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Analysis of Blood Pressure Variability Based on Frequency Spectrum

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Abstract
Blood pressure variability is an important quantitative tool to assess the antonomic activity, which plays essential role in cardiovascular disease prediction and monitoring. In order to improve the accuracy of blood pressure variability analysis, find the best BPV frequency domain analysis by using several different algorithms for blood pressure variability in frequency domain analysis, and compare their matlab simulation results to find out the best choice for the analysis of BPV. The peak and trough points of blood pressure were detected for SBP, DBP and SBP to get BPV. The five kinds of methods were used to analysis the frequency spectrum of BPV, which are Fast Fourier transform algorithm, Fundamental period graph algorithm, Bartlett average cycle diagram algorithm, Welch smooth average cycle diagram and the auto regressive model methods. From the results include the spectrograms and the values of the spectral peak calculated, it showed that the power spectrum of BPV using FFT method has no LF spectral peak, and Welch Periodogram (Hamming window, Blackman window) methods have no HF spectral peak, using Basic Periodogram and Bartlett Periodogram methods with clear peaks but the larger error, using AR model method has not only clear spectral peaks but also good resolution. Through the comparison for the analysis of results, that AR model method with smaller variance and higher resolution, more suitable for spectrum analysis of blood pressure variability.
Key words: BLOOD PRESSURE VARIABILITY, FREQUENCY DOMAIN ANALYSIS, PERIODOGRAM, AUTO REGRESSIVE MODEL, POWER SPECTRUM.

1. Introduction

Blood Pressure Variability (BPV) usually refers to the degree within a certain period of time frame fluctuations in blood pressure, which is the result arrived from the body neuroendocrine dynamically adjusting for the balance [1]. As one of the most fundamental human physiological characteristics of blood pressure, the research of BPV has an important pathophysiological significance for prediction of cardiovascular disease and monitoring. Even the European Society of Cardiology (ESC2010), which held in 2010, have had a special topic of BPV. From then on, BPV officially became a new indicator impact the treatment of hypertension in 21st Century [2].

Nowadays, the main methods for BPV analysis are time-domain and frequency-domain. Time-domain mainly reflects the variation of size. Ambulatory Blood Pressure Monitoring (ABPM) can provide continuous recording of blood pressure, but it depends on the application of ABPM technology in clinical [3, 4]. Frequency-domain analysis can directly reflect the speed of variation, to estimate the spectral of blood pressure variation signal can reflect the role of vagal nerve and sympathetic nerve tension, as well as respiration and temperature and so on, and thus this analysis method is widely used in clinical studies and analysis of the BPV [5]. This article aims to find the best BPV frequency domain analysis by using several different algorithms for blood pressure variability in frequency domain analysis, and compare their matlab simulation results to find out the best choice for the analysis of BPV. It would give us a direction for a good approach and new insights into analyzing BPV, which plays essential role in cardiovascular disease prediction and monitoring.

2. Methods and Principles

2.1 Data Acquisition and Preprocessing

Continuous blood pressure (BP) signal data used in this paper is downloaded from MIT-BIH, Physio-Bank ATM, BP in Salt-Sensitive Dahl Rats (BPSS-RAT) data bank. The original BP signal is shown in Figure 1. Before spectrum analysis of blood pressure variability, systolic blood pressure (SBP) and diastolic blood pressure (DBP) were calculated by method of difference. And then calculate the verage pressure of pressure relationship (MBP) through the integration of systolic and diastolic blood pressure to average pressure. Since the spectral analysis of these three parameters in the same manner, we just study the systolic blood pressure as an example. After extracting the SBP, since the blood pressure signal is affected by many factors and interference, it is inevitable. There will be some point to detect errors. Compare the blood pressure values before and after, if they are more than 20% of the deviation between the values of the current blood pressure values, then remove of the sequence. If the sequence of the detection error is less, the similar sequence of the original sequence will be obtained (see Figure 2).

