Calibration of Cutting Force Coefficients Using Surface Errors in Milling of Thin-Walled Workpiece

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Abstract
Cutting force coefficients are the key factors for efficient and accurate prediction of milling force. This paper presents a new method to calibrate the cutting force coefficients using the surface errors related to milling of thin-walled workpiece including effect of cutter runout. The surface error is separated into nominal surface error and perturbation component due to runout. By analyzing forming of surface error, cutter deformation and deflection of thin-walled workpiece, the result that cutter runout has no effect on the average surface error is achieved. Relationship between nominal surface error and cutting force coefficients is constructed, and also an approach for extraction of nominal surface error from measured surface error is proposed. Then, the cutting force coefficients are estimated conveniently. Milling tests are carried out to verify the proposed method. A good agreement between predicted results and experimental results is achieved, which shows that the method is efficient.
Key words: CUTTING FORCE COEFFICIENT, SURFACE ERROR, THIN-WALLED, RUNOUT, WORKPIECE, NOMINAL SURFACE ERROR

1. Introduction

With the advancement in networking and multimedia technologies enables the distribution and sharing of multimedia content widely. In the meantime, piracy becomes increasingly rampant as the customers can easily duplicate and redistribute the received multimedia content to a large audience. Insuring the copyrighted multimedia content is appropriately used has become increasingly critical [1].

Peripheral milling is one of the most widely used processes in the manufacture of aerospace, automobile and die parts. Precise prediction of milling force is required for various purposes including estimation of cutter deformation and tool wearing, reducing the shape error and prediction of stability. Therefore, it is necessary to pay more attention to milling force. To predict the milling force with mechanistic model, the cutting force coefficients must be calibrated firstly.

Existing cutting force coefficient models can be classified into two categories. In the first one, cutting force coefficients are taken as constants or exponential of average chip thickness, which are called average cutting force coefficients and determined with measured average cutting force[1-5]. The second model considered the size effect of the chip thickness in the cutting force coefficients model to meet the demand for more accurate milling force prediction, and express the cutting force coefficients as exponential functions of instantaneous uncut chip thickness, that is, instantaneous cutting force coefficients[6-11]. Among them, Yun[6] and Bhattacharyya[9] identified the coefficients using average milling forces with multiple experiments. Wan[8] calibrated the instantaneous cutting force coefficients using nominal milling forces which are extracted from experimental results with one experiment.

All contributions mentioned above are performed with measured milling forces, and special and expensive force meter is needed. Kim[12], Xu[13] assumed that the surface error is deduced by cutter deformation completely, and presented methods to calibrate cutting force coefficients with surface error of workpiece. Dotcheva[14] neglected the effect of the runout to the surface error, proposed a method to obtain the cutter deformation form surface error and calibrate instantaneous cutting force coefficients. Wang[15] presented a method to determine cutter deformation and instantaneous cutting force coefficients and taking account of runout. In these works[12-15], the deformation of workpiece was neglected, which will cause inaccurate calibrated results especially in milling of thin-walled Workpiece.

This paper presents a new approach to calibrate the cutting force coefficients using surface errors taking account of cutter runout in milling of thin-walled workpiece. A comparative study between predicted results and experimental results is made to validate the proposed method. It is different from the existing works is that a new method is proposed to extract nominal surface error from the measured results. The advantage of the proposed approach lies in that the calibration can eliminate influences of cutter runout, and also it does not require expensive force meter in practice.

2. Mechanistic milling force model

Suppose the cutter is discretized into a finite number of disk segments along the axis. When the cutter position is k, the tangential $dF_t(\theta(i,j,k))$ and radial $dF_r(\theta(i,j,k))$ cutting force components acting on j-th element of i-th flute are given by

$$\begin{cases} dF_t(\theta(i,j,k)) = K_T h(\theta(i,j,k))dz \\ dF_r(\theta(i,j,k)) = K_R h(\theta(i,j,k))dz \end{cases} \quad (1)$$

Where, K_T and K_R represent the tangential and radial cutting force coefficient. $\theta(i,j,k)$ is the angular position of the segment. dz is the axial length of discrete segment. $h(\theta(i,j,k))$ is the instantaneous uncut chip thickness.

