Chaos in weighted networks

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Abstract
In this paper, the transition from a non-chaotic state to a chaotic state in weighted networks is discussed. How does the weight between nodes in the network affect the dynamical behavior, which kind distribution of weight could translate more fast or easier on earth? They are the questions that concerned in this letter. We study various networks with different distribution both node degree and weight of edge. Both numerical simulations and theoretical analysis for networks show that the weight is more heterogenous then the transition from non-chaotic to chaotic states can be more fast.
Key words: CHAOS, WEIGHTED NETWORKS, COMPLEX NETWORKS

Introduction

Since Barabasi and Albert proposed the classical BA model [1], complex networks have recently attracted more and more interest of people to research. We also live in all kinds of networks for anytime, such as WWW, airport networks, metabolic networks and so on. A complex networks consists of nodes and edges. Furthermore, we defined weighted networks for the difference strength between two nodes. At the same time, dynamical behaviors of complex networks such an irreplaceable topic it is becomes the focus of study. In[2], Li and Chen studied the condition of transition from non-chaotic states to chaotic states in complex dynamical networks and the chaos of continuous time dynamical networks was discussed in [3]. While the transition to chaos in small-world was analyzed in detail by Yuan [4]. All above works are very interesting and meaningful, but the research very little involved to the chaos in weighted networks, even if involved is also the word, has not carried to the systematic characteristic and the theoretical analysis. In fact, the real network is usually present by weighted networks. The research of chaos in weighted networks is more useful in the application of complex network.

The rest of this paper is organized as follows: The conditions of chaos in the dynamical system are discussed in section 1. The chaos transition of weighted network is discussed in section 2. 3. In section 3, our main results are presented Finally, conclusion and discussion are given is section 4.

The condition of the transition from non-chaos in complex networks

We now consider each node which in a coupled dynamical network is an n-dimensional dynamical system which is described by $\dot{X}(t) = f(x(t))$ where $X(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ are the state variables of the node, and $f(\cdot)$ is a given nonlinear vector-valued function. Consider a general complex dynamical network consisting of N-linearly and diffusively coupled identical nodes. The network is specified by

$$X_i(k+1) = f(X_i(k+1) - c\sum_{j=1}^{N} a_{ij} f(X_j(k)), \ i = 1,2,\ldots,N \quad (1)$$

If there is an edge between node i and node j(i≠j), then $a_{ij} = a_{ji} = 1$, otherwise, $a_{ij} = a_{ji} = 0$, and

$$a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij} = -\sum_{j=1, j\neq i}^{N} a_{ji}, i=1,2,\ldots,N$$

We let $0 = \lambda_1 > \lambda_2 \geq \ldots \geq \lambda_N$ be the eigenvalues of coupling matrix $a = (a_{ij})_{N\times N}$. We assume each node in the network is in non-chaotic regions, which described by $x_i(k+1) = f(x_i(k), c)$ with coupling strength $c > 0$. Hence, all the Lyapunov exponent $h_i (i = 1,2,\ldots,n)$ of each node is non-positive. We let $0 \geq h_{\max} = h_1 \geq h_2 \geq \ldots \geq h_n$. Similar to [1-3, 5], we defined that the Lyapunov exponent u_i of the coupled dynamical networks as

$$u_i(\lambda_k) = h_i - c\lambda_k, k = 1,2,\ldots,N, \quad (2)$$

we have

$$u_1(\lambda_k) \geq u_2(\lambda_k) \geq \ldots \geq u_n(\lambda_k), k = 1,2,\ldots,N$$

$$u_i(\lambda_1) \geq u_i(\lambda_2) \geq \ldots \geq u_i(\lambda_N), i = 1,2,\ldots,n$$

In general, if the coupled network is chaotic, there is at least one positive Lyapunov exponent. So the maximum of h_i is $u_1(\lambda_N) = h_1 - c\lambda_N = h_{\max} - c\lambda_N > 0$, that is the emergence of chaos in dynamical network when $c > \frac{|h_{\max}|}{|\lambda_N|}$.

