HOC-Based Generalized Cross-correlation Radar Signal Detection Algorithm

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Abstract
The traditional signal detection algorithms analyze by adopting the second-order statistical magnitudes, such as correlation function and power spectrum, as the mathematical tool. They are extremely sensitive to the additive noise and can only be adapted to the signal detection environment of the additive white Gaussian noise (AWGN) and the high signal to noise ratio (SNR). However, the high-order cumulant (HOC) has sound Gaussian noise immunity, which can theoretically get rid of the influence of both AWGN and additive colored Gaussian noise (ACGN) to reach low SNR signal detection. This paper puts forward a HOC-based generalized cross-correlation radar signal detection algorithm, and conducts the Monte Carlo simulation. From the curve of receiver operating characteristic (ROC) and the curve of the detection probability to SNR, it can be seen that the algorithm's detection performance, (i) for the AWGN environment, is no inferior to that of the traditional correlation detection algorithm at high SNR and is superior to that of the traditional correlation detection algorithms at low SNR; (ii) for the ACGN environment, the detection performance of the algorithm put forward in this paper is much better than that of the traditional correlation detection algorithms.
Key words: HIGH-ORDER CUMULANT, RADAR SIGNAL DETECTION

1. Introduction

The signal detection under the noise environment is a basic problem of signal detection theories. The traditional binary hypothesis detection deduces based on the principle of the maximum likelihood ratio. The difference of various rules exists only in the difference of the threshold. It has been proved that, under the AWGN environment, the optimal detection function deduced by the maximum likelihood ratio is the correlation function [1]. However, correlation operations are only optimal in the AWGN, and have a strong reliance on SNR. Therefore, it is an issue of great concern about how to obtain the optimal detection performance at low SNR and under the ACGN environment. Especially for a band-limited radar receiver, the signal detection problem under ACWN is difficult to avoid.

The high-order statistics methods have achieved wide attention in the signal detection applications [6]-[10] for good Gaussian signal immunity [2]-[5]. D. Kletter and H. Messer employed the bi-spectrum symmetry and the generalized likelihood ratio to put forward a two-dimensional signal detection algorithm based on the bi-spectrum estimation in [8,9], and conduct theoretical deduction of the detection performance. When SNR is low, its performance is superior to that of the traditional second-order spectrum signal detection. However, it is hard for the algorithm to be applied to the current coherent radar system. One reason is that the huge computational comple- xity can hardly ensure the real-time signal detection. The second reason is that it lacks the consideration for the coherent treatment. Kai M. Hock put forth a signal detection algorithm based on the bi-spectrum and tri-spectrum slices in [10], thus can better detect signals at low SNR and the computational complexity can be greatly reduced. However, the signal detection algorithm based on the spectrum estimation calls for the selection of signals with good spectrum properties and can hardly be adapted to the coherent radar system.

This paper puts forward an HOC-based radar signal detection algorithm, which has good Gaussian noise inhibiting ability theoretically. On the one hand, the computational complexity can be reduced by a large amount by replacing the multi-dimensional operation with the cumulant slices. On the other hand, it is similar to the traditional cross-correlation operation in form, thus it can be called generalized cross-correlation operation, and can be easily implanted to the current coherent radar system.

2. Traditional detection algorithms

Currently, most radar systems are coherent. The traditional coherent radar detection system is shown in Figure 1. Correlation operation in (1) is at the core.

$$R_{sr} = E\{s(t+\tau)r(t)\} \qquad (1)$$

Where, $s(t)$ stands for the local signals and $r(t)$ for the echo signals.