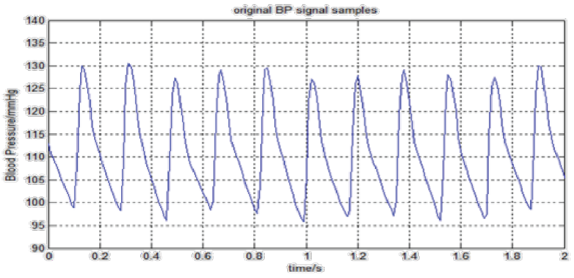

Figure 1. The original BP signal

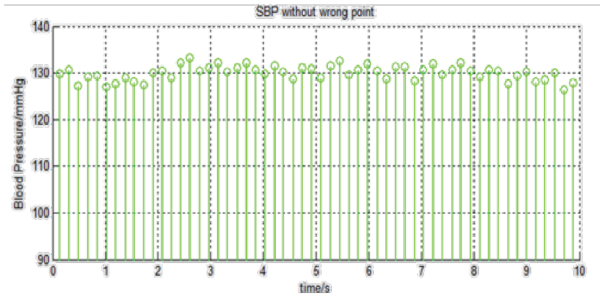

Figure 2. The SBP without wrong point

2.2 Cubic Spline Interpolation to re-sampling of SBP

After the above steps, it is the unequal interval sampling of the instantaneous blood pressure, and the power spectrum is also to be re-sampled into equal interval series. In this paper, we use the cubic spline interpolation method to carry out this step, in addition to the characteristics of the human body signal set the weight of the sampling frequency is 4Hz. The results are shown in Figure 3 and Figure 4.

The cubic spline interpolation usually used to solve the curve function group is a smooth curve through a series of points. In mathematics, generally get its function by solving the three moment equations.

Let $a \le x_0, x_1, \ldots, x_n \le b$ is a part of the interval [a, b], if the function S(x) in the interval [a, b] satisfies the condition on:

(1) S(x), S'(x), S''(x) in the interval [a, b] continuous, then $S(x) \in C^2$ [a, b];

(2) If function S(x) is satisfied in each cell $[x_k, x_{k+1}]$ is the three polynomial, then call S(x) is the cubic spline function for the interval [a,b];

(3) If the function f(x) at the nodes $x_0, x_1, \ldots, x_n$ the values were $f(x_j)=y_j$, j=0, 1, …, n, and cubic spline function S(x) also meets $S(x_j)=y_j$, j=0, 1, …, n , then called S(x) is the cubic spline interpolation function of f(x) in [a, b].

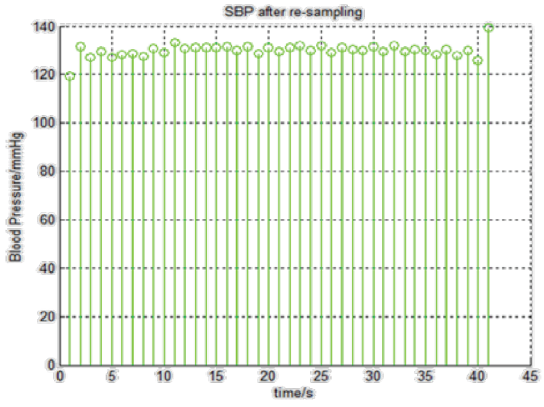

Figure 3. The SBP after re-sampling

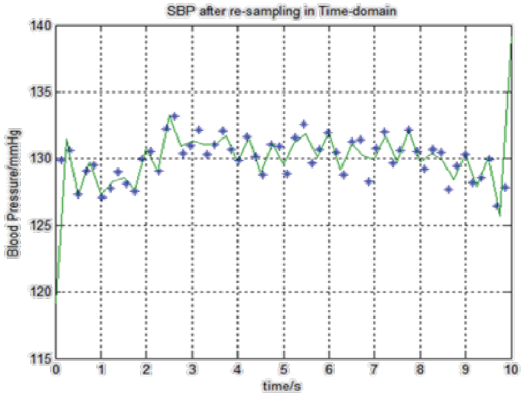

Figure 4. The SBP after re-sampling in Time-domain

In actual calculation of cubic spline interpolation method, it is also necessary to introduce boundary conditions to completing its computation. Usually use three kinds of it , natural boundary, border holding and non-twisted boundary [6]. The general calculation method in books did not explain the definition of non-twisted boundary, but the numerical calculation software, such as Matlab the one mainly used in this subject, take as default non-twisted boundary as the boundary conditions.

2.3 Several methods and Principles of frequency-domain analysis for BPV

After feature extraction and re-sampling, five kinds of common algorithms are used to analyze the variation of blood pressure in frequency-domain. They are Fast Fourier transform method, Fundamental period graph method, Bartlett average cycle diagram method, Welch smooth average cycle diagram and the auto regressive model method, which will be specialized below.

2.3.1 Principles of Fast Fourier Transform Method for BPV

Typically classical power spectrum estimation method is to assume that the unknown data outside of the work space is assumed as zero, which works just like windowing [7]. Fast Fourier Transform (FFT) is one of the classic power spectrum estimation methods.

