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The analysis of manufacturing errors effect on contact stresses distribution under the ring parts deformed asymmetrically

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Abstract
The analytical and numerical analytic methods using the expansion of simple layer for the problem solution of die with the basis in the form of ring deformed asymmetrically are suggested. The close agreement of results is achieved when calculations carrying out by two methods. The influence of ellipse eccentricity, centers shifts and roughness coefficient on the contact pressure distribution is investigated.
Key words: SPATIAL CONTACT PROBLEM, EXPANSION, DECOMPOSITION OF SIMPLE LAYER POTENTIAL, SMALL PARAMETER METHOD, CONTACT PRESSURE, DEFORMED RING

Introduction

The role of calculations on durability and durability of details of cars increases in mechanical engineering. The role of calculations of machine elements strength and durability is increased in machine building with the increase of requirements to accuracy and productivity. At the present time, the contact problems solution approach, which gives the opportunity to estimate the efficiency of individual design solutions and the production accuracy of contacting elements in each specific case, is required. The contact elastic displacements constitute a considerable part of the engineering constructions elastic displacements balance [1]. The contact displacement is up to 50% in turning and spindle units, and 80-90% in the overhands and support stands of the machine [2]. It is necessary to find out the distribution and concentration of pressure on a contact surface considering the deviations from the regular geometrical shape, deformations, production errors and influence of the aggressive medium, which changes the sizes of constructions details, and also to make recommendations about sizes optimum ratios selection.

The ring machines elements, which are of ellipsis form and deviate from the concentricity, are modelled by the asymmetrical ring die pressed into an elastic half-space. The consideration of roughness, friction and initial stress [1, 3] makes the model more adequate to real contact conditions.

The problem of a ring die is one of the relevant ones in the contact problems theory. A number of works are devoted to it, and papers [4-6] should be noted. In papers [7, 8], the method of problems solution of pressing-in of the doubly-connected die, which is close to the ring, with the use of a simple layer potential, is developed. However, the problems of an asymmetrical ring die were practically not considered. Rather wide range of various problems can be solved by modern numerical methods. The work objective is finding of the analytical and numerical analytic solution of contact pressure determination problems, turning angles and deepening of asymmetrically deformed ring die, using expansion of a simple layer potential.

Problem statement

Let us consider a spatial problem about pressing into a uniform elastic half-space of the rigid die of the form of nonconcentric ring in the plan. The die is affected by the vertical force Q, which line of action passes through the beginning of coordinates system, and the direction coincides with the direction of an axis z. The main integral equation of the problem is of the form [3, 4]

$$\delta + \beta_1 \rho_0 \sin\theta_0 - \beta_2 \rho_0 \cos\theta_0 = \frac{1-\nu}{2\pi G} \iint_S \frac{p(\rho,\theta)}{r} \mathrm{d}S \qquad (1)$$

where δ – die deepening; β_1, β_2 – turn vector projections; G – shear modulus; ν – Poisson's constant; $p(\rho,\theta)$ – unknown function of normal pressure distribution; $r^2 = \rho^2 + \rho_0^2 - 2\rho_0 \cos(\theta - \theta_0)$, (ρ,θ) – polar coordinates; $(\rho_0,\theta_0) \in S$ S– the contact area, which is of the form of nonconcentric deformed ring

$$S = \{\rho,\theta : \rho_{\Gamma_1} = a(1+f(\varepsilon,\theta)) \le \rho \le \rho_{\Gamma_2} = b(1+f(\varepsilon,\theta)),$$
$$0 \le \theta \le 2\pi\}$$

$a < b$ – the constant rings characterizing the width; $\varepsilon < 1$ – the small parameter characterizing a deviation form from a circular ring;

$$f(\varepsilon,\theta) = \sum_{i=1}^{\infty} \varepsilon^i f_i(\theta)$$

single-valued continuously differentiable function.