The emergence of chaos in weighted networks

In above study, we aim at the network with same coupling strength between each node. In fact, many realistic networks have different coupling strength between these nodes. So we described the dynamical network as follows:

$$\dot{x}_t = f(x_i(t)) = \sum_{j=1}^{N} c_{ij} x_j(t), i = 1,2,\ldots,N \quad (3)$$

Where we set $c = (c_{ij})_{N\times N}$, and let c is real symmetric matrix, with additional constraints that $\sum_{j=1}^{N} c_{ij} = 0, i = 1,2,\ldots,N$,

Furthermore, we suppose let

$b_{ij} = c_{ij} \cdot t^{\beta}, t = 1,2,\ldots,N, \beta = 0, \pm 1, \pm 2, \ldots, \pm N$

Without loss generality, let $t = 2, b_{ij} = 2^{\beta} \cdot c_{ij}$ the Lyapunov exponents of the coupled network can be denoted like in section 2 by

$u'(\lambda_k') = h_i + \lambda_k', i = 1,2,\ldots,N, k = 1,2,\ldots,N$

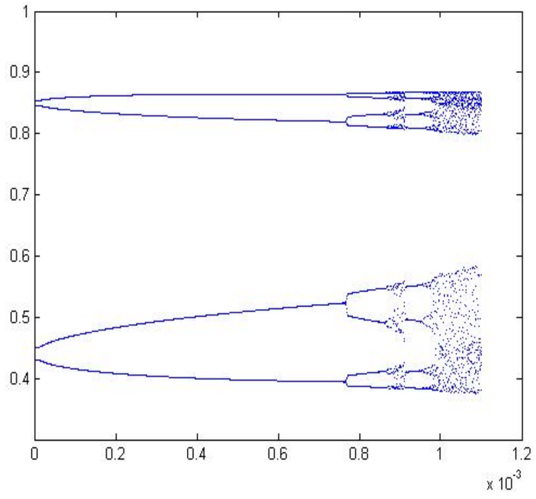
Where $\lambda_k', k = 1,2,\ldots,N$ are the eigenvalues of matrix $B = (b_{ij})_{N\times N}$, if $\beta = 0$ then $b_{ij} = c_{ij}$, and if $\beta = 1$ then $b_{ij} = 2\times c_{ij}$ where $\lambda_k^{€}, k = 1,2,\ldots,N$ are the eigenvalues of matrix $B = (b_{ij})_{N\times N}$.

We know $\lambda_k^{€} = 2\lambda_k'$, it also means that the variance of weight is bigger when the eigenvalues is bigger. In another word, it means the weight is more heterogenous.

With the increasing of the variable β, the variance of weight is increasing too. When $\beta = 1$ chaos more fast then $\beta = 0$, the coupled network will emergence more early.

If $\beta = 2$ then $b_{ij} = 4c_{ij}$.

We demonstrated our results by numerical on dynamical networks with globally coupled networks with n=100, e=5000 and $\beta = 0,1,2$ and the nearest neighbor coupled network, and star networks. We consider each node in a Logistic map.



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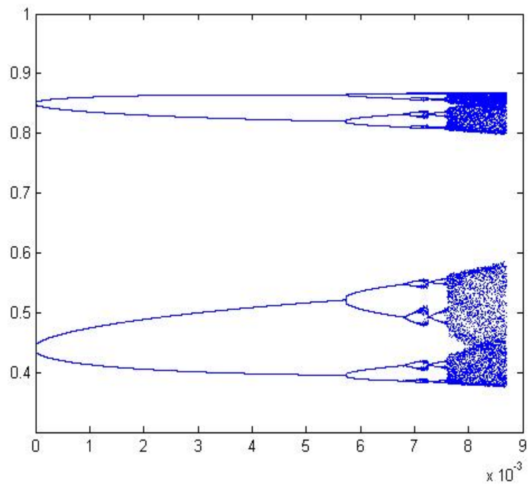

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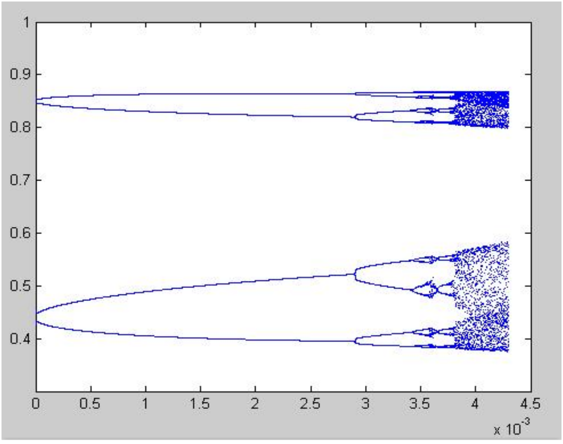

