Inversion and construction of nuclear radiation field

Min Zhang[1, 2, 3]

[1]School of Nuclear Science and Technology, University of South China, Hengyang 421001,China
[2]School of Math and Physics, University of South China, Hengyang 421001, China
[3]Hunan Provincial Key Laboratory of Emergency Safety Technology and Equipment for Nuclear Facilities, Hengyang 421001,China
E-mail:emin1217@sina.com

[*]Shuliang Zou[2, 3]

[2]School of Math and Physics, University of South China, Hengyang 421001, China
[3]Hunan Provincial Key Laboratory of Emergency Safety Technology and Equipment for Nuclear Facilities, Hengyang 421001,China
**Corresponding author Email: 3467593@qq.com*

Abstract
In this paper, the inversion of the particle field is put forward by a function constructed for solid radioactive sources. Based on the inverse proportion between concentration and the square of the distance from the radioactive source , the concentrations surface near the radioactive source is sharp, steep and concave. The concentration surface may be convex where the nuclear radiation field is restored with sparse data by interpolation or fitting. Hence we make use of mathematical construction techniques to construct concentration function tactfully. An inversion model is established which is based on the concentration of nodes in the grid. The concentration value of the grid lines is restored by the cubic spline interpolation. According to an idea of concentration dominance of adjacent point, the function of interior point of grid is structured, which is consistent with the characteristics of the radiation field. During the simulation experiment, concentration value of node is obtained by MCNP program based on the Monte Carlo method. It is verified that the concentration function is close to the real concentration field. Therefore inversion method is feasible, which provides theoretical basis for radioactive sources search and safe evacuation.
Key words: RADIATION CONCENTRATION FIELD , INVERSION , STRUCTURE ,CUBIC SPLINE INTERPOLATION,CONCENTRATION DOMINANCE OF ADIACENT POINT, MCNP PROGRAM SIMULATION

1. Introduction

Nuclear energy is one of the most promising energy in the future. Since human peaceful use of nuclear energy, nuclear power has been brought to mankind a lot of clean energy. Meanwhile it reduces the amount of CO_2 emissions for the fossil fuel burned. Nuclear power is widely used in civil non-power fields. However, due to no fixed workplace to nuclear radiation sources, the loss event of nuclear radiation source occurs occasionally. At the same time, using radioactive sources may cause nuclear leakage accident and nuclear terrorism, etc. Once lost or leaked, it will affect social stability. So it is very important to eliminate the potential hazard in the application of nuclear sources and nuclear technology and guide the decision of nuclear emergency response. The establishment of the radiation field concentration model is the research basis of the above problems. We consider that the radiation field is reduced by the structured approach of mathematical with the concentration value of grid node in the radiation field .

In the research of the distribution theory of nuclear radiation field, there are two methods of forward and inversion methods[1-5]. The forward model is mainly confined to the principle of the dispersion of the radioactive plume in the atmosphere. For the region of less than 2km, CFD (Computational fluid dynamics) turbulence model[6] can be used. For local diffusion within the range of 20 km, the Gauss model[7-12] is adopted. For the medium scale (20-200km) and large scale (more than 200 km), the Lagrange model[13-19] or the Euler model[20] is applicable. In order to avoid the limitations of the model itself, the use of nested model will be a good choice [21-22]. The above models are both known to the source location and the intensity of the radiation source. Climate change has a great influence on the establishment of the model. Though the climatic factors have been considered for the radionuclide diffusion model, there is still a large deviation. But in practice, for the location of radiation sources and source strength is unknown in many cases, so the study of the nuclear radiation inversion research is very significant.

Existing inversion method is relatively simple, the results less, the missing data filled with node data of the grid. Because of the inherent characteristics of the radiation field, interpolation or fitting cannot be used to restore the nuclear radiation field completely. Therefore, it is worthwhile to construct a function of the concentration distribution of the nuclear radiation field.

We make use of mathematical construction techniques to construct concentration function tactfully. An inversion model is established which is based on the concentration of nodes in the grid. It is verified that the concentration function is close to the real concentration field. Therefore inversion method is feasible, which provides theoretical basis for radioactive sources search and safe evacuation.