The periodicity and symmetry of WN are used in the calculation of FFT, and a N-point sequence is divided into two sub sequences of N/2. After this transformation, the N point DFT transform N^2 operation, only required for $N\log_2 N$ time operations. And the number of points needed to calculate the greater the amount of savings, which is also the embodiment of FFT's advantage. FFT is different from the theory of modern spectral estimation methods, but they are the same in fact and their results are similar. The physical meaning of FFT is clear, the algorithm is simple and fast, and it can be a good reflection of the data characteristics.

2.3.2 Principles of Basic Periodogram method for BPV

Direct method, also known as the periodogram method, is a kind of signal power spectral density estimation method based on the Fourier transform. The principle of periodogram is that N data of random sequence x(n) as a sequence of finite energy, directly calculate N-point data of x(n) DTFT (DFT) to obtain X(k), then take amplitude squared and divided by N as the true power spectrum estimation of a sequence x(n) [8]:

$$\hat{P}(\omega) = \frac{1}{N}\left|X_N(e^{-j\omega})\right|^2 \quad (1)$$

The sequence x(n) of DFT, which is obtained from the periodogram, is periodic, and its power spectrum is also periodic. Early statisticians use this method to find the hidden law of periodicity in the mass data. The variance of periodogram value is not zero when the length of the signal sequence increases to infinity, which is a partial estimation of signal power spectrum [9].

2.3.3 Principles of Bartlett average periodogram for BPV

Bartlett average periodogram is an improved method of basic periodogram. First, N-point signal sequence x(n) is divided into L segments, with each length M, each of which are calculated periodogram respectively, then take the average of each segment periodogram to get the value of the power spectrum as [8]:

$$\hat{P}(\omega) = \frac{1}{ML}\sum_{i=1}^{L}\left|\sum_{n=0}^{M} x_m^i(n)e^{-j\omega n}\right|^2 \quad (2)$$

Bartlett average periodogram reduce the estimated covariance. Obtaining the power spectral density of random signals estimated power spectrum in this way, is smoother than direct periodogram. Increasing the number of sub-segments can further reduce the estimated covariance, but too few points of each segment in turn greatly reduces image resolution [10,11].

2.3.4 Principles of Welch smoothing averaged periodogram for BPV

Welch smoothing averaged periodogram is also known as the weighting overlaps average method (WOSA). This method first make the segmented data windowing and becomes smooth, and then calculates the average of its periodogram. Welch method was modified by Bartlett method.On the one hand is before the periodogram calculated, choose the appropriate window function w(n) directly added to the signal, it makes no matter what kind of window function was used, the spectral estimation will be non-negative; on the other hand, the segments between paragraphs can have a certain degree of overlap, which could reduce

$$\hat{P}(\omega) = \frac{1}{MUL}\sum_{i=1}^{L}\left|\sum_{n=0}^{M} x_m^i(n)e^{-j\omega n}\right|^2 \quad (3)$$

Wherein, U = Σω (n), ω (n) is the window function. Firstly, divided the data into L segment, of each section length M, multiplied by the window function (windowing), respectively calculate the periodogram, and then the average. This processing method is the further improve of basic periodogram method and the Bartlett average periodogram method, and is more commonly used [28].

2.3.5 Principles of AR model method for BPV

Modern power spectrum estimation is also known as the method of parameter spectrum estimation. In this method, the model is estimated by means of observation data, and the power spectrum estimation of the signal is obtained by the method of calculating the output power of the parameters. The resolution and variance of the classical method are greatly improved. In signal analysis of the actual application of relatively large method is the maximum entropy spectral analysis method (AR model) more representative [13].

AR model (auto-regressive model) power spectrum estimation, also known as self-regression model. Any of a random signal having a power spectral density, can be seen by the white noise generated by a particular physical network [14], it can be used as a differential equation to represent:

$$x(n) = -\sum_{k=1}^{p} a_k x(n-k) + \omega(n) \quad (4)$$

Z transform of the formula, you can get AR model of the system transfer function is:

$$H(z) = \frac{1}{1+\sum_{k=1}^{p} a_k z^{-k}} \quad (5)$$

The transfer function H(z) only has pole, and there is no other zero except the origin. So it is known as the full pole model, and the power spectral density obtained by this method can be written as:

$$P(\omega) = \frac{\sigma_\omega^2}{\left|1+\sum_{k=1}^{p} a_k e^{-j\omega n}\right|^2} \quad (6)$$

Firstly solve the AR parameters, generally through the Levinson-Dubin recursive algorithm based on linear prediction or theory of Burg algorithm or Marple AR parameters determined by the Yule-Walker equations, in addition to the modified covariance method and the like can be used the AR model parameter estimation [15].