Taking runout into account, instantaneous uncut chip thickness of the segment (i,j) can be calculated by

$$\begin{cases} h(\theta(i,j,k)) = m_i f_z \sin\theta(i,j,k) + R_{i,j} - R_{i-m_i,j} \\ \theta(i,j,k) = \dfrac{2\pi k}{N f_z} - \dfrac{2z_{i,j}\tan\beta}{D} - \dfrac{2\pi(i-1)}{N} \\ R_{i,j} = \dfrac{D}{2} + \rho\cos(\lambda - \dfrac{2z_{i,j}\tan\beta}{D} - \dfrac{2\pi(i-1)}{N}) \end{cases} \quad (2)$$

Where, f_z is feed per tooth. $R_{i,j}$ denotes the actual cutting radius of the segment (i,j). m_i is a number that indicates the current tooth is removing the material left by the previous tooth. ρ is cutter runout offset. λ is the location angle of cutter runout defined as the clockwise angle between offset direction and the nearest tooth tip in the bottom of cutting tool. $z_{i,j}$ is the axial height of the segment (i,j). D and β are diameter and helix angle of the tool. N is flute number of the cutter.

It is well known that milling force in Y direction is the main factor inducing cutter deformation and workpiece deformation. Considering that milling force in X and Z directions have small effects on the deformations, they are neglected. Then, transforming the cutting force acting the segments into Y direction and summing cutting forces acting on the all segments, the total milling forces in Y direction can be given as

$$F_Y(k) = \sum_{i=1}^{N}\sum_{j=1}^{M}[K_T \sin\theta(i,j,k) - K_R\cos\theta(i,j,k)]h(\theta(i,j,k))dz \quad (3)$$

3. Calculation of total deformation

As shown in Figure 1, the cutter deformation is simplified as piecewise cantilever beam. Neglecting the axial cutter deformation, the total cutter deformation consists of arbor deformation and deflection of fluted part. So, the deformation of segment can be defined as

$$\begin{cases} \delta_{ct}(k,h) = \delta_s + \theta_s(h) + \delta_t(h) \\ \delta_s = \dfrac{F_Y(k)}{6EI_S}[-(L_{ct}-L_t)^3 + 3(L_{ct}-L_t)^2(L_{ct}-Z_F)] \\ \theta_s(h) = \dfrac{F_Y(k)}{2EI_S}[-(L_{ct}-L_t)^2 + 2(L_{ct}-L_t)^2(L_{ct}-Z_F)](L_t-h) \\ \delta_t(h) = \dfrac{F_Y(k)}{6EI_t}[(\max(0,Z_F-h))^3 - (L_t-h)^3 + 3(L_t-h)^2(L_t-Z_F)] \end{cases} \quad (4)$$

Where, $\delta_{ct}(k,h)$ is the total deflection of the cutter at h height when the cutter position is k. δ_s is the deflection due to the deformation of cutter arbor. $\theta_s(h)$ is the deviation due to deflection angle of cutter arbor. $\delta_t(h)$ is the deformation of fluted part of the cutter. Z_F denotes distance between simplified concentrated forces and cutter bottom. L_{ct} is overhang length of the cutter. L_t is the length of the fluted part. I_s and I_t are the moments of inertia for the arbor and fluted part. E is Young's modulus of elasticity.