2. Data collection

The rectangular area of containing the radiation source is divided into grid, and the corresponding grid node data is collected. We can structure concentration function to fill the missing part of the data with the limited data. Consider horizontal rectangular area with point source pollution $\Omega = \{(x, y) \mid 0 \le x \le a, 0 \le y \le b\}$ In the X axis , range $[0,a]$, insert the $n-1$ point $x_1, x_2, x_3, \cdots, x_{n-1}$,and let $x_0 = 0$, $x_n = a$ .In the Y axis ,range $[0,b]$,insert the $m-1$ point $y_1, y_2, y_3, \cdots, y_{m-1}$,and let $y_0 = 0, y_m = b$,so the search area is evenly divided into:(see Figure 1)

$$E_{n+1}: \ 0 = x_0 < x_1 < \cdots < x_n = a$$
$$x_{i+1} - x_i = a/n, \ i = 0, 1, \cdots, n-1$$
$$E_{m+1}: \ 0 = y_0 < y_1 < \cdots < y_m = b$$
$$y_{j+1} - y_j = b/m, \ j = 0, 1, \cdots, m-1 .$$

Then

$E_{n+1} \times E_{n+1} = \{(x_i, y_i) \mid i = 0,1,2\ldots,n; j = 0,1,2,\ldots.n\}$ is the rectangular point set(see Figure 1),the grid node value (x_i, y_j) is $f_{i,j}, i = 0,1,\cdots,n, j = 0,1,\cdots,m$ (see Table 1).

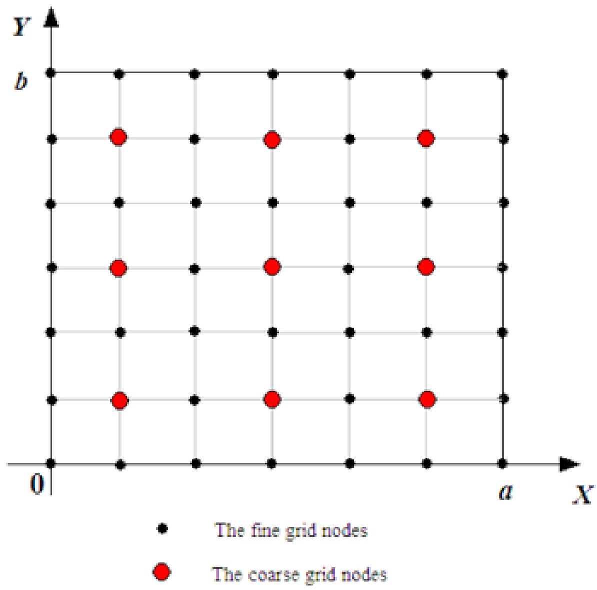


Figure 1. Division of the monitoring area

Table 1. Corresponding observation values of grid nodes

	y_0	y_1	y_2	n	y_m
x_0	$f_{0,0}$	$f_{0,1}$	$f_{0,2}$	$Q_3 = -\int_t^{t+\Delta t} \iiint P_0 dVdt$	$f_{0,m}$
x_1	$f_{1,0}$	$f_{1,1}$	$f_{1,2}$	$\dots$	$f_{1,m}$
x_2	$f_{2,0}$	$f_{2,1}$	$f_{2,2}$	$\dots$	$f_{2,m}$
$\iiint_V divqdV = divq \|_{(x',y',z')} \cdot V$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_n	$f_{n,0}$	$f_{n,1}$	$f_{n,2}$	$\dots$	$f_{n,m}$

3. Inversion by structure radiation concentration function

The idea of the inversion of the radiation concentration field is that the concentration value of any point is obtained by using the cubic spline interpolation with the data of grid node ,which is based on collected grid node data. For interior grid points, we can get the value of any point by structure .Therefore, the concentration of any point in the region can be obtained.

3.1. Fill the concentration value of grid line by cubic spline interpolation

The cubic spline[23] interpolation overcomes the non-convergence of the Lagrange interpolation [24], and improves the smoothness of the piece-wise linear interpolation function [25] at the nodes, so we choose the cubic spline interpolation to restore the grid line function.

Steps of the reduction in the grid line by cubic spline interpolation :

(1)According to the observed values of the nodes, determine the boundary conditions of the endpoints and establish the equations.

(2)Get the solutions of equations by chasing method, get the piece-wise cubic function.

(3)The value of each point given can be obtained by plugging into the corresponding interval function .

3.2. Get the grid interior point value based on the concentration dominant of adjacent point

For interior grid points, the characteristics of the radiation field are first considered. The concentration value of any point in the radiation field is inversely proportional to the square of the distance from the radiation source[26]. So the surface should be sharp and steep near the source of radiation and far away from the radiation source should be relatively flat.

Secondly, the maximum concentration of four vertex of the grid has great influence on the nearby interior points. According to the above factors, so we can construct the function to get the grid interior point value with the method of concentration dominance of adjacent point .

