Tumor in Ultrasound Image. *Biomedical Signal Processing and Control*, 8(6), p.p.688-696.
7. Mahnaz Etehadtavakol, E.Y.K. Ng, Vinod Chandran, et al. (2013) Separable and Non-separable Discrete Wavelet Transform Based Texture Features and Image Classification of Breast Thermograms. *Infrared Physics & Technology*, 61(11), p.p.274-286.
8. Yasmin, M. Sharif, I. Irum, S. Mohsin (2014) An Efficient Content Based Image Retrieval using EI Classification and Color Features. *Journal of Applied Research and Technology*, 12(5), p.p.877-885.
9. Ketan Tamboli, Sunny Patel, P.M. George, Rajesh Sanghvi (2014) Optimal Design of a Heavy Duty Helical Gear Pair Using Particle Swarm Optimization Technique. *Procedia Technology*, 14, p.p.513-519.
10. Amer Fahmy, Tarek M. Hassan, Hesham Bassioni (2014) Improving RCPSP Solutions Quality with Stacking Justification- Application with Particle Swarm Optimization. *Expert Systems with Applications*, 41(13), p.p.5870-5881.
11. Susmita Mall, S. Chakraverty (2014) Chebyshev Neural Network Based Model for Solving Lane–Emden Type Equations. *Applied Mathematics and Computation*, 247(15), p.p.100-114.
12. Tomás Rodríguez García, Nicoletta González Cancelas, Francisco Soler-Flores (2014) The Artificial Neural Networks to Obtain Port Planning Parameters. *Procedia-Social and Behavioral Sciences*, 162(19), p.p.168-177.
13. Amir Hossein Zaji, Hossein Bonakdari (2014) Performance Evaluation of Two Different Neural Network and Particle Swarm Optimization Methods for Prediction of Discharge Capacity of Modified Triangular Side Weirs. *Flow Measurement and Instrumentation*, 40(12), p.p.149-156.
14. Buse Melis Ozyildirim, Mutlu Avci (2014) Logarithmic Learning for Generalized Classifier Neural Network. *Neural Networks*, 60(12), p.p.133-140.

Trajectory Algorithm for Underwater Target based on Kalman Filter

Peng Wang

School of Electronic Information Engineering, Xi'an Technological University, Xi'an 710021, PR China

Abstract

To improve the accuracy of identification and the damage efficiency for underwater target, this paper researched the processing algorithm of moving trajectory for underwater target. This paper ameliorated the conventional Kalman filter algorithm in order to obtain true moving trajectory of underwater target, adopted state space description method and recursive algorithm to establish the signal's nonlinear state equation for the motion model of target, used the Unscented Kalman Filter (UKF) method to get rid of the influence of noise and then to get more accurate estimated value of the target location. The results of simulation calculation from multi–sensor data fusion show the real-time tracking data had high

accuracy, and then the tracking ability was very powerful. Multi-sensor data fusion can effectively reduce the error and achieve the purpose of rapid convergence. The results of simulation calculation verify the correctness and effectiveness of this method.

Key words: UNDERWATER TARGET, MOVING TRAJECTORY, KALMAN FILTER

1. Introduction

The detection and moving trajectory of underwater target is a scientific problem in the field of navigation [1-2]. Due to the complexity of the submarine environment, the refraction of the light and the characteristics of target, these factors affect of the detection result for underwater target [3]. Conventional processing algorithm is very difficult to detect or judge the moving trajectory of the underwater target, but the performance and stability of the underwater guided weapon is determined via the moving trajectory of the underwater target [4-5]. Therefore, the research on the processing algorithm of moving trajectory from underwater target is useful to improve the accuracy of identification and the damage efficiency for underwater target, which has very high application value.

On the basis of the output signals from the underwater sensors, the previous researchers mainly use the conventional Kalman filter method to deal with the signal of underwater target [6], but the conventional Kalman filter method can only be applied to stationary vector signal and noise signal. Mainly because the conventional Kalman filter method adopts the least mean square error estimate, the dynamic or state equation to describe the noise in the output signal of underwater sensor, then the real target information is obtained. This processing method of the target signal in complex environment is difficult to achieve satisfactory filtering result [7-8]. In order to actually reflect the moving trajectory of the underwater target, the conventional Kalman filter algorithm is needed to improve.

