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# Lengthwise movement dynamic boundary-value problem for trailing boundary ropes

**Andrey Syasev**
*Candidate of Physical and Mathematical Sciences,*
*Associate professor of Differential control department,*
*Dnipropetrovsk national university*

**Tatyana Zelenskaya**
*Post-graduate student*
*of Differential control department*
*Dnipropetrovsk national university*

Abstract
The statement and solution of mathematical physics initial boundary value problem of elastic wave motion in variable-length ropes as applied to the machines, which perform the load hoisting and lowering by using ropes, were considered. The primal problem boundary conditions accurately consider the state equation integration domain boundary changes of rope-load interaction character. The dynamic stress field amplitude value for steel-wire rope section was obtained and also maximum and minimum stress values throughout hoisting unit operating period were determined. The modified continuation method applying showed that the wave motion character in variable length environments has specific features and significantly differ from the wave motion character in fixed boundary environment.
Key words: WAVE EQUATION, TELEGRAPH EQUATION, TRAILING BOUNDARY

**Introduction**

In mathematical models, which are applied for wave propagation process description, the environmental resistance should be considered quite often. Particularly, this is divided to investigation of dynamic elastic displacement in rods and ropes considering frictional forces. Telegraph equation is the basis of such models [1-3]. This equation qualitative studies show that the functions, which are of propagating decaying waves nature, are its solution [2]. At that, it is important to know which solution components describe the wave propagation and which ones describe the wave distortion in order to compose accurate solution and for their physical interpretation. For this purpose, we will use modified continuation method for the task solution. The method presented considers the dilatational wave reflection from the variable end-point and residual oscillation occurring.

The research objective is the determination of modified continuation method applying values quantity for wave movement of reflected and residual waves in steel-wire ropes and also development of stress curve in rope sections throughout mine hoisting system operating.

The modified continuation method is based on longitudinal oscillations reflecting from moving boundary behaviour investigation. The combined stress function was being formed on the basis of numerous experimental investigation results of rope resistance point displacement with winding pulley and using analytical dependence for primal initial boundary value problem by means of modified continuation method.

**Problem statement**

This paper is devoted to the investigation of dynamic displacement and stress field in variable length steel-wire ropes of mine hoisting machines. The ropes are balancing; the main rope carries concentrated stress, which is in the low end of the main rope before the system movement [3].

Therefore, such tasks are solved by rope interpretation with certain equivalent surface appearance thread and, in general, they can be calculated by longitudinal oscillations motion dynamic since the torsion oscillations will not be significant for mine hoisting systems.

But the surface mechanics rope problems do not determine its real condition. For this purpose, the disposition of combined forces between rope individual elements must be known. This problem solving leads to necessity of optimum mathematical model development and investigation of rope individual elements and construction force interaction. In mathematical model, the wave processes movement at each time point is considered as analytical function, rather than by a discrete difference.

The evaluation of dynamic force in variable length perfectly elastic threads suggests that the forces in threads do not increase when only threads uplift without tipweights under non-integrable boundary conditions. In any other case, threads uplift forces increase (in inverse proportion to any given thread length). However, in practice, when moderate lifting speeds, this phenomenon is not observed inasmuch as along with dynamic forces amplitude increasing due to the length reduction, the oscillations amplitude is also reduced.

**Mathematical model development and the task solving**

Let us consider the main rope from the operation beginning moment to the total system stop. Let us investigate the boundary problem of dynamic field finding in variable length rope, where the variable end-point produces perturbation. We will solve the problems by integral $0 < x \le l$, considering that point $x = 0$ will move according to the law

$$v(t) = r\int_0^l\int_0^s \beta(\tau)d\tau ds + r\omega(0)t + v(0),$$

(1)

where $r$ - average radius of rope winding on the pulley;

$\beta(t)$ - angular pulley acceleration; $\omega(t)$ - angular rate; $s$ - the rope cross-sectional area.

The equation (1) describes the rope linear movement in the point of its contact with pulley. Vertically suspension rope, which initial length is $l$, begins to wind on the drum when $t = 0$.