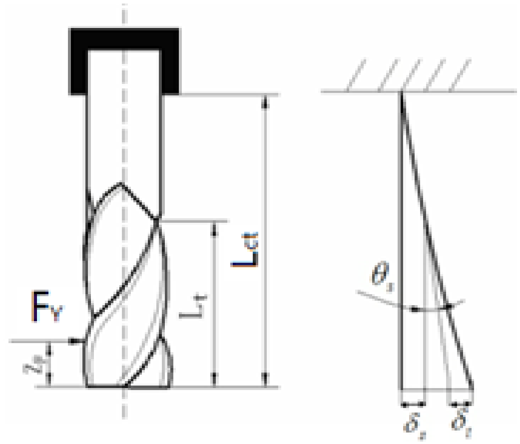


Figure 1. Cutter deformation

The Eq.(4) can be simplified as

$$\begin{cases} \delta_{ct}(k,h) = F_Y(k)A_Y \\ A_Y = A_{Y3} + A_{Y2} + A_{Y1} \\ A_{Y1} = \dfrac{1}{6EI_S}[-(L_{ct}-L_t)^3 + 3(L_{ct}-L_t)^2(L_{ct}-Z_F)] \\ A_{Y2} = \dfrac{1}{6EI_t}[(\max(0, Z_F - h))^3 - (L_t - h)^3 + 3(L_t - h)^2(L_t - Z_F) \\ A_{Y3} = \dfrac{1}{2EI_S}[-(L_{ct}-L_t)^2 + 2(L_{ct}-L_t)^2(L_{ct}-Z_F)](L_t - h) \end{cases} \quad (5)$$

In milling of thin-walled parts, deformation of workpiece is also significant and cannot be ignored. Because deformation of workpiece has coupling relationship with the deformation of cutter, it is calculated with the theory of elastic mechanics and the principle of minimum potential energy. Assuming that the workpiece is rectangular panel, stationary at three sides, free at one side and acted by linear load, the boundary conditions are defined as

$$\begin{cases} w\big|_{x=a} = 0, w\big|_{x=0} = 0, w\big|_{y=0} = 0 \\ \dfrac{\partial w}{\partial x}_{x=a} = 0, \dfrac{\partial w}{\partial x}_{x=0} = 0, \dfrac{\partial w}{\partial y}_{y=0} = 0 \end{cases} \quad (6)$$

Fitting the boundary conditions, deflection of workpiece at any position can be calculated by [16]

$$w(x,y) = \sum_{m=2,3,4...} \sum_{n=0,1,2...} C_{m,n} (\frac{y}{b})^m (1 + \cos\frac{\pi x}{a})(\frac{x}{a})^n \quad (7)$$

Where, $w(x,y)$ denotes deflection of rectangular panel at (x,y). $C_{m,n}$ is a coefficient. a and b are length and width of the workpiece.

Using the principle of minimum potential energy, that is, $\partial\Pi/\partial C_{m,n} = 0$, $C_{m,n}$ can be obtained. Substituting the $C_{m,n}$ to Eq.(7), the $w(x,y)$ can be calculated. So, the workpiece deformation $w(k,h)$ can be calculated when $x=k$ and $y=h$.

Then, the total deformation including cutter deformation and workpiece deflection can be expressed as

$$\delta_{total}(k,h) = \delta_{ct}(k,h) + w(k,h) \quad (8)$$

To make the subsequent analysis convenient, the Eq.(8) can be transformed into

$$F_Y(k) = \frac{1}{M}\delta_{total}(k,h) \quad (9)$$

Where, M can be obtained according to Eq.(5) and Eq.(7).

4. Calibration of cutting force coefficients

As we know, except the cutter deformation and workpiece deflection, runout also makes the actual machined surface deviate from the desired machined surface. Thus, it will have effect on the surface error. The forming of surface error is shown in figure 2. It is worth noting that the cutting traces of different flutes are not same under the effect of cutter runout. When the cutter position is k, the surface error of workpiece at height h were expressed by co-author[15] as

$$\begin{cases} e(k,h) = -e_\rho(k,h) + \delta_{total}(k,h) \\ e_\rho(k,h) = \rho\cos(\lambda - \dfrac{2h\tan\beta}{D} - \dfrac{2\pi(i-1)}{N}) \end{cases} \quad (10)$$

Where, $e_\rho(k,h)$ is error induced by cutter runout directly.