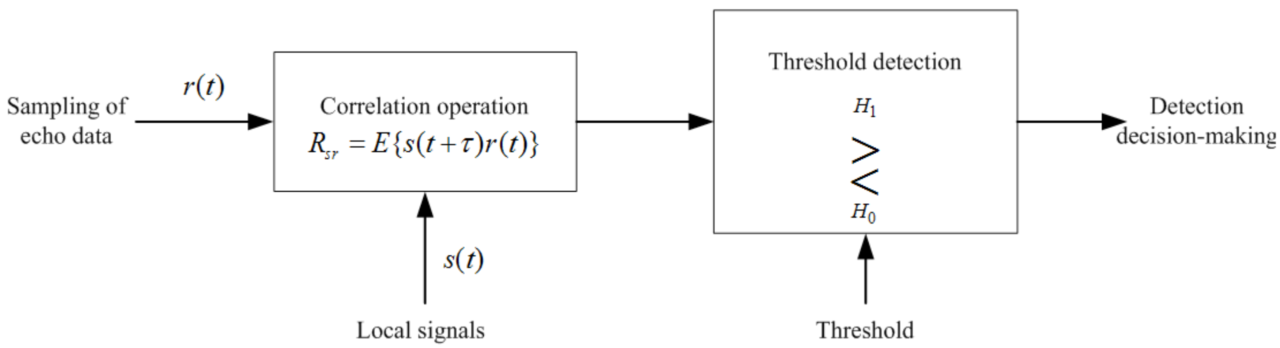


Figure 1. Traditional coherent detection block

For the simple binary hypothesis test [1], is assumed to be with signals; while H_0 to be without signals.

$$\begin{cases} H_1 : r(t) = s(t) + w(t) \\ H_0 : r(t) = w(t) \end{cases} \qquad (2)$$

Where, $w(t)$ stands for the noise signal. For the convenience of mathematical analysis, $w(t)$ is usually assumed to be the zero-mean white Gaussian noise.

The traditional radar signal detection process is shown in Figure 2, where $R(\bullet)$ stands for the correlation operation.

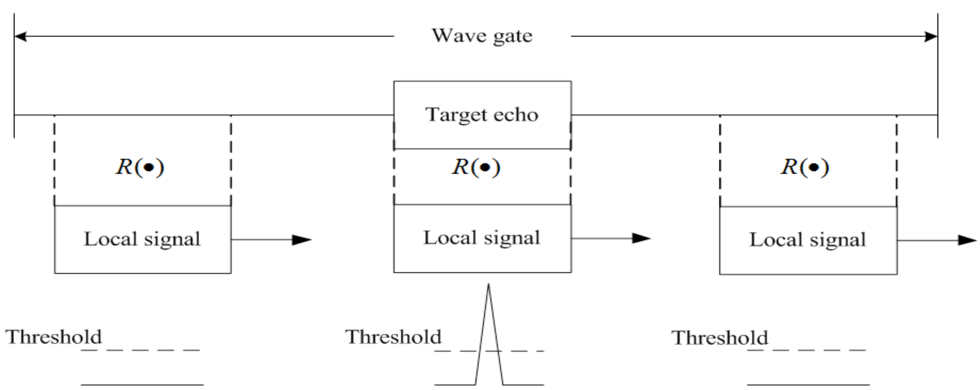


Figure 2. Traditional cross-correlation detection

Within the detection wave gate, the local signals and the echo signals undergo cross-correlation operation continuously. When the local signals are fully aligned the target echoes, the cross-correlation operation reaches the maximum value (when $\tau = 0$); when it exceeds the detection threshold, H_1 is proved to be correct.

3. HOC-based signal detection algorithm

3.1. Introduction of HOC

Define the first characteristic function as below:

$$\Phi(\omega) \overset{\text{def}}{=} E\{e^{j\omega x}\} = \int_{-\infty}^{+\infty} f(x)e^{j\omega x}dx \quad (3)$$

The natural logarithm of the first characteristic function is called the second characteristic function, which is expressed below:

$$\Psi(\omega) \overset{\text{def}}{=} \ln \Phi(\omega) \quad (4)$$

Define the k-order cumulant of the random variable, x, as below:

$$c_{kx} = (-j)^k \left. \frac{d^k \ln \Phi(\omega)}{d\omega^k} \right|_{\omega=0} = (-j)^k \Psi^{(k)}(0) \quad (5)$$

Proceeding from the above definition, the high-order (no lower than third-order) cumulant of the zero-mean Gaussian random variable, whose variance is σ^2, is identically equal to 0.

