve the continuous space optimization problem. The effectiveness of the proposed algorithm is tested through three typical test function and the comparison with other algorithms shows that it has the advantages of high precision and high efficiency.

## 2. ICACO' Structure

Assuming continuous space optimization problem is as follows:

$$\begin{cases}\min \quad f(x)\\ s.t. \quad x\in S\end{cases} \tag{1}$$

In the formula (1),$x=(x_1,x_2,\cdots,x_n)^T$ is N dimensional vector,S is the feasible region of N dimensional space, f (x) is defined to the real function in continuous space, everyone x of S is a feasible solution.

### 2.1. ICACO Algorithm Principle

Proposed ICACO algorithm in this paper, there are the following questions need to note:

(1)The form of the solution: ICACO algorithm proposed in this paper, retained the expression form of the continuous space optimization problem solution, every feasible solution with real number of the problem, without changing the original structure.

(2)The ant position initialization: The ant colony algorithm for function optimization when initialization mostly adopt the strategy of random distribution, so that the initial solutions to difficultly ensure better ergodicity. ICACO algorithm uses chaotic sequence to initialize, using Logistic map to produce a chaotic sequence, then mapped to the feasible region of solving problems. Initialization of Chaos sequence can improve the uniformity of the initial population in the search space, and be helpful to find the global optimal solution.

(3)Individual fitness value: Individual fitness value is defined as the objective function . If optimization problem is solving the maximum, achieve optimization problem is converted into solving minimum value firstly. Individual fitness value is used to guide the ant leaving the corresponding amount of pheromone in the process of optimization in the corresponding position.

(4)Initialization of pheromone: The size of initialization pheromone is related to the objective function of ants' location , the calculation formula as formula (5).

(5)Movement rules of ants: Reference to ideas of individual extremum and global extremum of particle swarm algorithm in the search mechanism update particle velocity and position of the guidance, in ICACO algorithm, the ant searching modes are divided to two: the global searching and the local searching. In a loop, the mobile process is known as global searching of failed to find the best solution then ants to find the optimal solution of the location. The process is referred to as local searching of finding the best solution of ants performing within the small neighborhood of itself to determine random searching.

### 2.2. ICACO Algorithm's Operating

(1) set $\min f(x)$ as the objective function, the variable$x=(x_1,x_2,\cdots,x_n)^T$'s value range is $[X_{ilower},X_{iupper}]$,$i=1,2,\cdots,n$,$Ant\_num$ shows ant' number. Using the formula(2) the Logistic map generating chaotic sequence:

$$L_i(t+1)=\mu L_i(t)\left(1-L_i(t)\right) \tag{2}$$

Formula (2), $\mu$ as a constant, $3.5\le\mu\le4$ , $L_i(t)\in(0,1)$, and that is mainly used to control system in a chaotic state.Then the chaotic sequence is mapped to the optimization problem of feasible domain, the mapping formula as follows:

$$X_i(t)=X_{ilower}+L_i(t)\left(X_{iupper}(t)-X_{ilower}(t)\right) \tag{3}$$

Formula (3), $X_i$ is the initial distribution of the ants $i$.The point from distribution of the initial is every ant searching initial point. $X_{ilower}$ is Minimum values for variables, $X_{iupper}$ is maximum values for variables. To the problem of continuous variables of the feasible region, set ant colony size $Ant\_num$ , per unit length of each variable quantum space as follows:

$$Len(i)=\frac{X_{iupper}-X_{ilower}}{Ant\_num},(i=1,2,\cdots,n) \tag{4}$$

(2) Initial distribution of pheromone: According to every ants' initial position, providing an initial pheromone of the location of ants, distribution is as follows:

$$\tau_0(i)=ka^{-f(x_0)} \tag{5}$$

Formula (5), $a$ and $k$ are the constant of greater than zero. If the objective function is solving minimum value,$a>1$,generally $a=e$.But if the objective function is solving maximum value, $0<a<1$. Usually according to the actual problem to select the size from a and k. If the objective function is solving minimum value, $f(X_i)$ is also smaller, by the formula (5), more and more the pheromone from the ants in the location.