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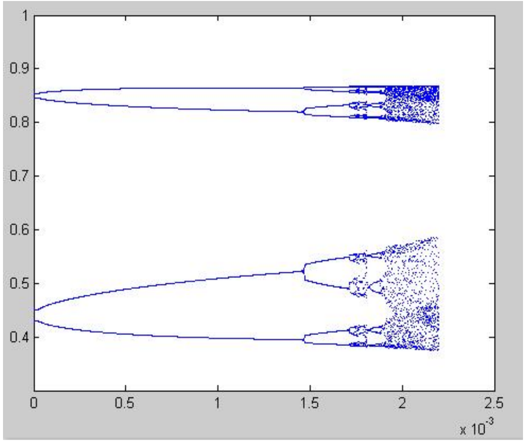

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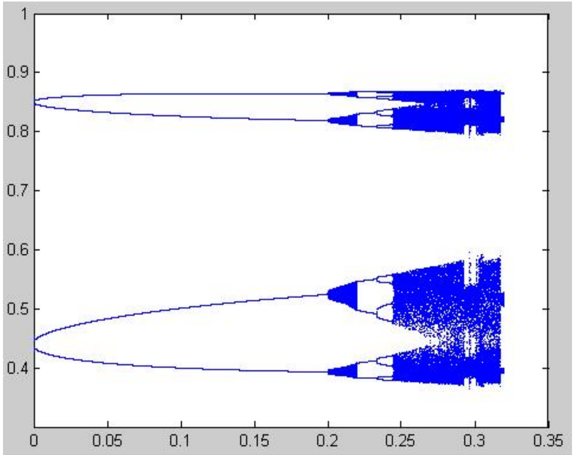
Figure 1. The globally coupled network with n=100, and the emergence of chaos in a globally coupled network with $\beta = 0,1,2,3$ in above maps



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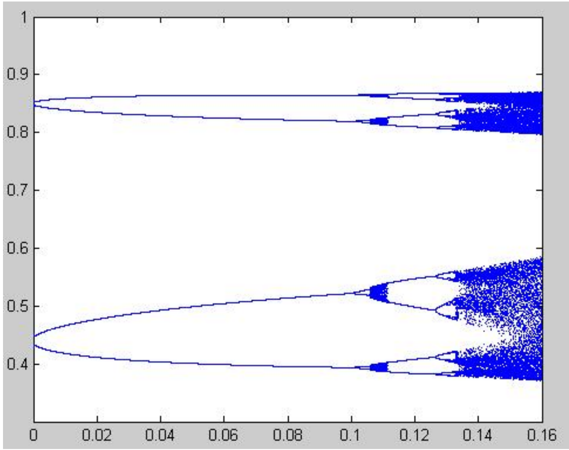

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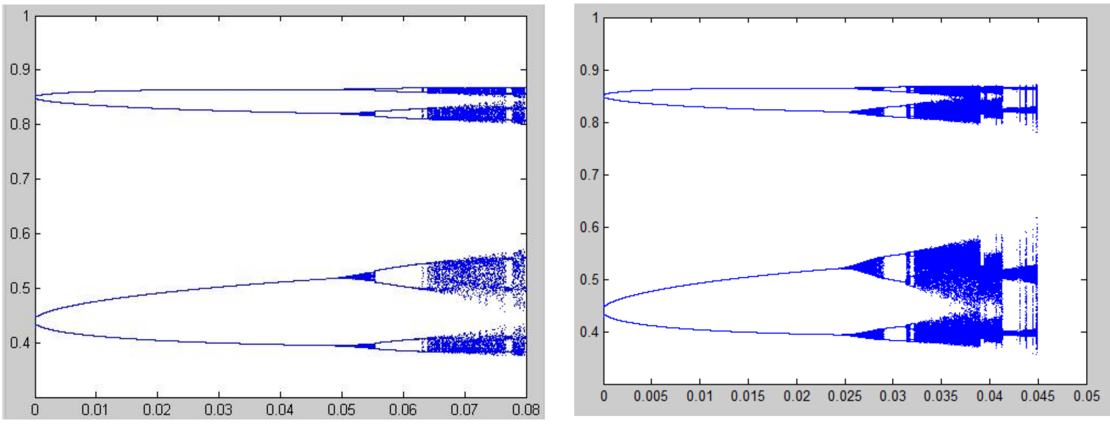

