Models and Algorithms of Uncertain Chance Constrained Programming with the Transportation Problem under Transport Capacity Constraints

Xinchun Wang

College of Science, North China University of Science and Technology, Tangshan 063009, Hebei, China

Lin Chen

College of Mechanical Engineering, North China University of Science and Technology, Tangshan 063009, Hebei, China

Lifang Chen

College of Science, North China University of Science and Technology, Tangshan 063009, Hebei, China

Wenying Ren

College of Mechanical Engineering, North China University of Science and Technology,Tangshan063009,Hebei,China

Abstract
This paper studies planning model constrained by a class of uncertain chance of objective function concerning the problem of transportation. It is given to solve the uncertain chance constrained programming model and to make the constraints to meet the given confidence level, meanwhile, to limit transport capacity under the uncertain environment. The model uses the simplex method;

numerical examples show that the establishment and the algorithm design of the model reasonable and effective.

Key words: TRANSPORTATION ISSUES, TRANSPORT CAPACITY CONSTRAINTS, UNCERTAIN CHANCE CONSTRAINED, PLANNING MODEL

1. Introduction

The basic form of uncertainty theory of uncertain programming model are: establishing the expected value of the objective function to achieve optimal mathematical programming under uncertainty in the given confidence level[1][3] .

However, in normal conditions stability of uncertain variables is poor, that is to say, when variance is high, mathematical expectation cannot reflect the important characteristics of random variables, at this point, the value and the best value of the objective function have an actual large deviation. In fact, we do not always care about the expected revenue maximization or minimization.

In real life, we are particularly concerned about how likely the possibility of events happen, that is to say, problem of degree of credibility.

Aiming to establish a class of uncertain chance constrained programming model to make the objective function to meet the given confidence level, the paper points at deficiency in achieving optimal expectations of the objective function using simplex method and numerical example to analyze its feasibility.

2. Preliminaries

Theorem 1: (Liu[2])Assume $\xi_1,\xi_2,\cdots,\xi_n$ to be regular independent uncertain variables, corresponding uncertain distribution functions are: $\Phi_1,\Phi_2,\cdots,\Phi_n$ .If a strictly monotonic function is strictly increasing on $x_1,x_2,\cdots,x_m$ and strictly decreasing on $x_{m+1},x_{m+2},\cdots,x_n$, then $\hat{\imath} = f(\hat{\imath}_1,\hat{\imath}_2,\cdots\hat{\imath}_n)$ is an uncertain variables and has an inverse uncertainty distribution as following:

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha),\cdots,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\cdots\Phi_n^{-1}(1-\alpha)) \quad (1)$$

Theorem 2: (Liu[2])Assume to be strictly increasing function for $\xi_1,\xi_2,\text{L},\xi_k$ and strictly decreasing function for $\xi_{k+1},\xi_{k+2},\cdots,\xi_n$. If $\xi_1,\xi_2,\cdots,\xi_n$ is regular independent uncertain variables, corresponding uncertain distribution functions are: $\Phi_1,\Phi_2,\cdots,\Phi_n$, then:

$$M\left\{g\left(x,\xi_1,\xi_2,\cdots,\xi_n\right)\lesseqgtr 0\right\} \quad \alpha \quad (2)$$

If and only if:

$$g(x,\Phi_1^{-1}(\alpha),\cdots,\Phi_k^{-1}(\alpha),\Phi_{k+1}^{-1}((1\leqslant\alpha)),\cdots,\Phi_n^{-1}(1-\alpha)) \quad 0 \quad (3)$$

Definition: (Liu[2]) $\hat{\imath}$ as uncertain variables, and, $a\in(0,1)$ then say:

$$\xi_{\inf}(\alpha) = \inf\left\{\gamma \middle| M\left\{\xi \le \gamma\right\} \ge \alpha\right\} \quad (4)$$

As pessimistic value of .

Theorem 3: Assume $\hat{\imath}$ to be uncertain variables and the uncertain distribution to be Φ . Then α pessimistic value of $\hat{\imath}$ is:

$$\xi_{\inf}(\alpha) = \Phi^{-1}(\alpha) \quad (5)$$

3. Establish uncertain chance constrained programming model example

A pointing at deficiency in achieving optimal expectations of the objective function and the requirements of credibility problem in real life, the paper deals with the objective function by the credibility. As a result, the objective function in decision-making programs is not less than a certain measure of credibility.