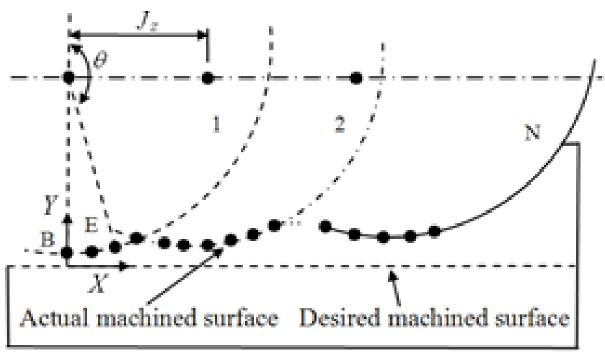


Figure 2. Formation of surface error

The total surface error also can be separated into two components as

$$e(k,h) = e^N(k,h) + e^D(k,h) \quad (11)$$

$$e^N(k,h)=\delta_{total}{}^N(k,h)=\begin{bmatrix}A^N & B^N\end{bmatrix}\begin{bmatrix}K_T\\K_R\end{bmatrix} \tag{12}$$

$$e^D(k,h)=-e_\rho(k,h)+\delta_{total}{}^D(k,h)=-e_\rho(k,h)+Mdz\begin{bmatrix}A^D & B^D\end{bmatrix}\begin{bmatrix}K\\K\end{bmatrix} \tag{13}$$

Where, $A^N=f_z dzM\sum_{i=1}^{N}\sum_{j=1}^{M}\sin^2\theta(i,j,k)$,

$A^D=\sum_{i=1}^{N}\sum_{j=1}^{M}\sin\theta(i,j,k)[(m_i-1)f_z\sin\theta(i,j,k)+R_{i,j}-R_{i-m_i,j}]$

$B^N=-f_z dzM\sum_{i=1}^{N}\sum_{j=1}^{M}\sin\theta(i,j,k)\cos\theta(i,j,k)$,

$B^D=-\sum_{i=1}^{N}\sum_{j=1}^{M}\cos(i,j,k)[(m_i-1)f_z\sin\theta(i,j,k)+R_{i,j}-R_{i-m_i,j}]$

. $e^N(k,h)$ is nominal surface error which is not affected by the runout. $e^D(k,h)$ is the perturbation components due to runout. $\delta_{total}{}^N(k,h)$ and $\delta_{total}{}^D(k,h)$ denote the nominal total deformation and deformation induced by runout.

Because nominal surface errors are not affected by the runout, the errors will change periodically with f_z duration. That is, $e^N(k,h)=e^N(k+gf_z,h)=\overline{e^N(k,h)}$ ($g=0,1,...,N-1$). Then, the average surface errors of $e(k+gf_z,h)$ can be expressed as

$$\overline{e(k,h)}=e^N(k,h)+\overline{e^D(k,h)} \tag{14}$$

Where, $k\in[0,f_z]$. $\overline{e^D(k,h)}$ are average perturbation components due to runout.

It well known that the cutter will rotate $2\pi/N$ when it moves forward f_z. So, the average of error induced by cutter runout directly can be calculated by

$$\overline{e_\rho(k,h)}=\frac{\rho}{N}\sum_{i=1}^{N}\cos(\lambda-\frac{2h\tan\beta}{D}-\frac{2\pi(i-1)}{N})=0 \tag{15}$$

For purposes of analysis, suppose the runout is small. That is $m_i=1$. A^D and B^D can be simplified to

$A^D=\sum_{i=1}^{N}\sum_{j=1}^{M}\sin\theta(i,j,k)(R_{i,j}-R_{i-1,j})$ and $B^D=-\sum_{i=1}^{N}\sum_{j=1}^{M}\cos\theta(i,j,k)(R_{i,j}-R_{i-1,j})$.