The united cumulant of multiple random variables is:

$$\text{cum}(x_1,...,x_k) \overset{\text{def}}{=} (-j)^k \left. \frac{\partial^k \ln \Phi(\omega_1,...,\omega_k)}{\partial\omega_1...\partial\omega_k} \right|_{\omega_1=...=\omega_k=0} \quad (6)$$

Where, $\Phi(\omega_1,...,\omega_k)$ is the first united characteristic function, which is defined as:

$$\Phi(\omega_1,...,\omega_k) \overset{\text{def}}{=} E\{e^{j(\omega_1 x_1+...+\omega_k x_k)}\} = \int_{-\infty}^{+\infty}...\int_{-\infty}^{+\infty} f(x_1,...,x_k)e^{j(\omega_1 x_1+...+\omega_k x_k)}dx_1...dx_k \quad (7)$$

The HOC of the continuous and stationary random signal, $x(t)$, can be defined as:

$$c_{kx}(\tau_1,...,\tau_{k-1}) = \text{cum}[x(t), x(t+\tau_1),...,x(t+\tau_{k-1})] \quad (8)$$

From (6), it can be seen that it is difficult to work out cumulants based on the definition. Thus, the matrix-cumulant (M-C) equation in (9) can be employed to calculate the cumulant.

$$c_x(I) = \sum_{\cup_{p=1}^{q} I_p = I} (-1)^{q-1}(q-1)! \prod_{p=1}^{q} m_x(I_p) \quad (9)$$

Where, I stands for the symbol set, $\{x_1,...,x_k\}$, of the random variable set, $I = \{1,2,...,k\}$; I_p stands for the disconnected and not empty set of I [11].

Based on the M-C equation, the calculation equation for the third-order cumulant can be obtained:

$$c_{3x}(\tau_1,\tau_2) = E\{x(t)x(t+\tau_1)x(t+\tau_2)\} - \mu_x R_x(\tau_2-\tau_1) - \mu_x R_x(\tau_2) - \mu_x R_x(\tau_1) + 2\mu_x^3 \quad (10)$$

Where, the correlation function, R_x, is the autocorrelation function; and μ_x stands for the mean value. Specially, when $\mu_x = 0$, (10) can be simplified into:

$$c_{3x}(\tau_1,\tau_2) = E\{x(t)x(t+\tau_1)x(t+\tau_2)\} \quad (11)$$

In the practical applications, in order to obtain the consistent estimation of k-order cumulant from the sample, $x(t)$ is 2k-order absolutely integrable, that is,

$$\sum_{\tau_1=-\infty}^{+\infty} \cdots \sum_{\tau_{m-1}=-\infty}^{+\infty} |c_{mx}(\tau_1,...,\tau_{m-1})| < \infty \quad (12)$$

Where, $m = 1,...,2k$. The HOC can be estimated based on the sample data. See (13) and (14).

Third-order:

$$\hat{c}_{3x}(\tau_1,\tau_2) = \frac{1}{N}\sum_{n=1}^{N} x(n)x(n+\tau_1)x(n+\tau_2) \quad (13)$$

Fourth-order:

$$\hat{c}_{4x}(\tau_1,\tau_2,\tau_3) = m_{4x}(\tau_1,\tau_2,\tau_3) - \hat{R}_x(\tau_1)R_x(\tau_3-\tau_2) - R_x(\tau_2)R_x(\tau_3-\tau_1) - \hat{R}_x(\tau_3)R_x(\tau_2-\tau_1) \quad (14)$$

Where,

$$\hat{R}_x(\tau) = \frac{1}{N}\sum_{n=1}^{N} x(n)x(n+\tau)$$

$$\hat{m}_{4x}(\tau_1,\tau_2,\tau_3) = \frac{1}{N}\sum_{n=1}^{N} x(n)x(n+\tau_1)x(n+\tau_2)x(n+\tau_3)$$

3.2. The HOC-based generalized cross-correlation radar signal detection algorithm

In order to make use of Gaussian immunity of HOC without losing the coherent and IQ dual-channel signal processing of radar system, the situation of the complex signal should be taken into consideration. The generalized cross-correlation function based on HOC is shown below:

$$C_{rrrs}(\tau_1,\tau_2,\tau_3) = E\{r(t)r(t+\tau_1)r^*(t+\tau_2)s^*(t+\tau_3)\} \quad (15)$$

Where, $r(t)$ stands for the received signal. When the received signal contains certain target, $r(t) = s(t) + w(t)$, where $s(t)$ stands for the zero-mean target signal; when the received signal has no target, $r(t) = w(t)$, which stands for the zero-mean Gaussian noise, either white or colored noise. Besides, the signal and the noise is independent from each other. * stands for the conjugate operation.