The mine hoisting skip installation with the main load $X_2$ and counterweight $-X_1$; the main rope length will be consist of

$l = h_y + H(t) + h_r$,

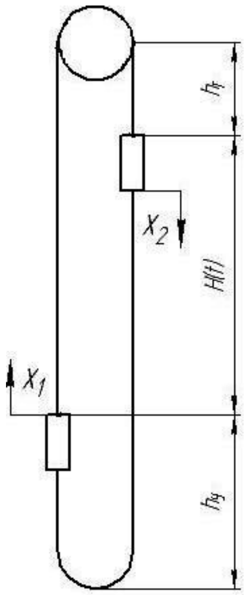


**Figure 1.** Hoisting unit model

where $h_y$ - the distance, on which the counterweight is located at the beginning of system operation; $h_r$ – the distance, which is equal to pulley circle perimeter; $H(t)$ – movable hanging main rope length change function.

Let us direct the axis $\xi$ down along the main rope axis; as zero point $\xi = 0$, we take the point of rope contact with pulley. The average radius of rope winding on the pulley is $r$. Before the rope winding on pulley, i.e. when $t < 0$, the loading had been already slung over the rope and static movements and voltages had been determined in the rope.

Let us divide the considered hoisting systems motion into absolute and relative ones. By absolute system motion, as solid body, considering the axis $\xi$ direction, is meant the motion of all system particles (except $\xi = 0$) with the speed

$$V(t) = r\int_0^t \beta(\tau)d\tau + V(0) \qquad (2)$$

By relative system motion is meant the motion of all system particles including $\xi = 0$) with the speed – $V(t)$. Let us connect the axis $x$ directed downward with the system in its relative motion; the axis $x$ zero is placed at the point $\xi = 0$ if $t = 0$. This axis also moves with system speed $V(t)$, i.e. all system points remain motionless about an axis $x$. From there, in relative motion, the considered system has the form of core of initial length $l$, which is contained in interval $0 \le x \le l$ (except $x = 0$) , all the points remain motionless and the point $x = 0$ moves with speed $V(t)$ changing the core length (2). The point $x = 0$ movement will be described by function (1). The rope elastic deformation under dynamic loading will be considered relative motion. Consequently, relative motion elastic deformation must satisfy the homogeneous wave equation (3).

In interval $0 < x < l + \nu(t), \quad t > 0$, it is required to find out the solution of equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{a^2}\frac{\partial^2 u}{\partial t^2} = 0, \qquad (3)$$

where $a$ - sound speed in steel-wire rope.

This solution must satisfy the initial conditions

$$u(x,0) = 0, \quad u_t(x,0) = 0, \quad 0 < x < l \qquad (4)$$

and boundary conditions

$$\left.\left(\frac{\partial u}{\partial x} + h(x,t)u\right)\right|_{x=0} = 0,$$

$$\left.\left(\frac{\partial u}{\partial x} + h(x,t)u\right)\right|_{x=l+\nu(t)} = \theta(t), \quad t > 0. \qquad (5)$$

In this task, we consider the interval with the length $l$ when $t = 0$; when $t > 0$, the right end of this interval (the rope and drum contact point) moves by the law $x = l + \nu(t)$ where $\nu(0) = 0$.

We find out the task solution by the form

$$u(x,t) = \chi(x - at) + \chi(x + at)$$

The environment non-degeneracy condition is fulfilled

$$l + \nu(t) > 0, \quad t > 0. \qquad (6)$$

Because of the quiescent state of law rope end in relative coordinate system,

$$u(x,t) = \chi(x + at). \qquad (7)$$

Let us use the modified continuation method for the area with moving boundary; let us continue the function $\theta(t)$ for the full axis $t$:

$$\tilde{\theta}(t) = \begin{cases} \theta(t), & t \ge 0, \\ 0, & t < 0 \end{cases} \qquad (8)$$

Inserting the solution form (7) into continuation (8) for the full axis $t$, the second boundary (5) considering (6) we obtain

$$\chi'(l + \nu(t) + at) + h(l + \nu(t), t) \cdot \chi(l + \nu(t) + at) = \tilde{\theta}(t). \qquad (9)$$

Let us introduce the independent variable transformation in equation (9)

$$g(t) = l + \nu(t) + at. \qquad (10)$$

If the equation (10) is unresolvable relating to $t$, the considered task does not possess the solution. There is nothing to do but consider the solution case of equation (10) relating to $t$.

The function $\nu(t)$, which is continuously differentiated when $t > 0$.