(3) The ant's global searching: In ICACO algorithm, after all ants had completed a searching cycle, will update the pheromone according to the corresponding rules, to guide the ants executing the next searching cycle. Failed to find the best solution in the last cycle of ants to find the optimal solution of ants moving process is named the global searching . The purpose of the global searching is expanding the searching space, in order to get a better solution. After

the completion of a cycle from ants, there will be an ant in the loop to finding an optimal solution X (Best). To the ant's position other ants look as a guide for the transfer of the next cycle. Other ants to the optimal position of ants transfer probability is shown in the following formula:

$$p(i.Best) = \frac{e^{\tau_{i,Best}}}{e^{\tau_{Best}}} \qquad (6)$$

Formula(6)

$$\tau_{i.Best} = \tau(Best) - \tau(i) \qquad (7)$$

Formula (7), $\tau(i)$ is the size of the pheromone when ants' place is i, but does not include the best position, $i \neq Best$.

When the ant moves toward the place from where the powerful pheromone, it may be on its way finding a better solution, thus define the mobile way that i move to the best ant:

$$X_i = \begin{cases} X_i + \lambda(X_{Best} - X_i) & p(i.Best) < p_0 \\ X_i + rand(-1,1)len(i) & otherwise \end{cases} \qquad (8)$$

Formulas (8) , $p_0$ and $\lambda$ are constant, usually $0 < p_0 < 1, 0 < \lambda < 1$.

(4) The ant's local searching: The local searching is to point to random searching in the recent cycle in the process of small neighborhood of the best ant's location scope , so as to find the better than the current solution. Set random searching new location $X(tempBest)$, if the new position of the objective function value is better than the original optimal value, replace it, else keep the optimal value of the original. During the process of random searching , searching step length w would decreases gradually with the increase of cycling times. It can be find a better solution within the more accurate scope .The rules of the local searching is as follows:

$$X(Best) = \begin{cases} X(tempBest) & \Delta\tau(tempBest) > \Delta\tau(Best) \\ X(Best) & otherwise \end{cases} \qquad (9)$$

Formular(9),

$$X(tempBest) = \begin{cases} X(Best) + wstep & rand(-1,1) < 0.5 \\ X(Best) - wstep & otherwise \end{cases} \qquad (10)$$

$$\Delta\tau(i) = k \cdot a^{-f(x_i)} \qquad (11)$$

$w$ updating the following formula:

$$w = w_{max} - (w_{max} - w_{min}) \frac{iter}{G_{max}} \qquad (12)$$

Formula (12), $w_{max}$ and $w_{min}$ are constant value, generally according to experience, $0.2 < w_{min} < 0.8$, $1 < w_{max} < 1.4$, $iter$ is the current cycle times, $G_{max}$ is the maximum cycle times.

(5)The Pheromone updating rule: after the global searching and the local searching, update the amount of pheromone that all the ants in the location . Update the rules as follows:

$$\tau(i) = \rho\tau(i) + \Delta\tau(i) \qquad (13)$$

Formula (13), $\rho$ is the coefficient of pheromone residues. Generally $0 < \rho < 1$, $\Delta\tau(i)$ is calculated according to the formula (11).

### 2.3. ICACO algorithm Implementation steps

The paper Proposed the ICACO algorithm that simulated ants foraging the principle in the biological nature. When an ant was searching for solving problem in the feasible region scope, after completion an ant would find the optimal solution in each loop (food source of ants foraging). After the ant pheromone was exchanged, other ants were moving in the direction of the ant, ants may find a better solution (food source) during the transferring process . Ants of find the optimal solution in the circulation do random searching near the original location, in order to find a better solution. ICACO algorithm implementation steps as follows:

Step 1: Initialization parameters, according to the specific optimization problem to determine the maximum of the cycle number and the number of ant colony ,other parameters initial value in the algorithm.

Step 2: According to the scope of variables in optimization problems, the formula (2), formula (3) to the initial distribution, the formula (5) to the corresponding initial pheromone of each position.

Step 3: Calculated value of the objective function value by the pheromone of the ants' location, calculated $X(Best)$ the best ant after a cycle that the optimal value of objective function from the ant location corresponding .By the formula (11) ,it has the most pheromone in the location.

Step 4: According to the formula (6) (7) (8) ,the ants do global searching, then update the location of ants.

Step 5: According to the formula (9) (10) (11)(12) $X(Best)$ do the local searching, update the best ant's position according to the rules.

Step 6: When all the ants completed the global searching and the local searching, according to the formula (13) updated pheromone.

