Calculation of parameters of cryolithic zone mine openings thermal protection coating

Aleksandr Galkin

D.Sc. in engineering
Saint-Petersburg, National Mineral Resources University,
E-mail: afgalkin@mail.ru

Abstract
The method of parameters calculation of mine openings thermal insulation in underground facilities of cryolithic zone was developed. The mine openings of spherical and cylindrical symmetry are considered. All the formulas are obtained in a dimensionless form that allows to select the necessary parameters of thermal protection coatings through Biot number. The case of thermal protection coatings characteristics selection was considered; this case does not allow thawing of rocks throughout the entire period of facility operation.
Key words: CALCULATION METHOD, UNDERGROUND MINE OPENING, FROZEN ROCKS, THERMAL INSULATION

The thermal protection coatings used in underground facilities fulfil various functions, on the basis of which their optimum parameters are selected. Conditionally, thermal protection coatings according to the functional area can be divided into four groups providing the following conditions [1, 2, 3, 4] a) a frozen condition of rocks during the facility service lifetime; b) thawing of rocks throughout the entire period of underground facility operation on the predetermined depth; c) the specified thermal mode in the facility; d) the specified thermal mode in the facility and rocks strength (the bearing coatings, which fulfill support function).

In this article, the theory of the first group coatings calculation is considered. For convenience, all the calculating dependences are reduced to the dimensionless form. The purpose of calculations is to determine the thermal resistance of a heat-insulation layer; this value is calculated by two stages. At first, the Biot number for a specific case is defined. Then, the necessary thermal resistance meeting the task statements is calculated according to determined Biot number. The calculating dependences are given for facilities with spherical (chamber type openings) and cylindrical (extended openings) symmetry. The obtained formulas serve mainly to calculate the thermal resistance of underground facilities coatings with small service lifetime at positive air temperature. Further, the following symbols are used: *Bi* _ Biot number, α *R/λ* ; *Fo* — Fourier number, $a_r \tau / R^2_0$; *T* — dimensionless temperature of rocks, T/T_e; *t* — dimensionless air temperature, t_{air}/T_e; δ — dimensionless radius of thermal influence, δ $/R_0$; R_0 — equivalent radius, 0.282 $\sqrt{F}$ and 0.564 $\sqrt{F}$ for constructions of spherical and cylindrical symmetry respectively; *F* — section area of facility, m^2; a_r, λ , C_r — rocks thermal diffusivity (m^2/c), heat conductivity (W/m·K) and thermal capacity (J/kg·K) respectively.

On the basis of dependences obtained in paper [1], let us calculate the thermal resistance of contacting throughout the entire period with rocks facility, which provides the "unthawing" of rocks, i.e. the fulfillment of the condition $T|_{R=R}$ T_{mel} for the facilities with spherical symmetry. Using the expression for rocks temperature determination under the following condition:

$$T|_{R=1} \leq 0, \tag{1}$$

providing a frozen condition of rocks during the whole service lifetime of a facility, we obtain the following expression for Biot number establishment:

$$Bi = (1-\delta)\ t\ , \tag{2}$$

where δ - the function of Fo and Bi, which is found from the equation

$$Fo = \frac{(\delta-1)^2}{12} + \frac{(\delta-1)}{3(Bi+1)} - \frac{2}{3(Bi+1)^2}\ell n\left[1+\frac{(Bi+1)(\delta-1)}{2}\right]. \tag{3}$$

Thus, we receive the transcendental equation concerning Biot number. If parameters of thermal protection are concretized and cannot be changed, and it is required to estimate time of safe existence of a design, the defined condition (1), the last can be unambiguously found from the equation:

$$Fo = \frac{1}{3}\left\{\frac{1}{(Bi\cdot t+1)^2} - \frac{2}{(Bi+1)(Bi\cdot t+1)} - \frac{2}{(Bi+1)^2}\ell n\frac{(t-1)}{(t+1/Bi)}\right\} \tag{4}$$

Example. To calculate the operating time of the facilities built in mine opening so as by the end of operation, the rocks temperature on the contact with coating was not higher than ice melting temperature equal to 0 °C. Natural temperature of rocks is -5 °C, while air temperature in a facility is +15 °C. Biot number is equal to 1. Parameter $t=t_0/T_e$=-3. Calculation by a formula (4) gives the value of *Fo* equal to 0.314. For the facility with an equivalent radius of 1.8 m, and which is placed in rocks with coefficient of temperature conductivity of 3.6· 10^{-3} m^2/h, this value of *Fo* corresponds to:

$$\tau = \frac{Fo\cdot R_0^2}{a} = \frac{0{,}134\cdot(1{,}8)^2}{3{,}6\cdot 10^{-3}} = 120 \text{ hours} = 5 \text{ days.}$$

The Biot number will be higher if the facility has a heat-protective design, which is more heat-conducting or less thick, made from other material. Let us consider a design at the same conditions when *Bi* = 3.0. The calculation according to formula (4) gives the value of *Fo* equal to 0.0091, that corresponds to 8 hours or 0.3 days. If the natural temperature of rocks was equal to -15 °C, i.e. t =-1, this value would be *Fo* = 0.121 or 109 hours, i.e. 4.5 days.