Consequently, A^D and B^D can be confirmed as

$$\begin{cases}\overline{A^D}=\frac{1}{N}\sum_{g=0}^{N-1}A^D=\frac{1}{N}\sum_{j=1}^{M}\sum_{g=0}^{N-1}\sum_{i=1}^{N}\sin(\theta(i,j,k)+g\frac{2\pi}{N})(R_{i,j}-R_{i-1,j})=0\\ \overline{B^D}=\frac{1}{N}\sum_{g=0}^{N-1}B^D=-\frac{1}{N}\sum_{j=1}^{M}\sum_{g=0}^{N-1}\sum_{i=1}^{N}\cos(\theta(i,j,k)+g\frac{2\pi}{N})(R_{i,j}-R_{i-1,j})=0\end{cases} \tag{16}$$

With the same method, the same conclusions also can be approved when runout is larger.

Substituting Eq.(15) and Eq. (16) into Eq. (14), the following equation can be obtained.

$$e^N(k,h)=\overline{e(k,h)}=\begin{bmatrix}A^N & B^N\end{bmatrix}\begin{bmatrix}K_T\\K_R\end{bmatrix} \tag{17}$$

Due to the runout, when different tooth is forming the surface at the same axial height, the surface errors are various accordingly. However, Eq. (17) confirmed that runout has no effect on the average surface errors and the nominal surface errors. Also, the equation reflects the relationship between surface errors and cutting coefficients, and gives a new approach to calibrate the cutting coefficients using surface errors. To obtain the nominal surface errors, measured surface errors at height h with certain horizontal interval must be needed with experiments, and average measured surface errors should be calculated. Then, Eq.(18) is modified as

$$\begin{bmatrix}\frac{1}{N}\sum_{i=1}^{N}e^M(k_1+gf_z,h)\\ \frac{1}{N}\sum_{i=1}^{N}e^M(k_2+gf_z,h)\\ ....\\ \frac{1}{N}\sum_{i=1}^{N}e^M(k_n+gf_z,h)\end{bmatrix}=\begin{bmatrix}A_1{}^N & B_1{}^N\\ A_2{}^N & B_2{}^N\\ ... & ...\\ A_n{}^N & B_n{}^N\end{bmatrix}\begin{bmatrix}K_T\\K_R\end{bmatrix}\quad 0\le k< . \tag{18}$$

Where, $e^M(k,h)$ is measured surface error.

The cutting coefficients can be calibrated with Eq. (18). Subsequently, regression method the runout can be estimated with iterative method according to the best fit between theoretical surface errors and measured results.

5. Experimental verifications

To verify the proposed method, peripheral milling of thin-walled workpiece is carried out to determine cutting force coefficients on DMC 70V hi-dyn. A 3-flute cylindrical end cutter with 45o helix angle and diameter of 12mm is used, which has 110mm total length, 45mm flute length and 83mm overhang length. The elastic modulus of the cutter is 206GPa. The material of workpiece is Al6061-T6. Thickness and height of the thin-walled workpiece are 2mm and 30mm as shown in Fig.3. Surface errors are measured by a coordinate measuring machine MISTRAL 070705 equipped by measuring face with diameter of 1mm as shown in Fig.4. A force meter Kistler9257B is also used to measure milling forces.

Conditions for the milling test are spindle speed n=1000r/min, radial depth ae=1mm, axial depth ap=3mm, dry cutting and down milling. Surface errors at height 0.5mm from bottom of milling are measured, and the step is set as 0.02mm. I this case, 10 points are measured per fz. The measured surface errors are shown in table1.