Set x as decision vector, $\hat{\imath}$ as uncertainty vector. $f(x,\hat{\imath})$ is objective function of decision vector x . Obviously, $f(x,\xi)$ is an uncertainty variable, then there are many possible values $\overline{f}$ making $M\left\{f(x,\xi)\lesseqgtr\overline{f}\right\}$ β .The goal is to minimize $\overline{f}$.

Establish uncertain chance constrained programming model as follows:

$$\begin{aligned} &\min \overline{f} \\ &s.t. \qquad\qquad\qquad\qquad (6) \\ &\quad M\{f(x,\hat{\imath}) \leqslant \overline{f}\} \geqslant \hat{a} \\ &\quad M\{g_j(x,\hat{\imath}) \leqslant 0\} \geqslant \acute{a}_j,\ j = 1.2,\cdots,p \end{aligned}$$

There into, $\beta,\alpha_j \in [0,1]$ respectively is credibility of object function and credibility of constraint condition. The measures of object function and constraint condition respectively cannot less than the minimum value of objective function in ensuring the confidence level is at least β .

A point is a feasible point,if and only if the likelihood of occurrence of $M\{g_j(x,\xi)\leqslant 0\}$ no less than α_j , or the possibility of violation of constraints less than $(1-\alpha_j)$.

4. Determined equivalent form of uncertain chance constrained programming model

For a given decision variable, $f(x,\xi)$ is obviously an uncertainty variable, then there are many possible values $\overline{f}$. Policymakers tend to focus on pessimistic or optimistic value. The paper by minimizing the pessimistic value convert model (6)to the corresponding certainty equivalent model.

4.1. Certainty equivalent form of the objective function

Theorem 3 gives objective function $M\left\{f(x,\xi)\lesseqgtr\overline{f}\right\}$ β of uncertain chance constrained model determined equivalent forms:

Step 1: Credibility $\hat{a}$ of given objective function;

Step 2: Establish $\xi_1,\xi_2,\cdots,\xi_n$ as uncertain distributed function of $\Phi_1,\Phi_2,\cdots,\Phi_n$; if $f(x_1,x_2,\cdots,x_n)$ is strictly increasing on $x_1,x_2,\cdots,x_m$ and strictly decreasing on $x_{m+1},x_{m+2},\cdots,x_n$ of monotonic function, then the inverse cumulative distribution function of $\xi_1,\xi_2,\cdots,\xi_n$ is $\Phi_1^{-1}(\beta),\cdots,\Phi_k^{-1}(\beta),\Phi_{k+1}^{-1}(1-\beta),\cdots,\Phi_n^{-1}(1-\beta)$;

Step 3: According objective function of, the inverse distribution function of objective function is obtained according to Theorem 1: $\Psi^{-1}(\beta)=f(\Phi_1^{-1}(\beta),\cdots,\Phi_k^{-1}(\beta),\Phi_{k+1}^{-1}(1-\beta),\cdots,\Phi_n^{-1}(1-\beta))$;

Step 4: Theorem 3 shows that $\Psi^{-1}(\beta)$ is the pessimistic value of $\overline{f}$.

4.2. Determined equivalent forms of constraint condition

According to Theorem 2, the certain equivalent form of $M\{g_j(x,\xi)\leqq 0\}$ $\alpha_j, j=1.2,\cdots p$ determined by uncertain chance is obtained, the following steps are:

Step 1: Credibility α of given objective function;

Step 2: Establish $\xi_1,\xi_2,\cdots,\xi_n$ as uncertain distributed function of $\Phi_1,\Phi_2,\cdots,\Phi_n$,

Get $(\acute{a})$ as inverse cumulative distribution function of $\xi_1,\xi_2,\cdots,\xi_n$;

Step 3: $g(x,\Phi_1^{-1}(\alpha),\cdots,\Phi_k^{-1}(\alpha),\Phi_{k+1}^{-1}(1-\alpha),\cdots,\Phi_n^{-1}(1-\alpha))\leqslant 0$ is the certain equivalent form of $M\{g_j(x,\xi)\leq 0\}\geqslant\alpha_j$;