In addition, considering the problem of computational complexity, the slice function of (15) is introduced to transfer multi-dimensional operation into one-dimensional operation, which is defined as:

$$C_{rrrs}(\tau,\tau,0)=\mathrm{E}\{r(t)r(t+\tau)r^*(t+\tau)s^*(t)\} \quad (16)$$

When, there is certain target contains in received signal, $r(t)=s(t)+w(t)$, then

$$\begin{aligned}&C_{rrrs}(\tau,\tau,0)\\&=\mathrm{E}\{r(t)r(t+\tau)r^*(t+\tau)s^*(t)\}\\&=\mathrm{E}\{s(t)s(t+\tau)s^*(t+\tau)s^*(t)\}+\mathrm{E}\{s(t)s(t+\tau)w^*(t+\tau)s^*(t)\}\\&+\mathrm{E}\{s(t)s^*(t)s^*(t+\tau)w(t+\tau)\}+\mathrm{E}\{s(t)s^*(t)w(t)w^*(t+\tau)\}\\&+\mathrm{E}\{s(t+\tau)s^*(t+\tau)s^*(t)w(t)\}+\mathrm{E}\{s(t+\tau)s^*(t)w(t)w^*(t+\tau)\}\\&+\mathrm{E}\{s^*(t)s^*(t+\tau)w(t)w(t+\tau)\}+\mathrm{E}\{s^*(t)w(t)w(t+\tau)w^*(t+\tau)\}\end{aligned}$$

Based on the mutual independence of the target signal and the noise, and that $\mathrm{E}\{w(t)\}=0$, then

$$\begin{aligned}&C_{rrrs}(\tau,\tau,0)\\&=\mathrm{E}\{s(t)s^*(t)s(t+\tau)s^*(t+\tau)\}+\mathrm{E}\{s(t)s^*(t)\}\mathrm{E}\{w(t)w^*(t+\tau)\}\\&+\mathrm{E}\{s(t+\tau)s^*(t)\}\mathrm{E}\{w(t)w^*(t+\tau)\}+\mathrm{E}\{s^*(t)s^*(t+\tau)\}\mathrm{E}\{w(t)w(t+\tau)\}\\&+\mathrm{E}\{s^*(t)\}\mathrm{E}\{w(t)w(t+\tau)w^*(t+\tau)\}\\&=\mathrm{E}\{s(t)s^*(t)s(t+\tau)s^*(t+\tau)\}+R_{ss}(0)R_{ww}(0)+R_{ss}(\tau)R_{ww}(\tau)\\&+R_{ss}(\tau)R_{w^*w^*}(\tau)+R_{s^*s}(\tau)R_{ww^*}(\tau)+\mathrm{E}\{s^*(t)\}c_{3w}(\tau,\tau)\end{aligned}$$

Where, $c_{3w}(\tau,\tau)$ stands for the third-order cumulant of the noise signal. Then, $c_{3w}(\tau,\tau)\equiv 0$, and:

$$\begin{aligned}C_{rrrs}(\tau,\tau,0)&=\mathrm{E}\{r(t)r(t+\tau)r^*(t+\tau)s^*(t)\}\\&+R_{ss}(0)R_{ww}(0)+R_{ss}(\tau)R_{ww}(\tau)+R_{ss}(\tau)R_{w^*w^*}(\tau)+R_{s^*s}(\tau)R_{ww^*}(\tau)\end{aligned} \quad (17)$$

Similarly, when there is no signal, then:

$$C_{rrrs}(\tau,\tau,0)=\mathrm{E}\{w(t)w(t+\tau)w^*(t+\tau)s^*(t)\}=\mathrm{E}\{s^*(t)\}c_{3w}(\tau,\tau)\equiv 0 \quad (18)$$