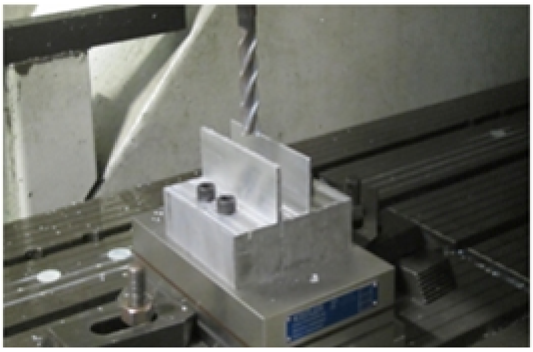


Figure 3. Milling test

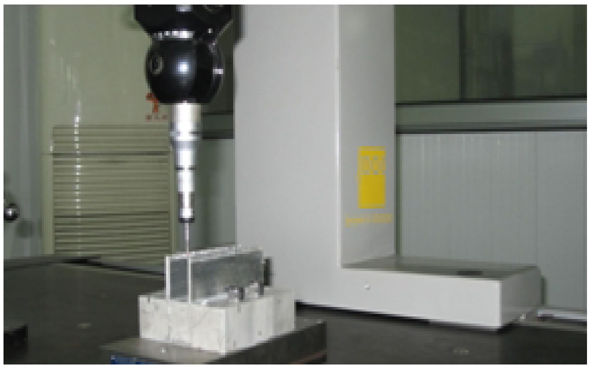


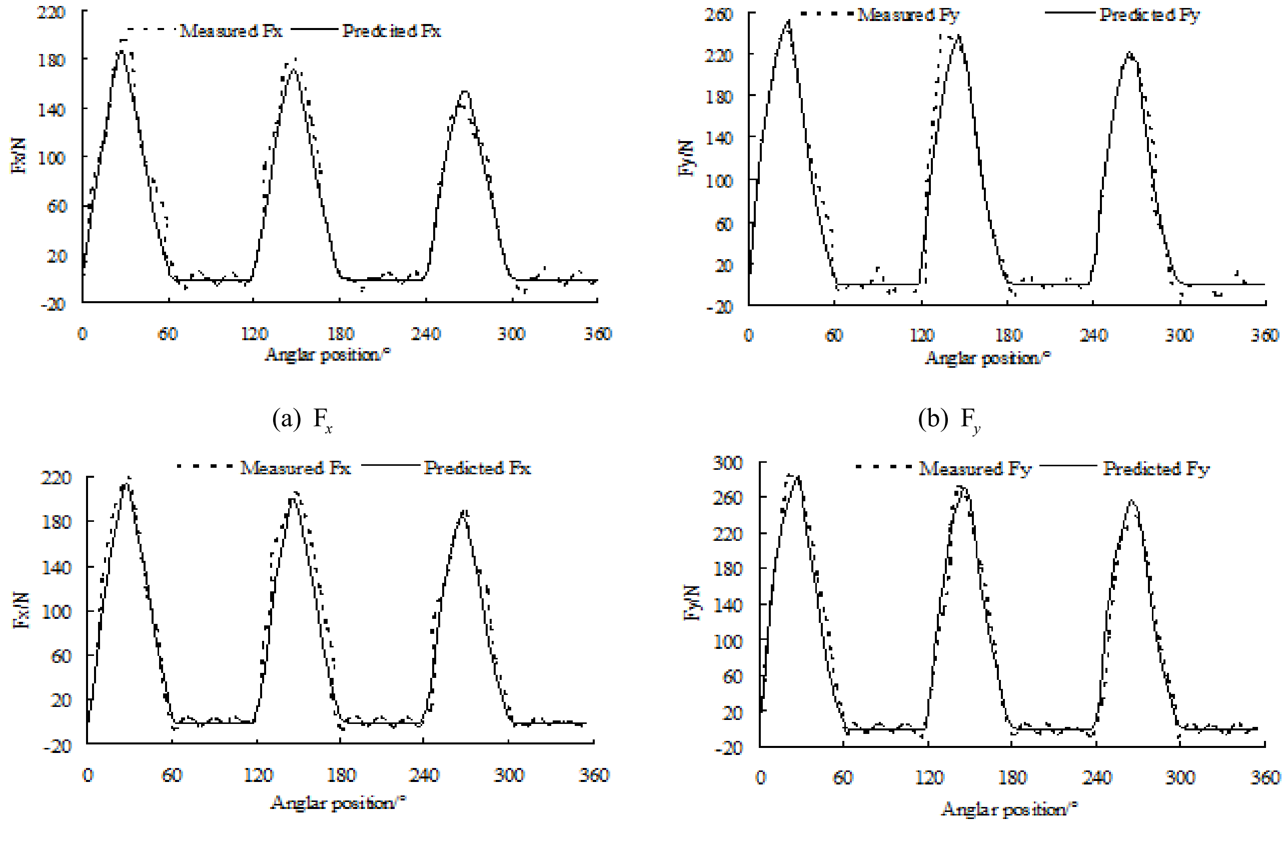
Figure 4. Measurement of surface error

Table 1. Measured surface errors and nominal surface errors

	k=0	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9
Measured results, mm	0	0.0135	0.0369	0.0288	0.0214	0.0102	0.0008	0.0001	0	0
Measured results (10+k),mm	0	0.0245	0.0358	0.0238	0.0170	0.0116	0.0002	0	0	0
Measured results (20+k), mm	0	0.0194	0.0342	0.0263	0.0191	0.0132	0.0019	0.0005	0	0
Nominal surface errors, mm	0	0.0191	0.0356	0.0263	0.0192	0.0117	0.00097	0.0002	0	0

With the measured results, the nominal surface errors are obtained. Further, the cutting force coefficients are evaluated with the proposed calibration method, and the results are KT=1535 and KR=1037. Also, the runout parameters are estimated by iterative method with good prediction accuracy, and the results are $\rho = 3\mu m$ and $\lambda = 71^\circ$.

Using the calibrated results, the milling forces for the milling test are predicted and compared with experimental results as shown in Fig.5. In addition, milling of thin-walled workpiece with thickness of 4mm is carried out with the same milling parameters, and the predicted milling forces and measured results are shown in Fig.6. Clearly, the predicted milling forces are in good agreement with measured results both in X-direction and Y-direction. It proves the effectiveness of the proposed method to calibrate cutting force coefficients and runout parameters. Also, comparing the Fig.5 with Fig.6, it can be seen that the forces in milling of the thin-walled workpiece with thickness of 4mm are larger than