In summary, the certainty equivalent form of available models (6) is:

$$\begin{cases}\min \Psi^{1}(\beta)\\ s.t.\\ \quad g(x,\Phi_1^{-1}(\alpha_j),\cdots,\Phi_k^{-1}(\alpha_j),\Phi_{k+1}^{-1}(\alpha_j),\cdots,\Phi_n^{-1}(\alpha_j))\leq\leq 0\\ \quad j=1,2,\cdots,p\end{cases} \tag{7}$$

5. Uncertain chance constrained programming model

Model (4) in literature [4] [5]is the model with the transportation problem under transport capacity constraints. the necessary and sufficient conditions for the existence of the model is:

$$\sum_{j=1}^{n} d_{ij}\geqslant a_i,\ \sum_{j=1}^{n} d_{ij}\quad b_j, i=1,2,\cdots,m; j=1,2,\cdots,n$$

Assuming this condition is satisfied all the time. Parameters of given model in Document are known, the objective function and constraint conditions in the model are clear and definite[6] [7]. While, in reality, the environment of the problem is complex, some of the data is uncertain. In the real environment, the unit price, the quantity of product and the demand of marketing place of products are not constants, but uncertain information. Therefore, the paper studies uncertain chance constrained programming model with the transportation problem under transport capacity constraints when the unite price c_{ij} of transport goods, the goods production a_i and quantity demanded of place of sale are uncertain variables.

6. Model with the transportation problem under transport capacity constraints--uncertain chance constrained programming model

Assuming traffic volume as the decision variables, the cost function is $\sum_{i=1}^{m}\sum_{j=1}^{n}\xi_{ij}x_{ij}$. The optimization objective of transportation problem under transport capacity constraints is to seek a minimum freight transport program and to establish uncertain chance constrained programming model as follows:

$$\begin{cases}\min \overline{f}\\ s.t.\\ \quad M\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\xi_{ij}x_{ij}\leqq\overline{f}\right\}\quad\beta\\ \quad M\left\{\sum_{j=1}^{n}x_{ij}\leqq\eta_i\right\}\quad\alpha_i, i=1,2,\cdots,m\\ \quad M\left\{\sum_{i=1}^{m}x_{ij}\geqslant\tau_j\right\}\quad\gamma_j, j=1,2,\cdots,n\\ \quad x_{ij}\leqslant d_{ij}, i=1,2,\cdots,m; j=1,2,\cdots,n\\ \quad x_{ij}\geqslant 0, i=1,2,\cdots,m; j=1,2,\cdots,n\end{cases} \tag{8}$$

Wherein, β,α_i,γ_j respectively is confidence level of objective function and constraints, $\overline{f}$ is the target value, that is to say, the minimum value of objective function when its confidence level is at least β . ξ_{ij} is transport unit supplies cost from place of origin A_i to place of sale B_j ; $\hat{o}_j$ is the sales volume (quantity demanded) of place of sales B_j . Wherein, ξ_{ij} , η_i , $\hat{o}_j$ are uncertain variables.

6.1. Solution of the model

Using inverse uncertainty distribution function method, according to calculation step of 4.1 and 4.2 in this paper, it obtain certainty equivalent form of the model (8). Taking into account the transport problems with transportation capacity constraints is a linear programming problem, it will be solved by using the simplex method after transformed into definite form.

At this time, certainty equivalent form of transport capacity constraints of transport problems is as follows:

$$\begin{cases}\min\sum_{i=1}^{m}\sum_{j=1}^{n}\Phi_{ij}^{-1}(\beta)x_{ij}\\ s.t.\\ \quad\sum_{j=1}^{n}x_{ij}-\Psi_{\eta_i}^{-1}(1-\alpha_i)\leq 0, i=1,2,\cdots,m\\ \quad\Psi_{\tau_j}^{-1}(\gamma_j)-\sum_{i=1}^{m}x_{ij}\leq 0\quad, j=1,2,\cdots,n\\ \quad x_{ij}\leq d_{ij},\quad i=1,2,\cdots,m; j=1,2,\cdots,n\\ \quad x_{ij}\geq 0,\quad i=1,2,\cdots,m; j=1,2,\cdots,n\end{cases} \tag{9}$$

Wherein, Φ_{ij} is uncertain distribution of uncertain variable ξ_{ij} ; Ψ_{η_i} : uncertain distribution of uncertain variable η_i ; Ψ_{τ_j} :uncertain distribution of uncertain variable τ_j .