Combine (17) and (18). It can be deduced that, during generalized cross-correlation detection, the judgment results of the binary simple hypothesis test are shown below:

$$\begin{cases}\mathrm{H}_1\text{:}\ C_{rrrs}(\tau,\tau,0)=\mathrm{E}\{s(t)s(t+\tau)s^*(t+\tau)s^*(t)\}+R_{ss}(0)R_{ww}(0)+R_{ss}(\tau)R_{ww}(\tau)\\ \qquad\qquad +R_{ss}(\tau)R_{w^*w^*}(\tau)+R_{s^*s}(\tau)R_{ww^*}(\tau)\\ \mathrm{H}_0\text{:}\ C_{rrrs}(\tau,\tau,0)\equiv 0\end{cases} \quad (19)$$

In summary, the generalized cross-correlation algorithm has following characteristics:

(a) Theoretically, it can totally eliminate the influence of the Gaussian noise;

(b) The energy of noise, in some extent, can be used to improve the detection performance at low SNR.

In consideration of the practical application, the model of HOC detection branch in parallel with the traditional correlation detection branch is designed as Figure 3.

From Figure 3, it can be seen that the detection model can be divided into three parts:

(a) Traditional correlation detection branch: The traditional correlation detection belongs to the optimal detection at high SNR and under the environment of AWGN;

(b) HOC-based detection branch: When SNR is low or under the environment of ACWN, HOC-based detection algorithm is superior in working out $C_{rrrs}(\tau,\tau,0)$;

(c) Comprehensive decision-making model: It mainly judges the detection results of the two-channel signals. Refer to Table 1 for the comprehensive decision-making logic.