3. Calculation and analysis results

FFT is directly used to estimate the power spectrum of the analysis, compared to other methods, which the variance is larger, but the results can be a good reflection of the peak, more intuitive and easy

to understand, especially to get the characteristics of high frequency components of the signal. The method of FFT is simple and can save more time, but the magnitude of the valley is not easy to analyze. The results are shown in Figure 5.

The results obtained by the basic periodogram method are rough, the fluctuation is larger, the peak amplitude is small, and the variance of processing results is very large. And the more the number of points obtained from the image is also more smooth and fine. The results are shown in Figure 6.

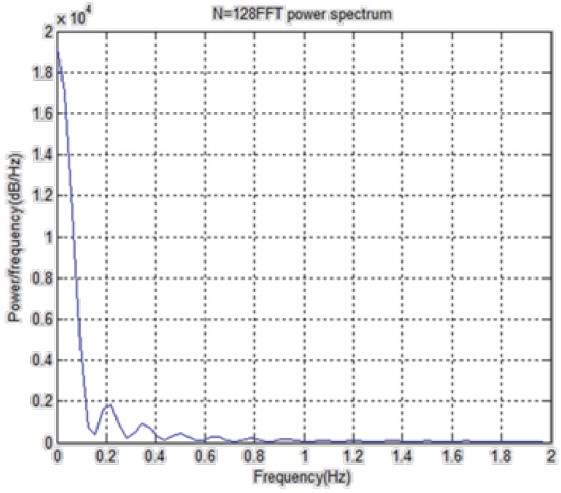

Figure 5. The FFT power spectrum of BPV in 128 points

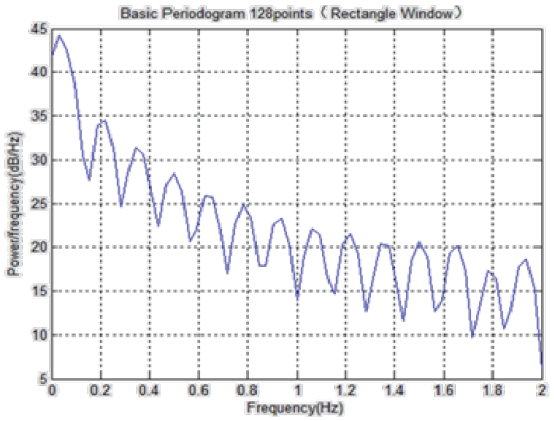

Figure 6. Basic Periodogram in 128 points (Rectangle Window)

The estimate result of the spectral lines in Bartlett average periodogram method is more smooth and convergence. It improves the variance of the spectral lines and decreases the random fluctuation. However, the main lobe is also increased, and the peak is not clear enough. The results are shown in Figure 7.

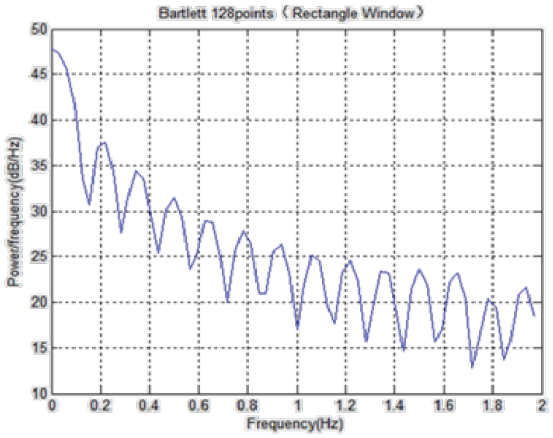

Figure 7. Bartlett Periodogram in 128 points (Rectangle Window)

The variance and convergence of Welch method is better. The spectral line is relatively rough, and the resolution of the windowing is more than that of Bartlett method, but the Welch method is not ideal for signal processing. The resolution of power spectrum can be improved by choosing appropriate window function according to the signal and the different processing purposes. The results are shown in Figure 8.