the forces in milling of the thin-walled workpiece with thickness of 2mm although the two tests are carried out with the same parameters. The reason is that deformation of workpiece with thickness of 2mm is more significant than workpiece with thickness of 4mm. The uncut chip thickness and entry angles decrease. Thus, the milling forces become smaller.



(a) F_x (b) F_y

(a) F_x (b) F_y

Figure 6. Comparison of measured milling forces and predicted milling forces (4mm)

6.Conclusions

A new approach is proposed to calibrate the cutting force coefficients using the surface errors related to milling of thin-walled workpiece including effect of cutter runout. Firstly, milling force, and cutter deformation and workpiece deflection are analyzed, and also forming of surface error is expressed. Further, according to induce reason, surface error is separated into nominal surface error and perturbation component due to runout. That cutter runout has no effect on the average surface error is confirmed with theoretical deduction. Thus, an approach for extraction of nominal surface errors from measured surface errors and a method to calibrate cutting force coefficients with nominal surface errors are presented. Finally, the proposed approach is tested with milling tests and validated by comparing the predicted results and experimental results. The main advantage of the approach is that calibration can eliminate influences of cutter runout because cutting force coefficients are calculated with nominal surface errors instead of total surface errors, and also it does not require expensive force meter in practice.

Acknowledgements

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