2. The estimation algorithm of actual moving trajectory for underwater target

On the basis of the Unscented Kalman Filter (UKF) algorithm, the target tracking was carried out via using the filtered signals. Assuming that the velocity of target was constant, with the influence of the seawater, the disturbance form slight velocity correction could be considered as a noise input [9], so when the target moved at the n moment, the velocity components in the X and Y direction were shown in formula (1):

$$\begin{aligned} v_x[n] &= v_x[n-1] + z_x[n] \\ v_y[n] &= v_y[n-1] + z_y[n] \end{aligned} \quad (1)$$

If the turbulent noise $z_x[n]$ and $z_y[n]$ did not exsit, the value of speed would be constant. Then It could be seen that the underwater target did the linear movement, which could be seen with dotted line in Fig.1. According to the equation of motion, the position at the *n* moment were shown in formula (2):

$$\begin{aligned} r_x[n] &= r_x[n-1] + v_x[n-1]t \\ r_y[n] &= r_y[n-1] + v_y[n-1]t \end{aligned} \quad (2)$$

In (2), t was time interval between the samples. In the discrete model of equation of motion, the underwater target moved at the speed of the preceding moment, and then the speed suddenly changed at the next moment, which was an approximation to the true continuous motion. The wanted signal vector was composed of position and velocity components, the wanted signal is shown as:

$$s[n] = \begin{bmatrix} r_x(\mathrm{n}) \\ r_y(\mathrm{n}) \\ v_x(\mathrm{n}) \\ v_y(\mathrm{n}) \end{bmatrix} \quad (3)$$

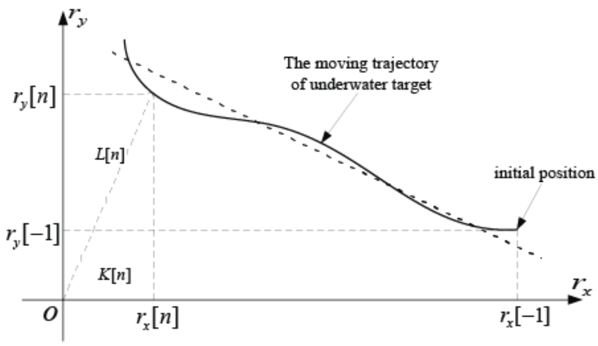


Figure 1. Typical trajectory in given direction of the underwater target with constant velocity motion

From formula (1) and (2), it could be seen that the *s[n]* met the following expressions.

$$\underbrace{\begin{bmatrix} r_x[n] \\ r_y[n] \\ v_x[n] \\ v_y[n] \end{bmatrix}}_{s[n]} = \underbrace{\begin{bmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} r_x[n-1] \\ r_y[n-1] \\ v_x[n-1] \\ v_y[n-1] \end{bmatrix}}_{s[n-1]} \underbrace{\begin{bmatrix} 0 \\ 0 \\ z_x[n] \\ z_y[n] \end{bmatrix}}_{z[n]} \quad (4)$$

The observation noise of distance and direction were shown in (5):

$$\hat{L}[n] = L[n] + w_L[n]$$
$$\hat{K}[n] = K[n] + w_K[n] \tag{5}$$

Where, $L[n] = \sqrt{r_x^2[n] + r_y^2[n]}$, $K[n] = \arctan \frac{r_y[n]}{r_x[n]}$.

According to the normal format, the observation equation of formula (4) could be shown:

$x[n] = \ell(s[n]) + w[n]$, where ℓ was a function and its expression was shown:. $\ell(s[n]) = \begin{bmatrix} L[n] \\ K[n] \end{bmatrix}$.

The Jacobi matrix could be obtained via the differentiation of observation equation:

$$\frac{\partial \ell}{\partial s[n]} = \begin{bmatrix} \frac{r_x[n]}{L[n]} & \frac{r_y[n]}{L[n]} & 0 & 0 \\ \frac{-r_y[n]}{L^2[n]} & \frac{r_x[n]}{L^2[n]} & 0 & 0 \end{bmatrix} \tag{6}$$

Finally, the covariance of the driving noise and the observation noise were appointed. If assuming that the changing extent of seawater and velocity correction in any direction was same, then the same value of variance σ^2 was given to $z_x[n]$ and $z_y[n]$, and they were independent, both of them were reasonable. Then there was the following expression.