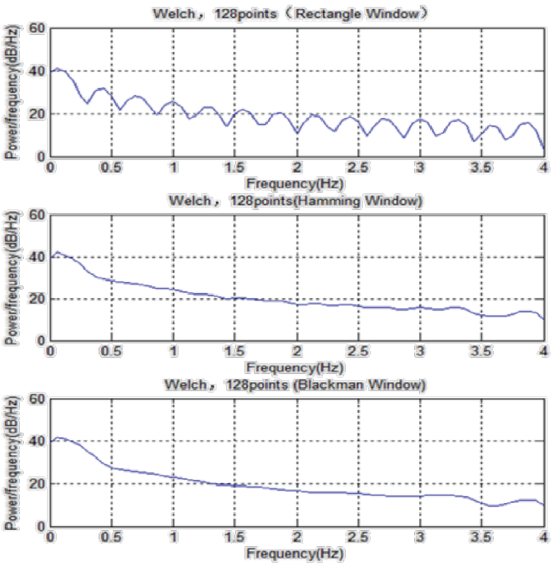

Figure 8. Welch Periodogram in 128 points with different windows

Based on the Bartlett method, Welch can be overlapped, and the correlation between each segment can be reduced, so that the variance of the power spectrum can be improved and the bias can be reduced. The variance and convergence of Welch method are better, and the spectral lines are relatively rough, but the resolution of the window is improved than Bartlett method. Furthermore, we can know that the Welch method is not ideal for signal processing.

At the same time, from the progress result it is clear that, according to the signal and the different processing purposes to select the appropriate window function, can improve the resolution of power spectrum. Using rectangular window to deal with the main lobe has the narrowest and high side lobe, resolution is the best, but it is the most of the ups and downs of variance. By Hamming window, main lobe of the results is widest, its resolution is relatively poor, but in this method the side lobe is smaller, and the variance is smaller too, greatly reduces the spectrum distortion. With Blackman window, the main lobe is width and the side lobe is small, its processing results with the Hamming window is somewhat similar, the resolution is not high, but in smaller variance, spectral line more smooth.

AR model use the Burg algorithm to estimate the power spectrum, the main lobe is narrow, the side lobe amplitude is smaller than the main lobe and the fluctuation is small, the spectral lines of this method are less variance and smooth, with good resolution, also the peak value is better. Processing results are shown in figure 9.

From the above analysis for power spectrum of BPV, the values of the spectral peak were calculated (see Table 1). The Standard ranges of BPV power spectrum were explained as follows: Low Frequency (LF): 0.03-0.15 Hz and High Frequency (HF): 0.15-0.40 Hz.

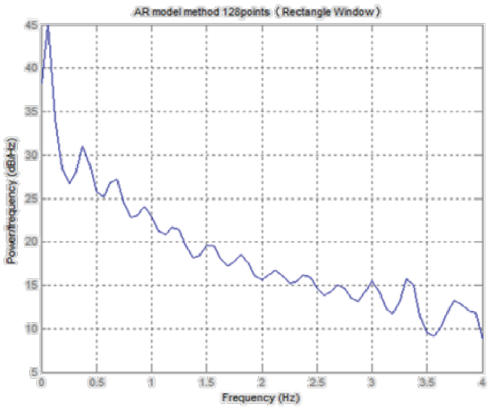

Figure 9. AR model in 128 points (Rectangle window)

According to the LF and HF standard values, the power spectrum of BPV using FFT method has no LF spectral peak, and Welch Periodogram (Hamming window, Blackman window) methods have no HF spectral peak, using Basic Periodogram and Bartlett Periodogram methods with clear peaks but the larger error, using AR model method has not only clear spectral peaks but also good resolution.

Table 1. The values of the spectral peaks in the spectrograms for BPV

The value of the spectral peak, Hz	The power spectrum of BPV using different methods						
	FFT	Basic Periodogram	Bartlett Periodogram	Welch Periodogram			AR model
				Rectangular window	Hamming window	Blackman window	
LF(0.03-0.15)	no peak	0.10±0. 02	0.05±0. 04	0.10±0. 05	0.10±0. 05	0.10±0. 05	0.10±0. 01
HF(0.15-0.40)	0.20±0. 05	0.20±0. 05	0.20±0. 05	0.40±0. 05	no peak	no peak	0.35±0. 01

4. Conclusions

In this paper, five kinds of frequency domain analysis methods were used to analyze the blood pressure variability. By comparing the results, the AR model is the best method. The resolution of the classical spectrum estimation is affected by many factors. Compared with the 5 kinds of power spectrum estimation methods, the resolution ratio of the modern spectrum is higher, and the spectral line is smooth. For different frequency domain analysis method, the more points, the result of the power spectrum estimation can be more exquisite. In addition, it can be concluded that with different windows, according to the characteristics of the signal and the different processing purposes, the quality of spectral estimation can be different. Choosing different window functions the width of the main lobe is different, so that the resolution of spectral estimation is also different. For example, if only require a high resolution and doesn't care about the accuracy, the Rectangle window which main lobe width

is narrow would be the best choice.

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