The idea of the concentration dominance of adjacent point is that the node of the maximum value in the four vertices has a great effect on the adjacent points in the grid. The ratio of the square of distance between two points and the maximum value point is equal to the ratio of the value difference of the corresponding points to the maximum point.

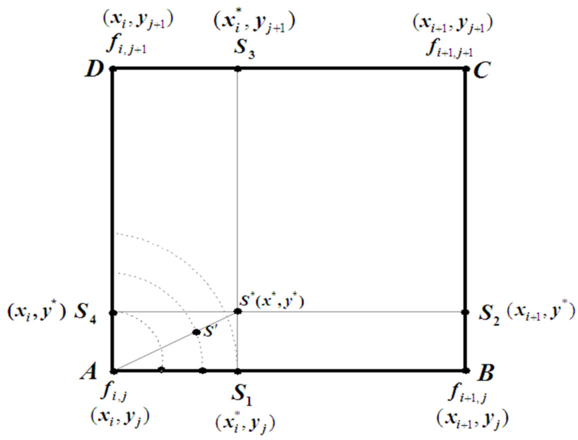
Within a unit grid (Figure 2), the four vertices are A, B, C, D , d is equal to the distance between $A(x_i, y_j)$ to an any point $S^*(x^*, y^*)$,

$$d = |AS^*| = \sqrt{(x_i - x^*)^2 + (y_j - y^*)^2}$$

Let

$$d_1^2 = (\frac{|x^* - x_i| + |y^* - y_j|}{2})^2,$$

Set S' as the corresponding point of line AS^*, $|AS'| = d_1$ .For any point $S^*(x^*, y^*)$ in the grid, the corresponding points on the four sides are S_1, S_2, S_3, S_4 ,the value of each point is $S_1^*, S_2^*, S_3^*, S_4^*$ respectively.(see Figure 2)



(1) If the value of A is the largest $f_{i,j} = \max\{f_{i,j}, f_{i,j+1}, f_{i+1,j}, f_{i+1,j+1}\}$,

$$\frac{d_1^2}{d^2} = \frac{\frac{S_1^* + S_4^*}{2} - f_{i,j}}{S^* - f_{i,j}}, \tag{1}$$

then

$$S^*(x^*, y^*) = \frac{\left(\frac{S_1^* + S_4^*}{2} - f_{i,j}\right) d^2}{d_1^2} + f_{i,j} \ . \tag{2}$$

(2) If the value of B is the largest $f_{i+1,j} = \max\{f_{i,j}, f_{i,j+1}, f_{i+1,j}, f_{i+1,j+1}\}$, then

$$S^*(x^*, y^*) = \frac{\left(\frac{S_1^* + S_2^*}{2} - f_{i+1,j}\right) d^2}{d_1^2} + f_{i+1,j} . \tag{3}$$

(3) If the value of C is the largest $f_{i+1,j+1} = \max\{f_{i,j}, f_{i,j+1}, f_{i+1,j}, f_{i+1,j+1}\}$, then

$$S^*(x^*, y^*) = \frac{\left(\frac{S_2^* + S_3^*}{2} - f_{i+1,j+1}\right) d^2}{d_1^2} + f_{i+1,j+1} . \tag{4}$$

(4) If the value of D is the largest $f_{i,j+1} = \max\{f_{i,j}, f_{i,j-1}, f_{i+1,j}, f_{i+1,j+1}\}$ then

$$S^*(x^*, y^*) = \frac{\left(\frac{S_3^* + S_4^*}{2} - f_{i,j+1}\right) d^2}{d_1^2} + f_{i,j+1} \ . \tag{5}$$

In practical operation, the more data collected, the more accurate for the reduction, if the less the node data, the effect of the inversion reduction will be rough. Due to the hazard and cost of collecting data, the grid data collected is limited ,in order to effectively restore the radiation concentration field, we need to balance the cost of data and the reduction of precision.

The structured approach of reducing the concentration field is not only suitable for large scale grid data but also sparse grid data within a unit grid. Regardless the value of the dense or sparse grid node , the concentration value of the grid points will follow the principle of the concentration dominance of adjacent point. So the structured approach will have better practical application value.