Model (9) is linear programming model, here we can use the simplex method to solve it.

6.2. Application examples

There are two places of produce A_1, A_2 , which supply for three places of sale B_1, B_2, B_3 .Assume unite price c_{ij} of transport goods, the goods production(Deliquty) a_i of place of origin A_i and sales volume B_j (quantity demanded) of place of sale B_j as certain distributed uncertain variables. Pre-give the objective function confidence level as 0.8, confidence level $\eta_i(i=1,2), \tau_j(j=1,2,3)$ of each constraint conditions are both 0.9, the relevant data as follows:

Table 1 schedule of charges of unit items

Unit price (RMB)	B_1	B_2	B_3	Output (t)
A_1	$L(3,25)$	$L(8,30)$	$L(14,30)$	$N(25,1.5)$
	$L(9,28)$	$L(5,22)$	$L(10,25)$	$N(30,1.5)$
Sales (t)	$N(10,1.5)$	$N(14,1)$	$N(22,1)$	$N(10,1.5)$

Table 2 limit table of transport capacity

Transport capacity (t)	B_1	B_2	B_3
A_1	6	10	14
A_2	13	7	11

In the following table, $L(a,b)$ shows the distribution of the elements obeying linear uncertain variables, $N(e,\sigma)$ indicates the element as distributed variables following Gaussian distributions.

Put the amount of each of these values in the model (8), and then converted it to equivalent deterministic model that is the equivalent form of model (9). By using the simplex method to solve its results it can be obtained as:

$x_{11} = 2.7601$; $x_{12} = 8.2114$; $x_{13} = 12.2114$; $x_{21} = 9.0570$; $x_{22} = 7.0000$; $x_{23} = 11.0000$

Final minimum total transportation costs is 1185.7148 RMB.

If use of expectation model Liu [2] were proposed to solve this problem, although the minimum value of transport costs is 917.1078, the likelihood that ξ_{ij} equals to the expected value is small for some variance of ξ_{ij} , $L(a,b)$ is the larger. so using Liu [2]'s expectation model to solve this problem there is a big risk. For such cases, the use of the model (8) is more realistic.

7. Conclusions

The paper on the strength of a class of uncertain chance constrained programming model based on uncertainty theory, gives the method that converting it into certainty equivalent form. It also studied the uncertain chance constrained programming model with the transportation problem under transport capacity constraints and gets the optimal solution of the model. Finally, a numerical example explains the validity of the model.

Acknowledgements

This work was supported by Fund Project of Hebei Province Natural Science (F2014209086) 2014.

References

1. Kissmann P, Edelkamp S. (2009) Solving fully-observable nondeterministrc planning problems via translation into a general game. *Lecture Notes in Computer Science*, 5803, p.p.1-8.
2. Liu B. (2002) *Theory and Practice of Uncertain Programming*. Springer.
3. Kuter U, Nau D, Reisner, et al. (2008) Using classical planners to solve nondeterministic planning problems. *Proc of the 18th Int Conf on Automated Planning and Scheduling*, p.p.190-197.
4. Santoso T, Ahmed S, Goetschalckx M , et al. (2005) A stochastic programming approach for supply chain network design under uncertainly. *European Journal Operational Research*, 167, p.p.96-115.
5. Pu Li, Harvey A G and Wozny G. (2008) Chance constrained programming approach to process optimization under uncertainty. *Computers and Chemical Engineering*, 34, p.p.699-708.
6. Zhang Y, Monder D and Forbes JF. (2002) Real-time optimization under parametric uncertainty: a probability constrained approach. *Journal of Process Contrl*, 12, p.p.373-389.
7. Cimatti M Roveri, P Traverso (1998) Automatic OBDD-based generation of universal plans in non-derministric. *Proc. of AAAI*, p.p.875-881.