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & \sigma^2 \end{bmatrix} \tag{7}$$

Due to $z_x[n] = v_x[n] - v_x[n-1]$, the use of accurate σ^2 depended on the variation of the velocity components from the sample to the sample, which just was the t times of acceleration, and it could be deduced from the navigation physics. When the variance of measurement noise was appointed, it was required to pay attention that measurement error could be seen as the estimation error of $\hat{L}[n]$ and $\hat{K}[n]$. Usually it was assumed that the estimation error $w_L[n]$ and $w_K[n]$ were zero mean, then the variance of w_R[n] might satisfy the expression: $E(w_L^2[n]) = E[(\hat{L}[n] - L[n])^2]$. Sometimes the variance could be derived, but in most cases it could not. The assumption, that the relation between $E(w_L^2[n])$ and the *PDF* of $L = [n]$ was irrelevance, was one possibility, so $E(w_L^2[n]) = E[(\hat{L}[n] - L[n])^2 \left| L[n] \right]$. Therefore $L[n]$ could be considered as a deterministic parameter, so that the variance of w_R[n] was just the variance of classical estimator. If $\hat{L} = [n]$ is *MLE*, then the assumption of long data record and high *SNR* could suppose that the variance reached *CRLB*. Using this method, the variance of distance and direction could be set according to *CRLB*. For uncomplicated, it was usually assumed that the estimation error was independent and the variance was time-invariance. Therefore, there was the following expression.

$$H[N] = H = \begin{bmatrix} \sigma_L^2 & 0 \\ 0 & \sigma_K^2 \end{bmatrix} \tag{8}$$

3. The tracking algorithm of target based on Unscented Kalman Filter

The essence of Kalman filter was that the state vector of system was reconstructed via measured value. Random errors was eliminated according to the measured values of the system and sequential recursion by “prediction - measurement – correction”, then the status of system was reappeared. The state equation of the conventional Kalman filter equation was too complex and the real-time performance was poor, so this paper used the *UKF* method. Based on Kalman filter, the *UKF* utilized probability density distribution to approximately describe of Kalman equation, namely, the *UKF* adopted sigma dot to weigh the sample mean value and covariance, and could reduce sharply the amount of calculation based on the guarantee of measurement accuracy, and the *UKF* had higher anti-interference ability.

Quantisty of state interfered by noise was a random quantity, then the exact value of this random quantity could not be accurately measured, but we could carry on a series of observations on these random quantities. Based on a set of observations, Kalman put forward the theory of recursive optimal estimation, which adopted state space description method and used recursive algorithm. The signal's nonlinear state equation was established for the motion model of target: $X[(n+1)/k] = f[X(n/n), U(n), n] + W(n)$ and measurement equation $Y(n) = h[X(n/n), n] + V(n)$, and the quantity of state X satisfied the condition: $X \in R^L$; the input U satisfied the condition: $U \in R^n$; the state noise W satisfied the condition: $W \in (0, P_k)$; the measurement noise V satisfied the condition: $V \in (0, Q_k)$.

First, the parameter mean value and the covariance were initialized: $X^a = [x^T, v^T, n^T]^T$; $\overline{X}_0 = E(\overline{X}_0)$; $P_0 = E[(X_0 - \overline{X}_0)(X_0 - \overline{X}_0)^T]$.

Noise parameter was added in the initialization process: $\overline{X}_0^a = E\begin{bmatrix} \overline{X}_0^T & 0 & 0 \end{bmatrix}$, $P_0^a = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{bmatrix}$.