3.3. Simulation experiment

In order to obtain the inversion results, we use the MCNP program based on Monte Carlo method to simulate the grid node data in the field. Radioactive source is placed in (08,10,0),The air inside the small ball with radius of 15cm is filled with a neutron source,2000 Curie radioactivity, gets the same height node value (x_i, y_j) from the ground, $E_{n+1} \times E_{n+1} = \{(x_i, y_i) \mid i = 0,1,\cdots,200, j = 0,1,\cdots 60\}$ According to the idea of the structure, the concentration surface is shown in Figure 3.

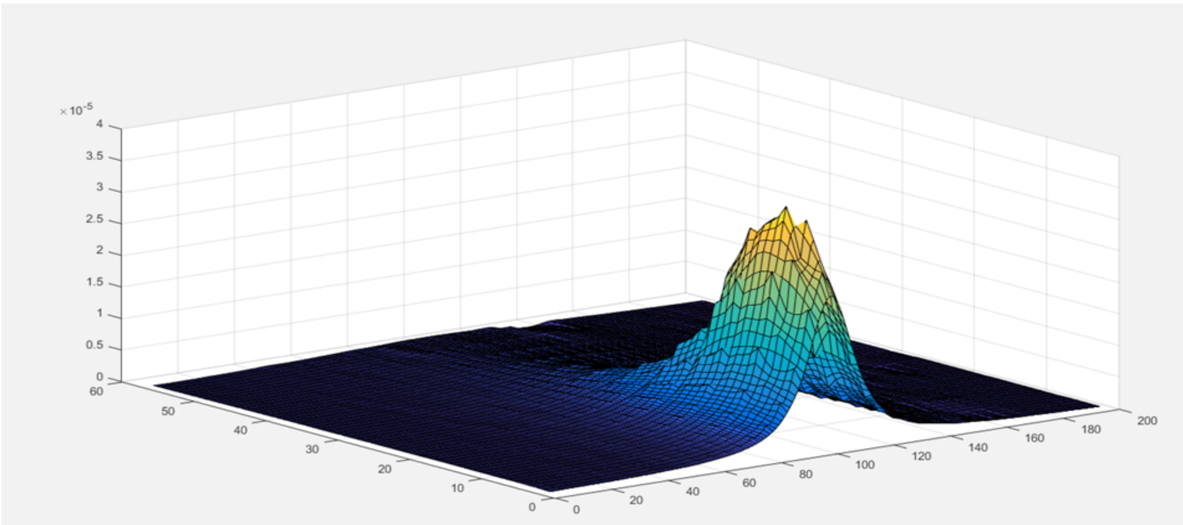


Figure 3. Reduction concentration surface

It can be seen that the concentration surface near the radiation source is obviously sharp and steep with a concave characteristics, which is more close to the characteristics of radioactive sources.

4. Generate the concentration surface by MATLAB

MATLAB is a product of the United States of America Math-Works commercial software, which can be used for the development of algorithms, data visualization, data analysis and scientific data visualization. MATLAB provides a series of powerful graphics functions, only some basic parameters given,the graph can be restored by interpolation or fitting with a few data .

Steps of drawing concentration surface by MATLAB:

Step 1: Input the grid node radiation concentration value into MATLAB;

Step 2:Run the surf or mesh command, output concentration surface graphics.