The following condition must be fulfilled:

$$|\nu'| < a, \quad t > 0, \qquad (11)$$

where $a$ - sound speed in steel-wire rope.

The function $t_0(g)$ opposite to (10) exists when and only when the function $\nu(t) + at$ is strictly monotone. Due to the statement (11), the function $t_0(g)$ is strictly monotone increasing and continuously differentiated. Consequently, the continued function $t_0(g)$ for the full axis $g$ exhibits the following properties:

$$t_0(g): \begin{cases} > 0, & g > l, \\ = 0, & g = l, \\ < 0, & g > l. \end{cases} \qquad (12)$$

Considering properties (12), the equation (9) can be presented in the form of

$$\chi'(g) + H(g)\chi(g) = \tilde{\theta}(t_0(g)). \qquad (13)$$

Indeed, the general solution of question (3) can be presented in the form

$$\chi(g) = \frac{C + \int_l^g \tilde{\theta}(t_0(y))E(y)dy}{E(g)}, \qquad (14)$$

where $H(g) = h(l + \nu(t_0(g)), t_0(g))$; $E(g) = e^{\int H(g)dg}$, $C$ - arbitrary constant.

The task solution for the initial time period

$$u(x,t)=\frac{1}{E(x+at)}\int_{l}^{x+at}\tilde{\theta}(t_0(y))E(y)dy \qquad (15)$$

as from the first initial equation, it follows that $C=0$.

Considering that $E'(g)=H(g)E(g)$, for function (15) we obtain

$$u_x(x,t)+h(x,t)u(x,t)=-H(x+at)\cdot\frac{1}{E(x+at)}\cdot\int_{l}^{x+at}\tilde{\theta}(t_0(y))E(y)dy+\tilde{\theta}(t_0(x+at))+$$

$$+\frac{h(x,t)}{E(x+at)}\cdot\int_{l}^{x+at}\tilde{\theta}(t_0(y))E(y)dy.$$

The function (15) satisfies the initial condition, the second boundary condition (5) and the equation (3) when $t>0$, but it satisfies first initial condition only when $t<\frac{l}{a}$. For this reason, the following additional problem is solved by modified method: finding of the solution to equation (3) in the area $0<x<l+\nu(t)$, $t>\frac{l}{a}$, which satisfies the initial conditions

$$\rho\left(x,\frac{l}{a}\right)=0,\quad \rho_t\left(x,\frac{l}{a}\right)=0,\quad 0<x<l+\rho\left(\frac{l}{a}\right) \qquad (16)$$

and boundary condition

$$\rho_x(0,t)+h(0,t)\cdot\rho(0,t)=(H(a\cdot t)-h(0,t))\times\times\frac{1}{E(a\cdot t)}\int_{l}^{at}\tilde{\theta}(t_0(y))E(y)-\tilde{\theta}(t_0(a\cdot t))=\varphi_0(t),\ t>\frac{l}{a};\ \varphi_0(t)=0$$

when $t<\frac{l}{a}$. (17)

It follows from initial conditions (16) that

$$\rho(x,t)=-E_1(at-x)\cdot\int_{l}^{at-x}\varphi_0\left(\frac{y}{a}\right)\frac{dy}{E_1(y)}$$

In this case, the main problem solution will be function

$$\psi(x,t)=u(x,t)+\rho(x,t)=\frac{1}{E(x+at)}\cdot\int_{l}^{at+x}\tilde{\theta}(t_0(y))E(y)dy-E_1(at-x)\int_{l}^{at-x}\varphi_0\left(\frac{y}{a}\right)\frac{dy}{E_1(y)}. \qquad (18)$$

**The optimized model results numerical representation**. In order to provide the fixed rope hoisting unit operation safety in all technically feasible productive situations in producing mine and pit shafts, it is necessary to know the main dependences between amplitude-frequency dynamic characteristics of rope longitudinal oscillations, operating and inertial parameters of hoisting unit. Considering the obtained solution (18), let us show the time moments of waves reflected from rope variable end-point appearing (Table 1).