From the given example, it is obvious that one of the effective measures, which allows increasing of service life of facility with a small thermal resistance design, is preliminary cooling of a massif in a place of facility construction.

Let us consider a selection of Biot number for the contacting constructions of cylindrical symmetry. For determination of Biot number, let us use the dependences given in papers [1, 5]. After the intermediate processes similar to given above for a case of spherical symmetry, we obtain the equation similar to (3):

$$Fo = \frac{1}{4}\left\{\frac{(Bi+\theta)(Bi-1)-\theta}{Bi(\theta+1)} + \left[1-\frac{Bi}{\theta+1}\right]\exp(2\theta/Bi)\right\} \tag{5}$$

The equation includes the new parameter θ, which is connected with the introduced earlier parameter *t* by the ratio:

$$\theta = (T_s - T_{air})/(T_{air}\cdot t - T_s) \tag{6}$$

where T_s — temperature surface. In this specific case, T_s is equal to zero, as it was calculated when the formula (4) development, and then θ and *t* are connected by a ratio:

$$\theta = -1/t \tag{7}$$

The formula (5) does not allow obvious finding of Biot number value, therefore it is convenient to use it for evaluation of time of facility safe operation at the specified parameters of thermal protection, i.e. if the Biot number is known, and it is necessary to determine the Fourier number. For the inverse problem solution, i.e. finding of values Bi by the known Fo, the nomogram (Fig. 1) is constructed. On the horizontal scales, we laid off the values not of Fo numbers but logarithms of these numbers in order to extend the nomogram scope. The Fo parameter can be easily obtained by recalculation.

$$Fo = \exp(\ell nFo) \tag{8}$$

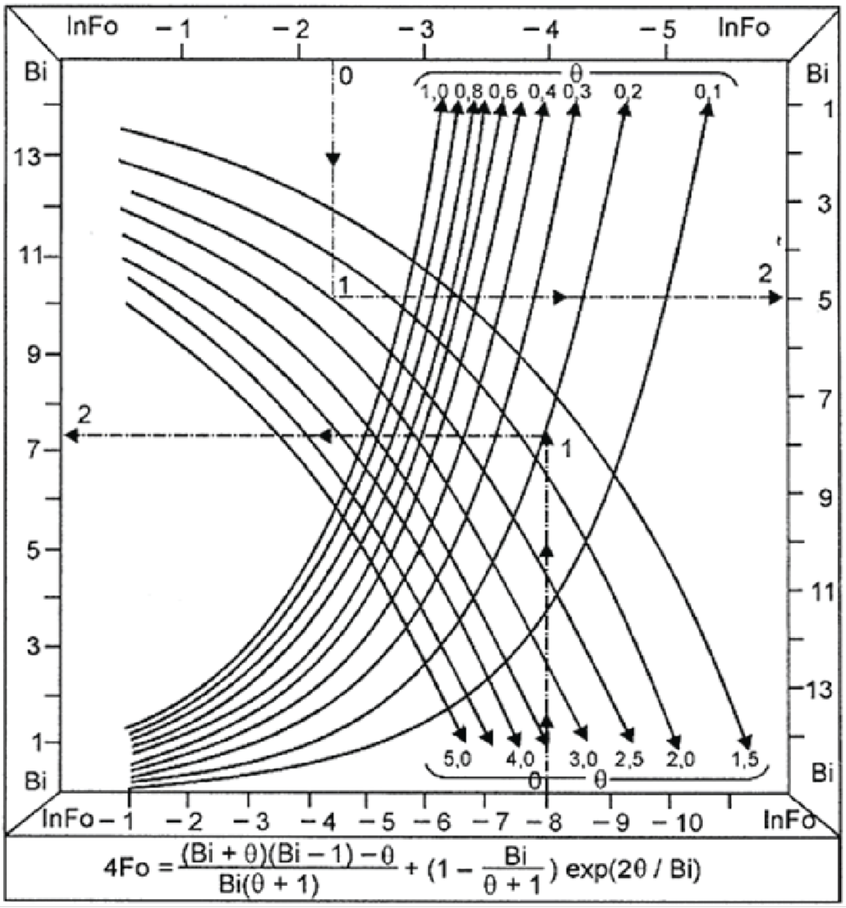

Figure 1. Nomograms for determination of Biot number of the contacting facilities, which are not allowing the rocks thawing

Example. To determine the Biot number for the contacting constructions of cylindrical symmetry for Fo = 0.10; t = –0.4. By the formula (7), we find θ = 2.5; the logarithm of Fourier number is equal to lnFo=ln0.1=-2.28. According to the nomogram, we find the Biot number, which is equal to five, on route 0+1+2.