The unknown values δ, β_1, β_2 are determined from the die balance equation

$$Q = \iint_S p(\rho,\theta) dS, \qquad M_1 = \iint_S \rho \sin\theta \cdot p(\rho,\theta) dS\ ,$$

$$M_2 = \iint_S \rho \cos\theta \cdot p(\rho,\theta) dS \qquad (2)$$

where Q, M_1, M_2– the main vector and the main moments of forces applied to the die.

Analytical problem solution

Let us consider a case, when the area S is limited by two lines Γ_1 and Γ_2 representing the nonconcentric deformed circles with the equations in polar system of coordinates $\rho_{\Gamma_1} = a(1+\varepsilon\cos\theta)$, $\rho_{\Gamma_2} = b(1+\varepsilon\cos\theta)$.

It is supposed that the distribution function of normal pressure can be presented as the series in terms of powers of ε:

$$p(\rho,\theta) = \sum_{k=0}^{\infty} \varepsilon^k p_k(\rho,\theta)$$

Unknown δ, β_1 and β_2 are also presented as the series in terms of powers of ε.

Let us apply one-to-one and continuously differentiable transformation of variables

$$\rho = R[1+f(\varepsilon,\varphi)], \quad \varphi=\theta \quad f(\varepsilon,\varphi)=\sum_{i=1}^{\infty}\varepsilon^i f_i(\varphi) \tag{3}$$

which transfers the area S to a circular ring $\Omega = \{(R,\varphi): a \le R \le b,\ \ 0 \le \varphi \le 2\pi\}$.

In this case, functions $f_i(\theta)$ in the expressions (3) of borders of area S are of the form: $f_1(\theta)=\cos(\theta)$ $\quad f_k(\theta)=0, \quad k \ne 1$.

Representing density of a simple layer potential considering the transformation of variables (3) in a form

$$p(\rho(R,\varphi,\varepsilon),\varphi) = \sum_{i=0}^{\infty} P_i(R,\varphi)\varepsilon^i \tag{4}$$

the expansion of potential in the series in terms of powers of ε, found in papers [7-9] is possible.

After that, let us present the integrals in the equations of balance in the form of the series in terms of powers of ε [7]. Comparing the expressions at identical powers ε in the equations (1), (2), we obtain the recurrent systems similar to them for determination of $P_k(\rho,\theta)$, δ_k, β_{1k}, β_{2k}, which include the integrals distributed on a circular ring.

Let us write down normal pressure limited by first power ε, in decomposition (4), in a case, when $b=2a$. We obtain that pressure under an asymmetrical ring die in points (ρ,θ_*) of the beam emerging from the center of coordinates and crossing external and internal contours of contact area in points $(\rho_{\Gamma_1},\theta_*)$ and $(\rho_{\Gamma_2},\theta_*)$ respectively, is defined by dependence

$$p(\rho,\theta_*) = P_0 + \varepsilon P_1 \quad , \tag{5}$$

where

$$P_0 = \frac{Q}{2\pi b^2}\sigma_0 \quad ;$$

$$P_1 = \frac{Q}{2\pi b^2}[2\sigma_0 + (0{,}4658\sigma_0 + 0{,}9873\sigma_1)\cos\theta_*] \quad ;$$

$$\sigma_i = \frac{a}{2}\gamma\sum_{k,p=0}^{\infty}\left[\alpha_{pk}^{(i)}\left(\frac{\rho}{\rho_{\Gamma 2}}\right)^{2k} + \beta_{pk}^{(i)}\left(\frac{\rho_{\Gamma 1}}{\rho}\right)^{2k+3}\right]\left(\frac{a}{b}\right)^{p}.$$

Coefficients, $\alpha_{pk}^{(i)}$, $\beta_{pk}^{(i)}$ $(i=0,1,2...)$ are determined for any values of indexes from recurrence equations with the initial data [9].