| Wave number | Origin time |
|---|---|
| 0 | 0 |
| 1 | 0,0198019801980198 |
| 2 | 0,0392118419762768 |
| 3 | 0,0582373500559545 |

| No | Time, s | Displacement of p.1 (m) | Displacement of p.2 (m) | Stress at p.1 (Pa) | Stress at p.2 (Pa) |
|---|---|---|---|---|---|
| 0 | 0,00105573120531471 | 0,001001 | 1,000001 | 50 | 49,9999999999945 |
| 1 | 0,00211146241062941 | 0,00102002824330224 | 1,0000168837813 | 49,9992575887123 | 49,9999999999945 |
| 2 | 0,00316719361594412 | 0,00103929371377833 | 1,00003287902501 | 49,9992576023233 | 49,9999999999945 |
| 3 | 0,00422292482125883 | 0,0010587815629372 | 1,00004898573113 | 49,9992576159327 | 49,9999999999945 |

The experimental researches were conducted on the basis of operating mine-hoisting machine of MK5x4-16K type on the ground of plant static-kinematical parameters and load-lifting capacity indexes. The results were compared with theoretical calculations of Scientific Institute "Krivbassproekt". The statical loading moment: main ropes with the average load index 110 kNm; outhaul ropes with maximum loading 110.04 kNm. By modified continuation method, the objective lengthwise movement indexes in the main rope were determined. And also, static and dynamic stress changing amplitude has been deduced during a full working cycle of hoisting system. In production conditions, the diagram was plotted (Fig. 1). It shows the jerk amplitude in the contact point of main rope to the winding pulley; by this amplitude we can determine the interval time where the stress will reach the maximum value and fracture probability will increase.

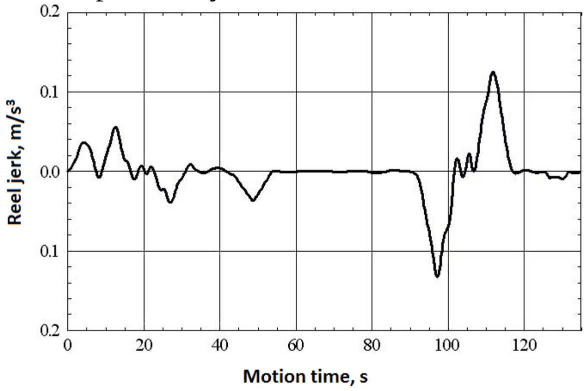


**Figure 1.** The diagram of drum jerk change under the influence of wave processes

Using power interpolation, we find the analytical form of the drum rope point displacement function, which corresponds to the initial-boundary conditions (4), (5) and (19) (20). Previously, we determine the discrete series of inverse function (10) values using the practical data of point displacement speed of hanging main rope.

Considering the obtained results, the diagram of stress field throughout unit operating period is demonstrated. Let us compare the experimental indexes of dynamic stresses using the modified continuation method with theoretical researches data of strength margin (Fig. 2).

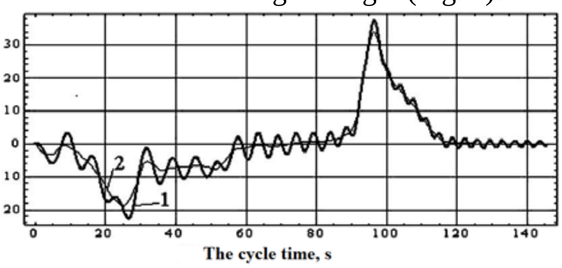


**Figure 2.** The voltage at the point of rope contact to the winding pulley.

1. The stresses function considering reflected and residual waves from moving boundary (practical results).
2. The stresses function without considering of oscillatory displacement at the point of rope contact to the pulley.

In the last diagram, we can see that considering the influence of reflected and residual waves from moving boundary, the minimum and maximum oscillating amplitude objective values can be determined. This method allows selecting of the strength margin optimal parameters conforming to working plant operating parameters in real conditions.

**Conclusions**

The solution of initial boundary value problem allows making of more valid elastic waves propagation conception and distribution of dynamic stress field in the rope of variable length. It must be noted that solution form corresponds exactly to the trajectory of longitudinal oscillations in the rope under the load of main rope low end.

The use of modified continuation method showed that the wave propagation character in the trailing boundary and real resistant environment is of the specific nature, which differs significantly from waves motion character in the fixed boundary. As consequence, the stress jumps occurring because of Heaviside functions in boundary conditions are propagated along the rope with movable boundary differently than in the permanent length rope.

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