An important factor that influenced the result of the Kalman filter was the sampling method used by sigma dot. Let $X_i = \overline{X} \pm \sqrt{(k_x + \lambda) P_x}$, sampling weights W_i^m satisfied the condition: $W_i^m = W_i^c = 1/\{2(k_x + \lambda)\}$, and λ was scale parameter. The distance between the sigma dot and the mean dot was adjusted. The sigma dot set was obtained by nonlinear transformation: $X_i^a(n/n) = g(x_i), i = 0,...,2k_x$. The transformation sigma dot set was carried out weighted processing, and then the predicted mean value of the sigma sampling points was calculated, the weight was W_i.

$$\bar{X}_i^x[(n+1)/n] = \sum_0^{2k_x} W_i^m X_i^x[(n+1)/n], i = 0,...,2k_x \quad (9)$$

the quantisty of state X contained a variety of information, such as target coordinates, speed and so on. The weighted processing could be adjusted according to the actual situation in order to adapt to the requirements of different measurement systems. But the measured value on position, velocity, and acceleration of target often were mixed up with noise at any time. The Kalman filter used the dynamic information of the target to get rid of the influence of noise and get a good estimated value of the target location. The predicted covariance of sigma sampling points could be calculated by (10).

$$P\left[(n+1)/n\right] = \sum_0^{2k_x} W_i^c \left[X_i^x\left[(n+1)/n\right] - \bar{X}_i^x\left[(n+1)/n\right]\right]\left[X_i^x\left[(n+1)/n\right] - \bar{X}_i^x\left[(n+1)/n\right]\right]^T + P_n, i = 0,...,2k_x \quad (10)$$

Let $Z_i\left[(n+1)/n\right] = h[X_i^x(n/n), U(n), X_i^w(n+1)]$ was the predictive measurement sampling equation, the mean value and covariance of the measuring point could be predicted by the predictive measurement sampling equation:

$$\bar{Z}_i[(n+1)/n] = \sum_0^{2k_x} W_i^m Z_i[(n+1)/n] \quad (11)$$

$$P_{mm}\left[(n+1)/n\right] = \sum_0^{2k_x} W_i^c \left[Z_i\left[(n+1)/n\right] - \bar{Z}_i\left[(n+1)/n\right]\right]\left[Z_i\left[(n+1)/n\right] - \bar{Z}_i\left[(n+1)/n\right]\right]^T + \kappa Q_n \quad (12)$$

$$P_{mo}\left[(n+1)/n\right] = \sum_0^{2k_x} W_i^c [X_i^x[(n+1)/n] - \bar{X}_i^x[(n+1)/n]][Z_i[(n+1)/n] - \bar{Z}_i[(n+1)/n]]^T \quad (13)$$

Where, $i = 0,...,2k_x$. The state estimation was an important part of Kalman filter. According to the multi-source data, the quantitative estimation of the random quantity was an estimation problem, especially, the state estimation of the dynamic target could achieve the estimation and prediction function of real-time running state.

The state estimation was very important for understanding and controlling a target system[10]. The renewed mean value and variance and Kalman gain could be calculated:

$$W[(n+1)/n] = P_{mo}[(n+1)/n]P_{mm}^{-1}[(n+1)/n] \quad (14)$$

$$\bar{X}(n+1/n+1) = \bar{X}[(n+1)/n] + W(n+1)[Z(n+1) - \bar{Z}[(n+1)/n]] \quad (15)$$

$$P(n+1/n+1) = P[(n+1)/n] - W(n+1)P_{mm}[(n+1)/k]W^T(n+1) \quad (16)$$

Through the algorithm establishment of the *UKF*, it could be seen that the algorithm did not need the system equation and predictive system function which were requirements of the ordinary Kalman filter algorithm, and then the amount of calculation could be greatly reduced. Moreover, the covariance of the algorithm had high degree of freedom, which could effectively reduce the systematic error.

4. Simulation calculation and analysis

According to the above calculation algorithm, the extended Kalman filter was used to process the output signal of the sensor for underwater target, it was shown in Fig.2. At first, because of the position estimation error, the signal error was relatively large. However, the value of extended Kalman filter was close to the value of true trajectory after more than 20 samples. In addition, it would be noticed that the minimum of *MSE* was not monotonically decreasing; conversely, it was increasing in some time. This could be explained by the contrast between Kalman filter and sequential *LMMSE*. For the latter, although the received data was more and more, the estimations were the same parameters. And when the new data samples were received, the new parameters were estimated. Due to the input influence of the drive noise, the uncertainty of new parameters were increased, which could be possible large enough to offset the information obtained from the observation of the new data sample, and it could make the minimum of *MSE* to increase.