The problem numerical analytic solution

Let us reduce the equation (1) to the form, where various approximate methods under certain conditions are applicable. Let us introduce the regularizing parameter B, which can be compared with the roughness coefficient of elastic half-space [3, 10]. When $B=0$, we obtain the problem of die pressing into a smooth half-space. Let us consider the numerical analytic solution of regularizing control (1), which can be considered as the main integrated equation of a problem of rigid die pressing into a rough elastic half-space [3]:

$$B\cdot p(\rho_0,\theta_0) + \frac{1-\nu^2}{\pi E}\cdot\iint_S \frac{p(\rho,\theta)}{r(\rho_0,\theta_0,\rho,\theta)}dS = g(\rho_0,\theta_0) \tag{6}$$

where

$$g(\rho,\theta) = \delta + \beta_1\rho\sin\theta - \beta_2\rho\cos\theta$$

Similarly to the above and the papers [8, 10], we obtain that the integral equation (6) of the problem and the equation of balance will pass into recurrent systems similar to them for expansion of

$P_k(R,\varphi)$, δ_k, β_{1k}, β_{2k},

where there are integrals, which are widespread on the area only in the form of a circular ring. Let us write down the recurrent system, into which the equation (6) will pass after the use of potential decomposition [11]

$$BP_k(R_0,\varphi_0) + \frac{1-\nu^2}{\pi E}\left[\iint_\Omega \frac{P_k(R,\varphi)}{r(R,R_0)}d\Omega + \Phi_k(P_0,P_1,\ldots,P_{k-1})\right] = G_k(R_0,\varphi_0) \tag{7}$$

Thus, the task for the area S, which is close to a circular ring, is reduced to the recurrent sequence of similar problems for the area Ω limited by circles ρ=a and ρ=b.

Let us introduce the values: $B_1 = \dfrac{B\pi E}{(1-\nu^2)\cdot b}$;

$P(\rho,\theta) = \dfrac{1-\nu^2}{\pi E}\cdot p(\rho,\theta)$ [3]; $\dfrac{B_1}{2\pi} = 1-\alpha \quad 0<\alpha<1$ [11].

The solution of the obtained systems (7) can be found by numerical methods, which converge to the solution when performing of Fredholm conditions

$\frac{2\pi}{B_1} < \frac{1}{4bK(a/b)}$, $K(a/b)$ –full elliptic integral of the first kind. In the case of small B_1, weak roughness, the replacement $B_1/(2\pi) = 1-\alpha$, where $0<\alpha<1$ [11] is suggested, and at values α, which are close to 1 the figure of one, we obtain the approximate solution of the equation (1). In that case, the equation (6) is reduced to the form

$$(1-\alpha)\cdot P(\rho_0,\theta_0) + \iint_S \frac{P(\rho,\theta)}{2\pi b\cdot r(\rho_0,\theta_0,\rho,\theta)} dS =$$
$$= \frac{\delta - \beta_2\rho_0\cos\theta_0 + \beta_1\rho_0\sin\theta_0}{2\pi b}. \qquad (8)$$

Using the calculations of simple layer potentials for asymmetrical distribution of potential density [8] and after transformations of the similar in the work [11], we obtain the system of the one-dimensional integrated equations in a circular ring. The obtained one-dimensional integrated equations are replaced with the system of the algebraic equations using quadrature formulas, and they are solved by approximation method.

The description of the results of the numerical research

Calculations were conducted for the dimensionless quantities. The value P/P* where $p^* = \frac{Q}{2\pi b^2}$ was considered for functional relation of normal pressures. Accordingly, the equations of the inner and outer boundaries of the contact area *S* are of the form

$$\rho_{\Gamma_1} = a\sqrt{1-\varepsilon_1^2}\,/\sqrt{1-\varepsilon_1^2\cos^2\theta};$$

$$\rho_{\Gamma_2} = b\sqrt{1-\varepsilon_1^2}\,/\sqrt{1-\varepsilon_1^2\cos^2\theta}$$