Final expression for Biot number determination is of the form:

$$Bi = \theta / \ell n\delta(Fo, Bi) \,, \tag{9}$$

where parameter δ (Fo, Bi) can be found from expression obtained in the paper [5]:

$$Fo = \frac{\delta^2}{4} - \frac{\delta^2}{4(\ell n\delta + 1/Bi)} + \frac{Bi^2 - 2Bi + 2}{4Bi^2(\ell n\delta + 1/Bi)} + \frac{Bi - 2}{4Bi} \,, \tag{10}$$

i.e. finally, we obtain the transcendental equation relating to Biot number which, requires the computational solution. To avoid this, we have analyzed the expression (10) for the purpose of obtaining of simple dependence of the form without accuracy loss.

The general nomogram, which is given in Fig. 2, for determination of the parameter δ by the equation (10) was constructed; it allows determining of value δ in the wide range of change of Fourier and Biot numbers.

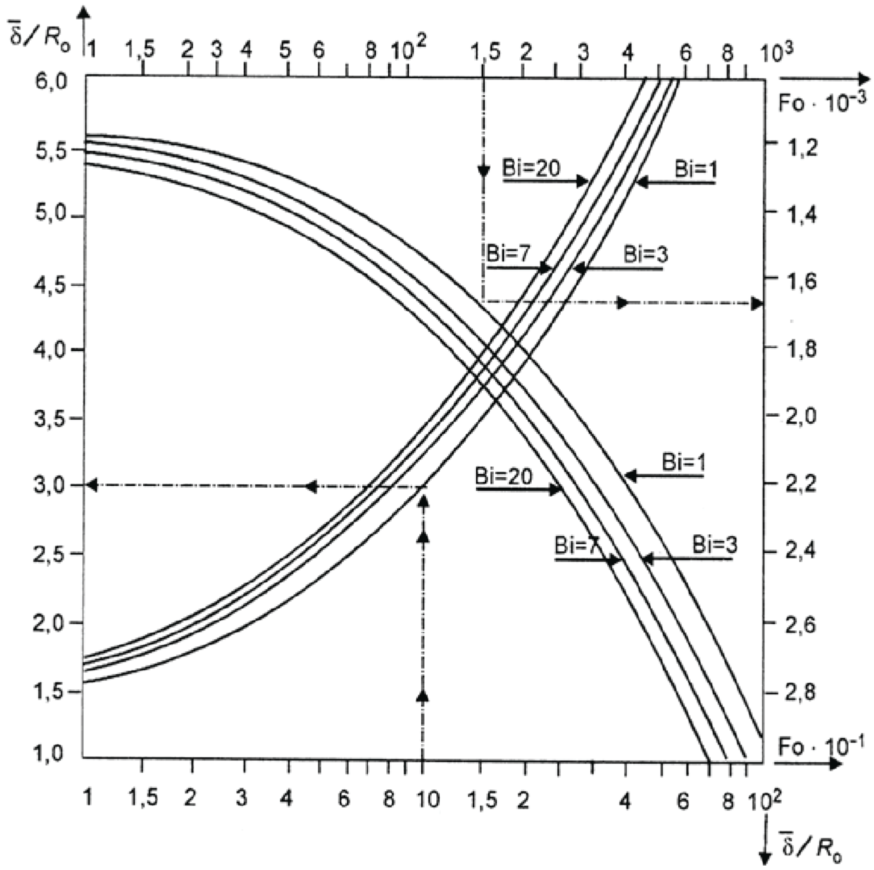

Figure 2. The nomogram for determination of parameter $\delta = \delta / R_O$ when changing of value F_O

From the diagrams presented in Fig. 2, it is seen that at great values of Biot numbers, the value δ does not practically depend on their change. For example, the value δ is not practically changed when Biot number changing from 7 to 20 at constant value of Fourier number. The conducted numerical analysis allows recommending of the following simplified dependences for determination of the parameter δ. They give close agreement with calculations result for a formula (10) in the wide range of change of Fourier and Biot numbers:

$$\delta = 1{,}05 + 2\sqrt{Fo}, \quad 0{,}000625 \le Fo < 1{,}0 \tag{11}$$

$$\delta = 1{,}25 + 2{,}1/\sqrt{Fo}, \quad Fo \ge 1\,. \tag{12}$$

In Fig. 3, the results of the comparative calculations executed from formula (10) (shaped line) and formulas (11) and (12) designated by lines 3 and 4 respectively are presented.