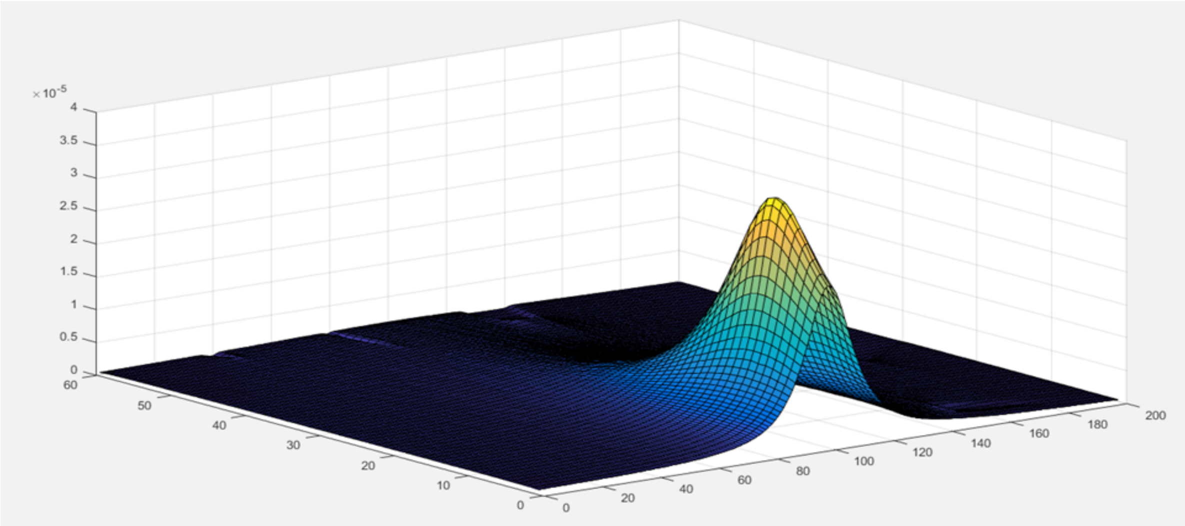


Figure 4. Concentration surface by MATLAB

Obviously, when the radiation field is restored by MATLAB, the surface is almost always smooth and convex (see Figure.2).

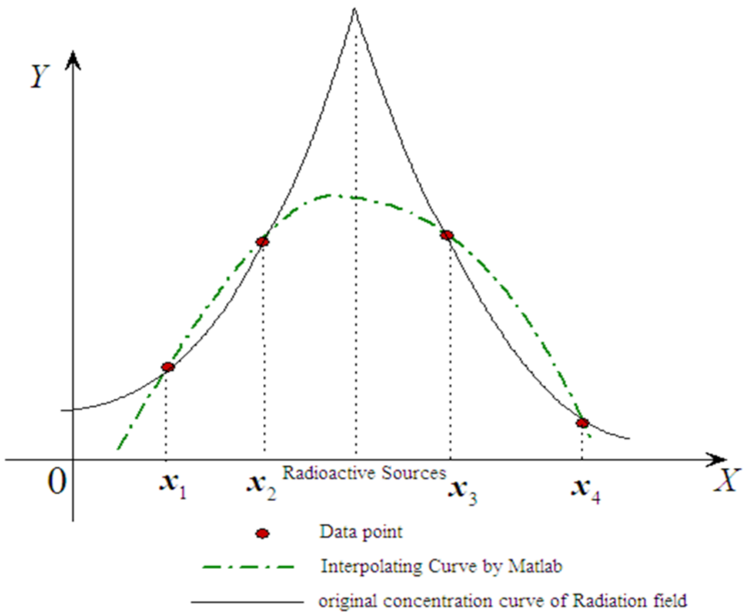


Figure 5. MATLAB interpolation curve and actual curve deviation

When drawing by MATLAB, the data node value is denser, the graph is more accurate, but the data is sparser, the graph is not accurate. In Figure 6, in the interval $[x_1, x_2]$, $[x_3, x_4]$, the concave function is interpolated into a convex function .The value interpolated cannot be up to the peak (as shown in $[x_2, x_3]$).For inversely proportional to the square of the distance from the radiation source, the surface should be sharp and steep near the source of radiation ,but the curve increase slowly with the full shape in the Fig.5. There is a large deviation between them. Due to the surface distortion by MATLAB, it would affect the nuclear emergency response decision and searching radiation sources .

5. Constructor is also suitable for sparse data

In order to show that the concentration field of constructors is applicable to the sparse data, we use nine sparse node data, including the radioactive sources, Drawing by MATLAB and constructors respectively(see Figure 6, 7).

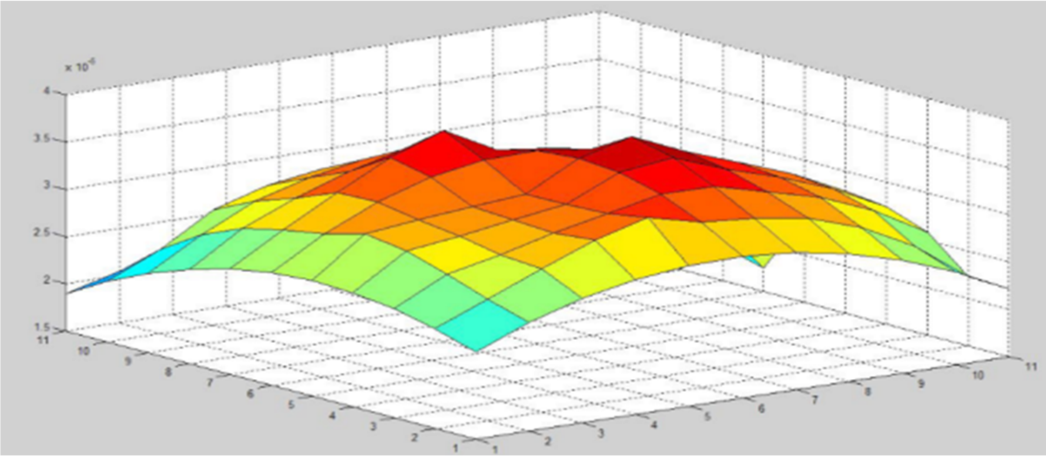


Figure 6. Reduction by concentration dominant of adjacent point with nine node data