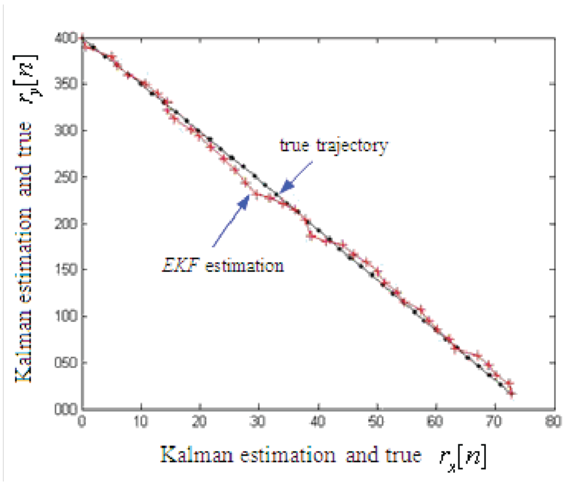
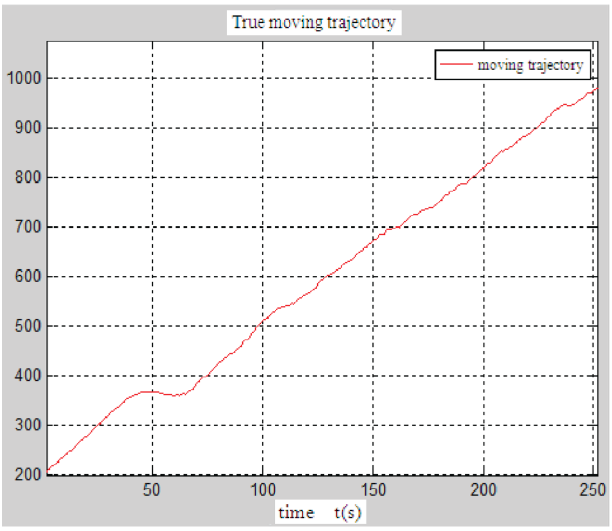
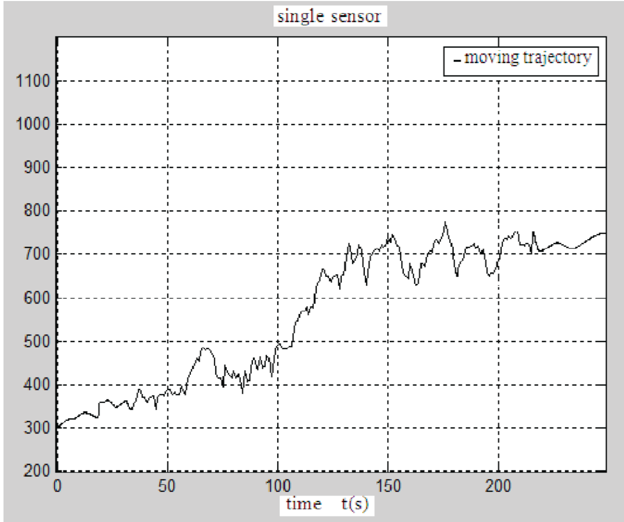


Figure 2. Extended Kalman filter estimation trajectory and the true trajectory of the underwater target

The Fig.3 (a) shown the true moving trajectory of target when the time is in 0~250s, it could be seen that the target had a maneuver near 50s. The overall trajectory of the target was stable. The curve in Fig.3 (b) was recurrent moving trajectory of target measured via a single sensor. And the recurrent moving trajectory of target was smooth before 50s, when target had a maneuver near 50s, the output of system arose severe wobble and the errors of signal was large. Meanwhile, the output error was difficult to converge, the system became unstable. The curve in Fig.3 (c) was recurrent moving trajectory of target measured via multi–sensor. It could be seen that the moving trajectory of target, which was filtered via *EKF* method and output via multi–sensor, was more stable than the moving trajectory of target measured via a single sensor. Especially, when there was a maneuver from target, the output did not arise severe wobble.