As a comparison, in this Fig., the known approximate formulas for calculation of the parameter δ obtained by A. N. Salamatin (1) and I. A. Charnyy (2) are shown [6].

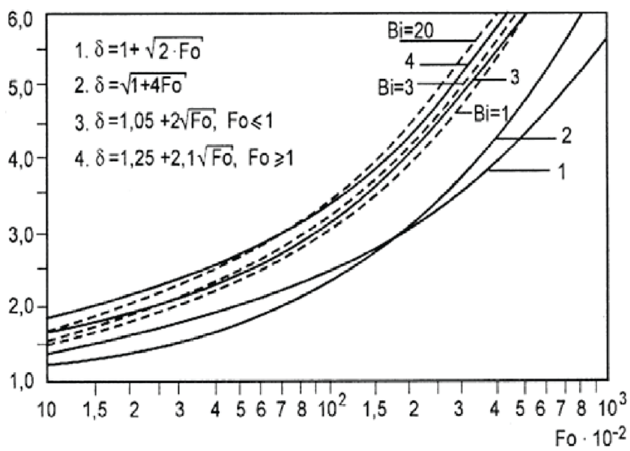

Figure 3. Comparison of dependences for determination of parameter δ (1 – 4) formulas: 1 – A. N. Salamatina, 2 – I.A. Charnyy, 3, 4 – (11) and (12).

Apparently from the Figure, the suggested formulas give closer agreement with formula (10) than known dependences in the entire considered range of Fourier number change. Thus, final expression for determination of Biot number of the contacting construction, which does not allow thawing of rocks, is of the form (for a case of $F_0 < 1$, which is most real):

$$Bi = \frac{\theta}{\ell n\left(1{,}05 + 2\sqrt{Fo}\right)} = \frac{1}{-\ell n\left(1{,}05 + 2\sqrt{Fo}\right)t} \quad (13)$$

If the considered case is $Fo \geq 1.0$, it is necessary to substitute expression (12) in a formula (9) instead of (11). Comparison of calculation results by a formula (13) with numerical calculations by formulas (9) and (10) showed close agreement of results. Thus, for conditions of the previous example, the value of Biot number found according to the nomogram was equal to five, and calculated by a formula (3) was 4.81. The error is equal to only 3.8%, and towards Biot number decreasing, i.e. design margin takes place. All the thermal calculations given above are reduced to determination of Biot numbers for designs of various symmetry. When knowing of Biot numbers, it is easy to pass on to finding of thermal protection parameters, which are usually used in design, namely, the construction thermal resistance, the thermal conductivity coefficient of coating materials, thickness of separate layers, etc. The dependence of constructions thermal resistance on time is a distinctive feature of the obtained calculating dependences, from ones, which are usually used when designing of thermal protection, for example, of buildings. In this regard, all the parameters of operation safety of an underground facility must be calculated for the most admissible service lifetime. Thus, if facility repeated use with small breaks is supposed, the calculations should be carried out for the total period of operation, excluding the period when air temperature in an excavation is lower than melting temperature of ice in rocks. Otherwise, so-called "thermal equilibrium time" of facility must be summarized with an estimation time of facility operation and the parameters of thermal protection for the total period must be selected. If in this case, the parameters of thermal protection are structurally impossible, it is reasonable to provide intermediate, or preliminary initial cooling of rocks in a construction site. Let us consider the general case of change from Biot number to thermal resistance calculation. The formula for changing is of the form:

$$R_{eq} = \frac{\delta_{eq}}{\lambda_{eq}} = \frac{R_0}{\lambda_{\bullet} \cdot Bi} - \frac{1}{\alpha_0} - \frac{1}{\alpha_1} \quad (14)$$

where δ_{eq} — equivalent thickness of thermal protection structure, m; λ_{eq} — equivalent coefficient of protective structure heat conductivity, W/m²·K; α_1 - heat-exchange coefficient from air in a construction to its internal surface, W/m²·K; α_0 - heat-exchange coefficient from air in the opening to an external surface of a construction, W/m²·K.