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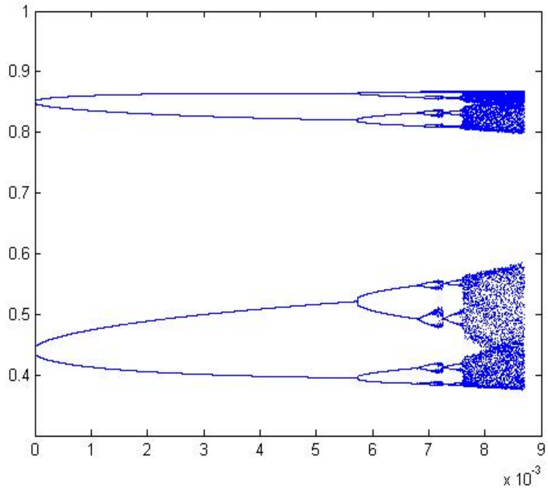
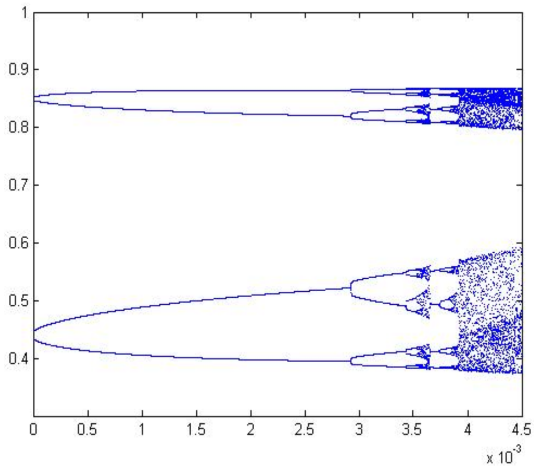
Figure 2. The nearest-neighbor coupled network with n=100, and the emergence of chaos in a nearest-neighbor coupled network with $\beta = 0,1,2,3$ in above maps

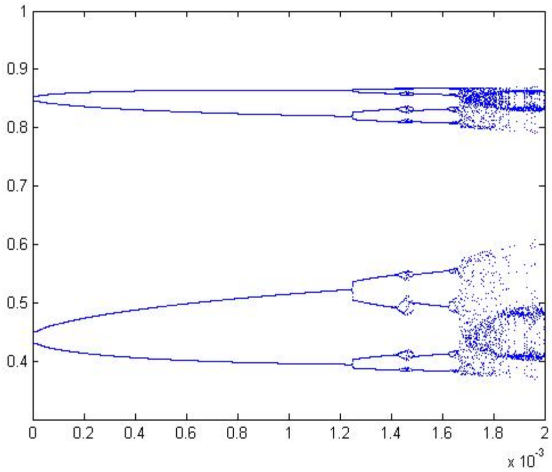
From the two groups of maps, both numeric and simulation results on a 100-nodes globally coupled network and nearest-neighbor coupled network show that the weight is more heterogenous then the transition from nonchaotic to chaotic states can be more easy.



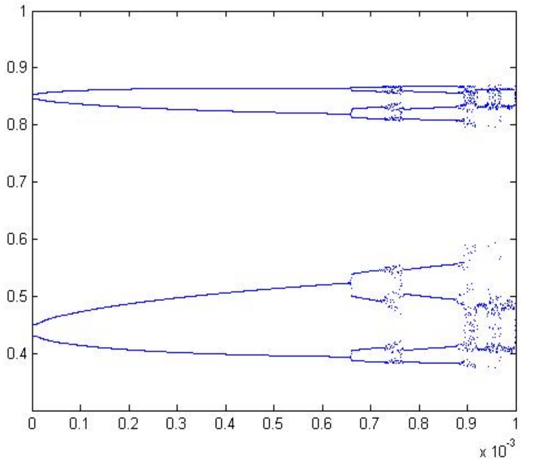
n = 100



n = 200



n = 500



n = 1000

Figure 3. The globally coupled network with n=100,200,500,1000 and the emergence of chaos in a globally coupled network in above maps

Conclusions

In this work, we have studied the transition from nonchaotic to chaotic states in two typical types of coupled networks: the globally coupled network and the nearest-neighbor coupled network. In these networks, we have found that the weight is more heterogenous then the transition from nonchaotic to chaotic states can be more fast. At the same time, our simulation results have verified that the size of the networks more large then the transition from nonchaotic to chaotic states can be more easy.

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