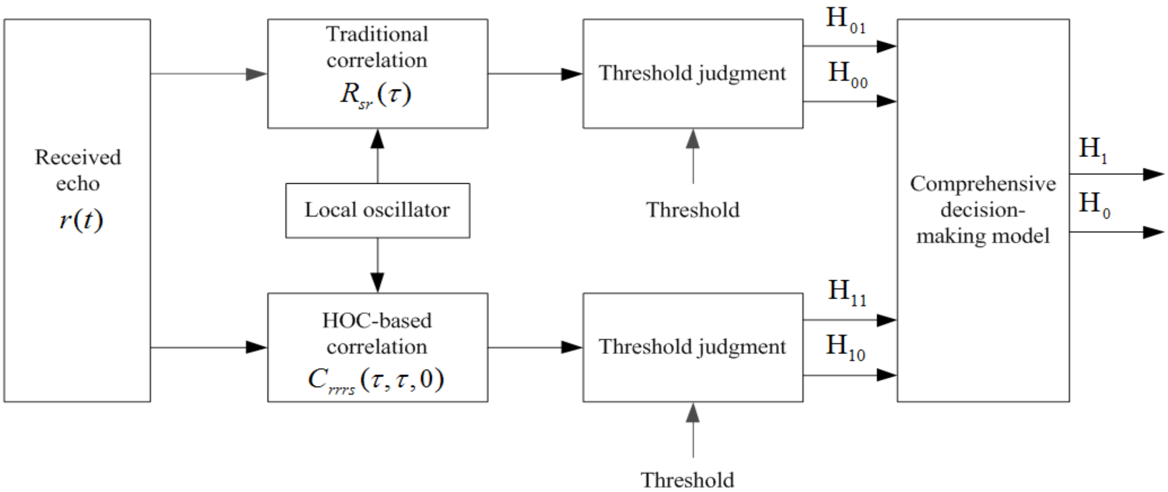


Figure 3. Signal detection using combined traditional/HOC-based analysis

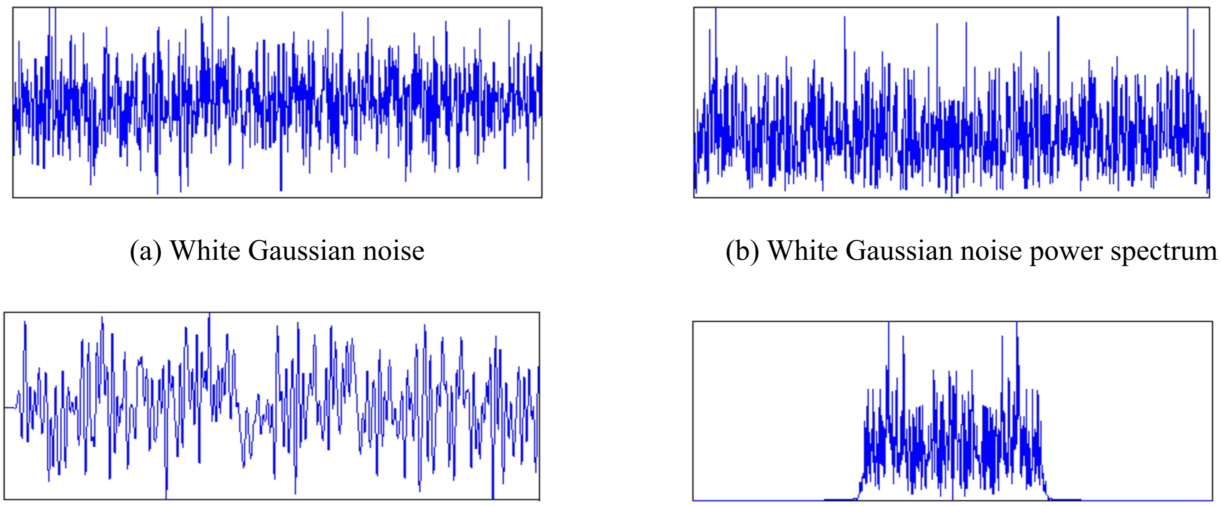
Table 1. Comprehensive decision-making logic

Situation	Traditional detection	HOC-based detection	Final results
1	H_{00}	H_{00}	H_0
2	H_{01}	H_{10}	H_1
3	H_{00}	H_{11}	H_1
4	H_{01}	H_{11}	H_1

4. Algorithm simulation and performance analysis

4.1. Simulated environment

The linear frequency modulation (LFM) signal is a commonly-used signal of the current radar system. It has sound pulse compression characteristics. Therefore, this paper employs the LFM pulse signal as the transmitted signal for simulation. The performance of the HOC-based detection algorithm put forward in this paper is compared with the traditional correlation detection algorithm through the Monte Carlo experiment, which carried out both the additive white Gaussian noise(AWGN) and the additive colored Gaussian noise(ACGN). The noise and its power spectrum is shown in Figure 4.



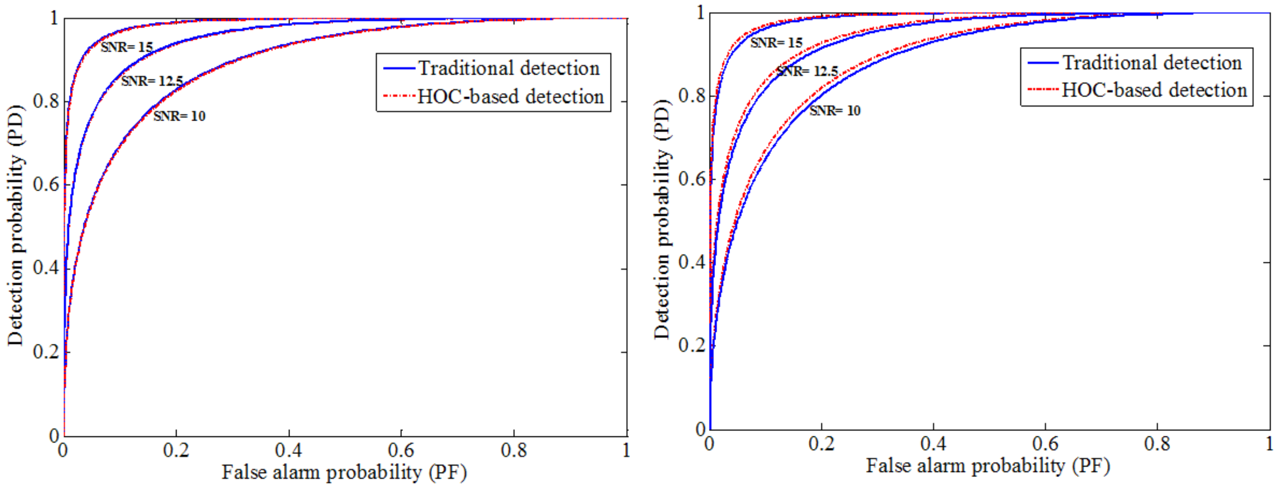
(a) White Gaussian noise (b) White Gaussian noise power spectrum