Values of α_0 for the contacting constructions are equal to infinity. The heat-exchange coefficient from air to the surface in a construction consists of convective and radiation components:

$$\alpha_1 = \alpha_{rad} + \alpha_{con}, \quad (15)$$

which are determined by the formulas obtained on the basis of the qualitative and quantitative analysis of the calculating dependences used for calculation of heat exchange intensity in residential buildings [7]:

$$\alpha_{con} \approx 4 + 0{,}05\left(t_{\text{air}} - \Delta t_{\text{air}}\right) \quad (16)$$

$$\alpha_{rad} \approx 1{,}66\sqrt{\Delta t_{\text{air}}} \quad (17)$$

where Δt_{air} — admissible difference of temperatures between surface temperature and air temperature in a construction, °C.

The Δt_{air} parameter is specified and should not be higher than 6 °C according to [7], therefore it is necessary to base upon this value in practical calculations. Thus, for evaluating calculations, it is possible to assume the following formula of determination of heat-exchange coefficient value:

$$\alpha_1 \approx 7{,}8 + 0{,}05t_{air} \quad (18)$$

For a case of plane wall, the ratio of δ_{eq}/λ_{eq} consists of the simple sum of thermal resistance of separate layers:

$$R_{eq} = \frac{\delta_{eq}}{\lambda_{eq}} = \sum_{i=0}^{n} \frac{\delta_i}{\lambda_i} \quad (19)$$

where δ_i — thickness of i-th layer of a protective structure, m; λ_i — thermal conductivity coefficient of i-th layer, W/m·K.

For a case of a cylindrical wall, the value of equivalent thermal resistance can be determined by a formula:

$$R_{eq} = \frac{R_0}{\lambda_{eq}} \ell n\left(1 + \delta_{eq} / R_0\right) \quad (20)$$

For a cylindrical design with an internal radius, the expression (20) can be written down in the form:

$$R_{eq} = \sum_{i=1}^{n} \frac{R_0}{\lambda_i} \ell n \frac{R_0 + \sum_{i=1}^{n} \delta_i}{R_0 + \sum_{i=1}^{n-1} \delta_i} \quad (21)$$

For a single-layer design of cylindrical symmetry:

$$R_{eq} = \frac{R_0}{\lambda_{in}} \ell n \frac{R_0 + \delta_{in}}{R_0} \tag{22}$$

In practical calculations, there is an interest of finding of thickness of
properly heat-insulating material layer at specified general thickness of a construction (δ_c) without isolation and properties of insulating material. In this case, calculation should be conducted by a formula:

$$\delta_{in} = \frac{R_0\left(1 + \frac{\delta_{eq}}{R_0}\right)\lambda_{in} / \lambda_{eq}}{\prod_{i=2}^{n} \frac{R_0 + \sum_{i=1}^{n} \delta_i}{R_0 + \sum_{i=2}^{n-1} \delta_i}} \tag{23}$$

where $$\lambda_{eq} = \frac{R_0}{R_{eq}} \ell n \left(1 + \frac{\delta_{eq}}{R_0}\right) \tag{24}$$

As δ_{eq} is dependent on δ_{in}, the calculation must to be conducted according to the iterative scheme, i.e. when setting of δ_{eq}, to determine δ_{in}, then to find the specified δ_{eq} by a formula:

$$\delta_{eq} = \sum_{i=2}^{n} \delta_i + \delta_{in} \tag{25}$$

and to repeat calculation again up to achievement of the specified accuracy in determination of δ_{in}. Evaluating calculations show that iterative process coincides quickly enough if δ_{eq} is chosen as initial value calculated from expression (19) by a formula for a plane wall.

The selected construction thermal resistance must be checked for a condition of condensate formation on an internal surface. It is of the form:

$$t_{IC} / t_{DP} > 1 \tag{26}$$

where t_{IC} — temperature of walls surface in a construction, °C; t_{DP} — dew-point temperature, °C.

According to [7], it is possible to write down:

$$t_{DP} = 20{,}1 - \left(5{,}75 - 0{,}00206 E \varphi_{air}\right)^2 \tag{27}$$

where $$E = 477 + 133{,}3\left(1 + 0{,}14 t_{air}\right)^2 , \tag{28}$$

φ_{air} — relative humidity of air, unit fraction.

Temperature of a construction wall can be determined by a formula:

$$t_{IC} = \frac{\left(T_{air} / R_{EF}\right) + \alpha_1 t_{air}}{\alpha_1 + 1 / R_{EF}} \tag{29}$$

If the condition (26) is satisfied, condensate on walls will not be formed. Otherwise, it is necessary to increase the thermal resistance of a construction.

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