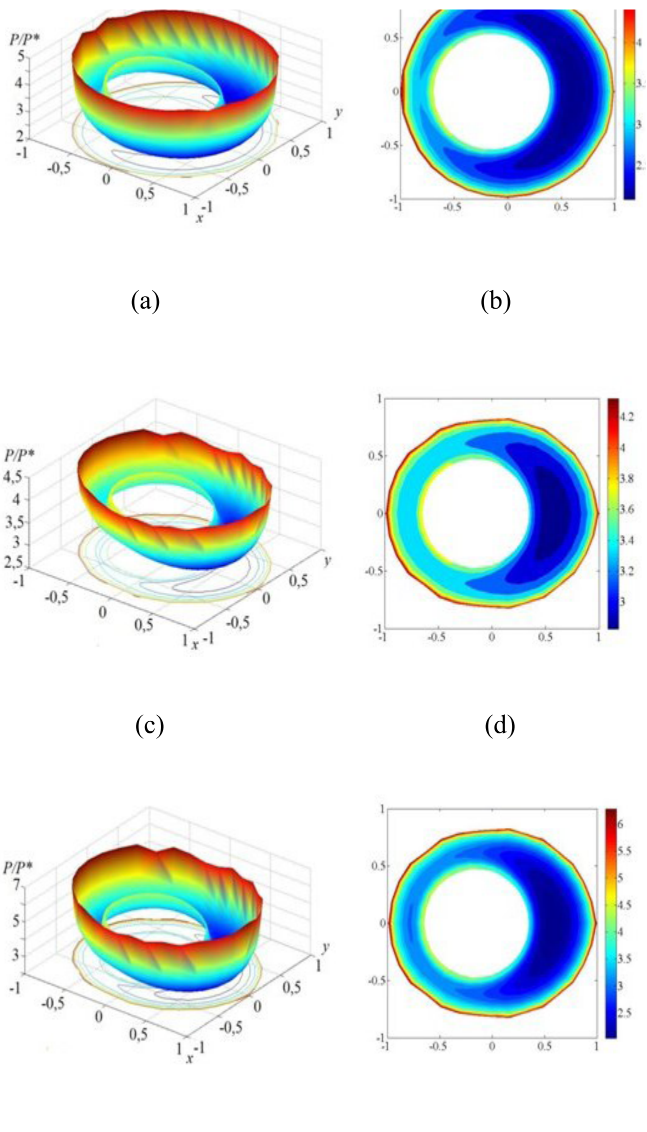
where ε_1 - an eccentricity of the ellipse, $\varepsilon_1^2 = 1-a_1^2/a^2 = 1-b_1^2/b^2$, α, *b* – focal, a_1, b_1 – minor axes of the ellipses.

Fig. 1 shows the surface and distribution of normal pressure P/P* on the contact area for the following values of the dimensionless parameters: a, b - $\alpha = 0{,}999$; $\varepsilon_1 = 0$; $\varepsilon_2 = 0{,}15$ – displacement of the center of the inner boundary against the center of the outer boundary along an axis Ox; c, d – α = 0,65 $\varepsilon_1 = 0{,}3$; $\varepsilon_2 = 0{,}15$; e, f – α = 0,999; $\varepsilon_1 = 0{,}3$; $\varepsilon_2 = 0{,}15$; g, h – $\alpha = 0{,}9$; $\varepsilon_1 = 0{,}6$, $\varepsilon_2 = 0{,}24$.

The results shown in Figure 1 a and b were obtained in two ways: by formula (5) when $\varepsilon = 0{,}3$ using analytical method and numerically according to formula (8) by a numerical analytic method. With the same values, the discrepancy was observed in the third decimal place only that confirms the adequacy of the developed methods.

As is seen from Fig. 1, the offset of the centers introduces a significant asymmetry in the distribution of normal pressure on the contact area; and a larger displacement results in a greater difference between the lowest and highest values of the normal pressure in the contact area, which should be considered in the calculations on the contact strength.