(c) Colored Gaussian noise (d) Colored Gaussian noise power spectrum

Figure 4. Gaussian noise and its power spectrum

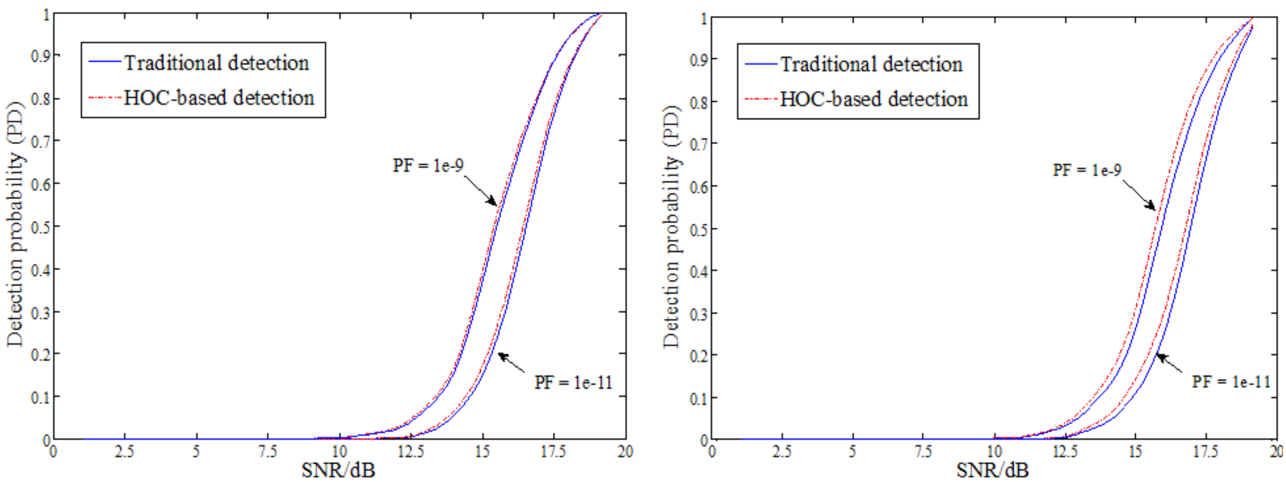
4.2. Analysis of the algorithm performance

ROC curve of the traditional correlation detection and HOC-based detection under the environment of AWGN and ACGN are shown in Figure 5 (a) and Figure 5 (b) respectively, as well as the curve of detection probability(PD) to SNR in Figure 6. To sum up, under AWGN environment, the performance of HOC-based detection is no inferior to the traditional correlation detection. Besides, the performance of HOC-based detection algorithm is superior to the traditional correlation detection algorithm at low SNR. Under ACGN environment, the performance of the HOC-based detection algorithm is obviously better than that of the traditional correlation detection algorithm.



(a) Curve of ROC under AWGN (b) Curve of ROC under ACGN

Figure 5. Curve of ROC



(a) Curve of PD to SNR under AWGN (b) Curve of PD to SNR under ACGN

Figure 6. Curve of detection probability(PD) to SNR

5. Conclusions

The radar signal detection algorithm based on HOC has obvious advantages in terms of the performance and has good Gaussian noise immunity. Theoretically, it can achieve a better detection performance under the condition of low SNR. Meanwhile, through the slice function can largely reduce the computational complexity and the real-time signal detection can be reached. Besides, it is much easier for the generalized cross-correlation operation to be applied to the coherent radar system.

However, HOC-based detection algorithm also has its defects. The excellent Gaussian immunity of HOC is based on the statistical analysis, which calls for a large number of sampled data in practical applications to reach full performance. Besides, the simulation of the algorithm put forward in this paper is conducted under the prerequisite of stationary target, which means no Doppler frequency shift. However, if we carried out the simulation under the prerequisite of moving target, it can be found that the detection performance of LFM signals will be reduced by a large margin. Therefore, the follow-up research work should be concentrated on how to employ the HOC to improve the detection capability of the radar signals under the situation of fewer sampled data and moving target.

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