(a)



(b)

Figure 3. The contrast among the true moving trajectory, the moving trajectories measured via single sensor and multi–sensor

The Fig.3 (a) shown the true moving trajectory of target when the time is in 0~250s, it could be seen that the target had a maneuver near 50s. The overall trajectory of the target was stable. The curve in Fig.3 (b) was recurrent moving trajectory of target measured via a single sensor. And the recurrent moving trajectory of target was smooth before 50s, when target had a maneuver near 50s, the output of system arose severe wobble and the errors of signal was large. Meanwhile, the output error was difficult to converge, the system became unstable. The curve in Fig.3 (c) was recurrent moving trajectory of target measured via multi–sensor. It could be seen that the moving trajectory of target, which was filtered via *EKF* method and output via multi–sensor, was more stable than the moving trajectory of target measured via a single sensor. Especially, when there was a maneuver from target, the output did not arise severe wobble.

Synthesizing results from three figures of Fig.3, the measured moving trajectory of target via a single sensor had larger amplitude wobble. Especially, when the target maneuvered to speed up, the measurement error of single sensor was obviously larger, and the anti-noise ability was not enough; from the multi–sensor data fusion figure, the real-time tracking data had high accuracy, and the tracking ability was very powerful. And in particular, it was still able to effectively track the target's maneuvering segment, in the case of large acceleration, it was still capable of rapidly converging, which could reduce the measurement error of the target's maneuvering segment.

5. Conclusions

On the basis of the Unscented Kalman Filter(*UKF*) algorithm, this paper adopts state space description method and uses recursive algorithm to establish the signal's nonlinear state equation for the motion model of target. And then the dynamic information of the target is used to get rid of the influence of noise and then to get a better estimated value of the target location,which can achieve the estimation and prediction function of real-time running state. Through the algorithm establishment of the *UKF*, this algorithm can greatly reduce the amount of calculation. Moreover, the covariance of the algorithm has high degree of freedom, which could effectively reduce the systematic error. The result from multi–sensor data fusion show the real-time tracking data had high accuracy, and then the tracking ability was very powerful. Multi-sensor data fusion can effectively reduce the error and achieve the purpose of rapid convergence. Therefore, the research on the processing algorithm of moving trajectory from underwater target is useful to improve the accuracy of identification and the damage efficiency of underwater target,which has very high application value.

Acknowledgements

This work was supported by key science and technology program of Shaanxi province of China (Grant No. 2015GY078).

References

1. Rao S.K., Murthy K.S.L., Rajeswari K.R. (2010) Data fusion for underwater target tracking. *Radar, Sonar & Navigation, IET*, 4(4), p.p.576-585.
2. Guo L.h., Wang D., Ding S.Q. (2005) Extraction of features of underwater target. *Shengxue Jishu,* 24(3), p.p.148-152.
3. James R., Solberg, Kevin M., Lynch, Malcolm A. (2008)Active Electrolocation for Underwater Target Localization. *The International Journal of Robotics Research*, 27(5), p.p.529-548.
4. Feng, W. (2002) Research of High Precision Underwater Target Simulation and Responding. *Chinese Journal of Scientific ,* 23, p.p.464-466.
5. Salazar J., Azimi-Sadjadi M. R. (2004) Adaptable Image Retrieval with Application to Underwater Target Identification. *Asilomar Conference on Signals Systems And Computers,* 2, p.p.1540-1545.
6. Zheng-guang X., Jin-xia W. (2012) Pattern moving trajectory: a new dynamics description method. *International Journal of Modelling, Identification and Control ,* 17(4), p.p.370-379.
7. Jung-Hwan Ko, Eun-Soo Kim. (2006) Stereoscopic video surveillance system for detection of target's 3D location coordinates and moving trajectories. *Optics Communications,* 266(1), p.p.67-79.
8. Hua-Tsung Chen, Kuo-Lian Ma, Jen-Hui Chuang. (2013) Recognizing jump patterns with physics-based validation in human moving trajectory*,* 24(7), p.p.1191-1203.
9. El-Hawary F., Aminzadeh F., Mbamalu G.A.N. (1992)The generalized Kalman filter approach to adaptive underwater target tracking. *Oceanic Engineering, IEEE Journal* , 17(1), p.p.129-137.
10. Jaehwei P., Jaemu Y., Jangmyung L. (2007) Trajectory estimation of a moving object using Kalman filter and Kohonen networks. *Robotica*, 25(5), p.p.567-574.