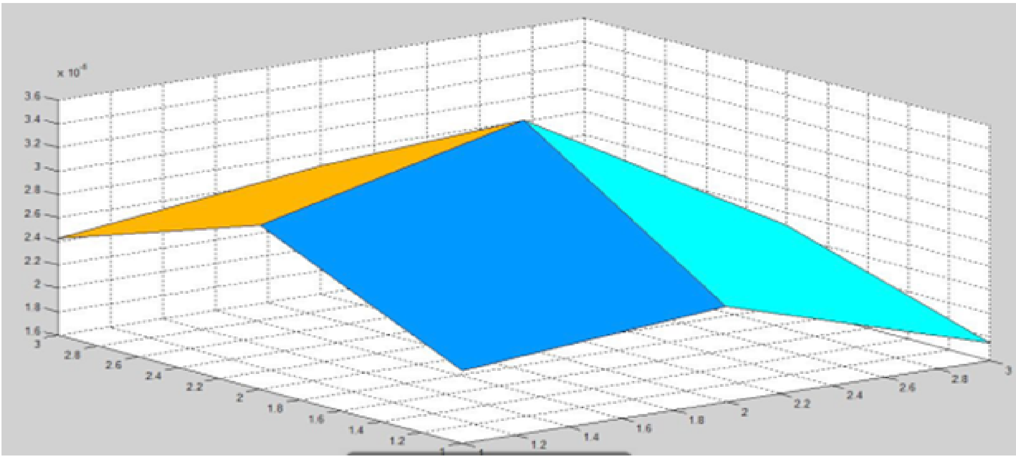


Figure 7. Reduction by MATLAB with nine node data

Under the same data, we can see that the structure of the concentration of surface is more refined in the Fig. 6, Graph is bump and sharp. In the Fig.7, Concentration surface get from the four planes joined together ,which lack the characteristics of surface. the image is rough. The adjacent point constructor is better than MATLAB with a few of node data. When reduction surface with the same data, the construction method would be better than the MATLAB.

6. Error analysis between MATLAB reduction and constructors

In order to show that the concentration surface constructed is close to the real concentration field, we will make the error analysis of the concentration function value and the concentration value from MCNP simulation.

In the case of existing data, we consider that taking a once in four nodes from the grid node data, the coarse grid data were obtained. The value of the corresponding fine mesh node is obtained from the concentration constructors. Make the difference between the fine nodes value from constructors and fine node value from MCNP simulation. If the difference is little error, the constructors are feasible.

Simulation generated grid nodes data value are $f_{i,j}, i=1,2,\cdots,n$, $j=1,2,\cdots,m$,The data values of the constructed surface nodes are $S_{i,j}$.

A formula of the relative error analysis of the value from constructors function and the value from MCNP simulation

$$W=\frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\| S_{i,j}-f_{i,j}\|}{\sum_{j=1}^{m}\sum_{i=1}^{n} f_{i,j}} \qquad (8)$$

among $\|\cdot|$ represents 1- Norm.

Steps for error analysis:

Step 1:Get coarse mesh data. Take a value at intervals of 4 points from the node data by the MCNP simulation.

Step 2: Using the coarse grid data, the concentration of the surface and the small mesh node value is obtained by constructor.

Step 3: Make the error W of the fine grid data get from the MCNP simulation and the corresponding mesh node data generated by the neighbor point concentration dominant .

Finally, The relative error value is 3.19% by simulation. There is very small error between the reduction field and the real field. Therefore the constructed concentration field is very close to the real radiation field. The reduction method of concentration of the adjacent point is feasible.

7. Conclusions

The concentration value of any point is inversely proportional to the square of the distance from the radiation source. It is not only suitable for large-scale grid data but also a few amount of sparse data. In the complex case of multi-point radioactive sources, the concentration of the nodes value is the superposition values of corresponding node, which obey the rule of concentration dominance of adjacent point.

The idea of constructors have been improved the non-authenticity of the smooth features of MATLAB and the distribution of the uniformity ,which have been improves the accuracy of the concentration field inversion. It is of great theoretical significance to improve the accuracy of nuclear emergency response decision.

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