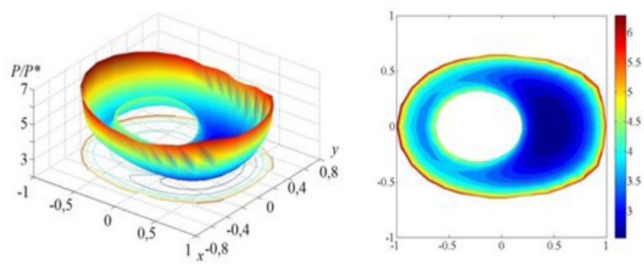
The lines of equal pressure closes near the points of minimum pressure as the distance to the boundaries takes the form of curves similar to the contact area boundaries.



(a) (b)

(c) (d)

(e) (f)



(g) (h)

Figure 1. Distribution of pressure on contact area

Points with a minimum pressure are located closer to the inner boundary. The surface of a hollow pressure rises at the approach to the external border. The points at the lowest pressure values are located on the axis Ox, along which the centers were displaced.

The normal pressures take the highest values on the outer boundary of the contact. However, keeping the surface roughness leads to a finite values at the boundaries of the contact area, while the problems for perfectly smooth contact pressure value increases indefinitely when approaching the limits of the field.

Comparison of Fig. 1 a-h shows that increasing of the eccentricity of a fixed value of the major axis of the ellipse increases the pressure at all the points of the contact area. Due to the fact that an increase in the eccentricity means less contact area, this conclusion is consistent with the existing general concepts of contact interaction.

In the Fig. 2, the diagrams of distribution of dimensionless pressure p/p^* along an axis Ox for different values of roughness coefficient $\alpha = 0{,}999;\ 0{,}850;\ 0{,}650$ respectively are given.

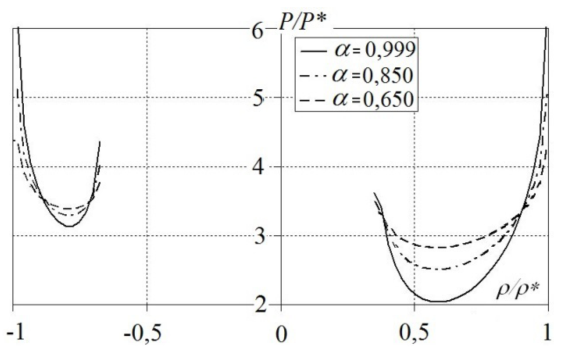


Figure 2. The pressure distribution along the axis Ox

The given numerical examples show that the value of normal pressure on borders of the contact area is reduced with the increase of roughness coefficient, and it rises in the middle, that confirms the conclusions drawn in the papers [4, 10, 11].

Conclusions

According to the methods developed in this work, numerical calculations of concrete examples for research of influence of a ring width, eccentricity ε_1, centers shift ε_2 of a ring borders and roughness coefficient on distribution of normal pressure under the presenting of cylindrical die, which is nonconcentric elliptic ring in the plan, were carried out.

The analytical and approximate numerical analytic solution of a die pressing-in problem in a smooth half-space was obtained. For this problem, the function of normal pressure has a rupture of the second kind in the border points. The solution of problem considering a roughness at values B_1, which are close to zero, comes closer to the solution of a problem for smooth contact. The pressure on borders of contact area grows with reduction of roughness coefficient values, but keeps final. Thus, the result of normal pressure extremity on contact area borders is confirmed when considering of a roughness of an elastic half-space. Due to the roughness distribution of pressure, it becomes more even, the greatest value decreases and the smallest increases.

The methods suggested by authors for the solution of problems on pressing into elastic as well as rough half-space for doubly-connected dies are possible to be applied for dies of more difficult configuration and with consideration of friction, coupling, adhesion. Research on determination of contact pressure under asymmetrically deformed ring die with and without consideration of roughness is conducted in this paper.

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