Step 7: Transferred to step 3, until meet the termination conditions, namely the maximum cycle times.

### 3. ICACO Algorithm Testing and Performance Analysis

In order to verify the validity of ICACO algorithm, select test functions in the table 1 by Matlab software simulation test. Parameter is set in the ICACO algorithm: the number of ants Ant_num = 100, the largest number of iterations Gmax = 100.

**Table1.** Test Functions

| **Symbol** | **Name** | **Function(F)** | **Interval of $x_i$** |
|---|---|---|---|
| $f_1$ | Griewangks | $f(X) = 1 + \sum_{i=1}^{n}\left(\frac{x_i^2}{4000} - \prod_{i=1}^{n}\left(\cos\frac{x_i}{\sqrt{i}}\right)\right)$ | $[-100,100]$ |
| $f_2$ | Schaffer | $f(X) = 0.5 + \frac{\sin\sqrt[2]{x_1^2 + x_2^2} - 0.5}{[1 + 0.001(x_1^2 + x_2^2)]^2}$ | $[-100,100]$ |
| $f_3$ | Sphere Model | $f(X) = \sum_{i=1}^{n} x_i^2$ | $[-5,5]$ |
| $f_4$ | Martin&Gaddy | $f(X) = (x_1 - x_2)^2 + (\frac{x_1 + x_2 - 10}{3})^2$ | $[-20,20]$ |
| $f_5$ | Rastrigin | $f(X) = 10 \cdot n + \sum_{i=1}^{n}(x_i^2 - 10\cos(2\pi x_i))$ | $[-5,5]$ |
| $f_6$ | $B_2$ | $f(X) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) - 0.4\cos(4\pi x_2) + 0.7$ | $[-100,100]$ |
| $f_7$ |  | $f(X) = (x_1^2 + x_2^2)^{0.25}(1 + \sin^2 50(x_1^2 + x_1^2)^{0.1})$ | $[-100,100]$ |

In order to further verify the validity of the algorithm in this paper, that puts forward the algorithm with the document [19] NFCACO algorithm for each test function independently tested 10 experimental . comparison, comparing the results are shown in table 2. Err is for solving error. Err, Min, Max are respectively the error average, minimum, maximum .Times is number of iterations of finding the optimal solution

**Table2**. Algorithm comparison

| **F** | **ICACO** |  |  |  | **NFCACO** |  |  |  |
|---|---|---|---|---|---|---|---|---|
|  | **Err** |  |  | **Times** | **Err** |  |  | **Times** |
|  | **Eve** | **Min** | **Max** |  | **Eve** | **Min** | **Max** |  |
| $f_1$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| $f_2$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| $f_3$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| $f_4$ | 3.254e-08 | 1.485e-07 | 0 | 23 | 7.234e-07 | 7.27le-08 | 9.967e-07 | 38 |
| $f_5$ | 0 | 0 | 0 | 1 | 8.395e-08 | 0 | 8.395e-07 | 1 |
| $f_6$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| $f_7$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |

The precision from ICACO algorithm and the document [111] are all high, but the ICACO algorithm performance is better than the algorithm in the document [19], the optimization speed is faster. The first generation most can find the optimal solution of function. Some the main reasons from good performance of ICACO algorithm: use of the ergodicity of chaos characteristics when use chaos initialization of the initial solution, increased the initial population in search space distribution uniformity. The global searching and local searching methods not only ensure the searching accuracy, but also accelerate the searching speed.

## 4. Conclusions

In the study of continuous space optimization problems, the author proposed in this paper ICACO algorithm according to latest research results of real ants, using chaotic sequence to initialize, using Logistic map to create a ergodicity method of chaotic sequence, the global searching and local searching, updating all the ants pheromone on the location. This strategy reduces the searching space and time greatly and the optimal speed and precision are improved significantly. The research findings of the simulation experiment shows that the convergence speed of algorithm increases significantly. Hence the high precision of optimization and the optimization are suitable for continuous space optimization problems.

Now, the research of solving the continuous space optimization problems does not get rid of ideas from a discrete domain application framework , and does not for convergence from continuous space optimization algorithm theory analysis. Therefore, further studies are needed in the area. ICACO algorithm proposed in this paper, the parameter setting needs certain experience, so the algorithm parameter setting need further research.

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