Fuzzy evaluation method of physical fitness sports subjective fatigue based on prospect theory

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Abstract
For physical fitness sports fatigue of subjective evaluation information which is intervened by the fuzziness and randomness may result in the misjudgment of experts, the author proposes an evaluation methodology which is based on the prospect theory. Firstly, the method defines the concepts of the triangle fuzzy number and intuitionistic fuzzy number and fuzzy number and fuzzy random variables. Secondly, the method obtains the prospect value function, and calculates the future value of self-assessment information, and builds the prospect decision making matrix. Finally, it maximizes deviations established programming model for solving the weights, and then according to the comprehensive prospect value of sportier determiner the sports fatigue state.
Key words: SPORTS FATIGUE, FUZZINESS AND RANDOMNESS, MULTIPLE ATTRIBUTE DECISION

Introduction

George L. Engel at the University of Rochester Medical Center put forward the modern medical model (Biopsychosoeial Model) of "bio - psycho - social." which has changed people's health awareness. Personal health not only refers to the physical presence of the disease, but includes the psychological, social, economic, cultural and many other factors. The understanding of health makes people realize the importance of fitness .Moderate exercise can not only promote the health of physical and mental , but also can enhance the function of body's heart and lung and the endurance and flexibility of muscle. What’s more, it haves great effect for the treatment of various chronic diseases of civilization. In the last century, because of the United States launched the "health plans", the incidence of diseases of modern civilization began to decline. In china, it is very important for us to enhance physical fitness, reduce the poverty caused by diseases therefore and have a good life are more important, so individual sports fitness began to receive attention.

Now, in order to make the residents do sports with the right workout places more convenient. Our country builds and opens all kinds of sports grounds and facilities. However, since the forms of personal fitness movement are varied, the environment of decision-making is complicated and the guidance of fitness is inefficiency, people cannot do scientific and subjective evaluation to sports fatigue when they are in the fitness, this gives rise to sports fatigue, and even the movement of injury. So, how to portray subjective information in fitness properly? How to use reasonable methods to help people make the subjective evaluation? All these problems are urgent to be solved.

According to the evaluation made by the subjects or by others, it can be divided into subjective self-assessment and subjective evaluation of others which is based on the motion process of the participants to estimate the level of fatigue by experts. The subjective self-assessment of sports fatigue come mainly from the self-feeling of muscles and cardiopulmonary system in sport, such as fatigue, pain, etc. Borg and Ahlstrom call these feeling which is generated in the sports as perceived exertion and designed the RPE scale. But in fact, the sportier often showed a more complex and mix condition that its vagueness and its randomness exist side by side and this result in the judgment problem of the experts of subjective evaluation of others. In order to simulate and describe the hybrid information accurately, as well as helping the experts of evaluation of others make the decision-making evaluation more effective, this paper uses the intuitionistic fuzzy random variables to describe the information of self-evaluating, and combines the prospect theory to find out the new method of evaluation of sports fatigue of subjective feeling.

The concepts of intuitionistic fuzzy number and intuitionistic random

The definitions of triangular fuzzy number

Definition 1 Let Y be a non-empty finite set, $A=\{<y,\mu_A(y),\ \ v_A(y)>|\ y\in Y\}$ is called intuitionistic fuzzy set, where $\mu_A(y)$ and $v_A(y)$ express the membership degree and non-membership degree of the element y of Y belong to A, thus $\mu_A: Y\to[0,1]$, $v_A: Y\to[0,1]$, and $0\le\mu_A(y)+v_A(y)\le 1, \forall y\in Y$.

Definition 2 Let Y be a non-empty finite set, $A=\{<y,\tilde{\mu}_A(y),\tilde{v}_A(y)>|\ y\in Y\}$ is called triangular fuzzy number intuitionistic fuzzy set, where $\tilde{\mu}_A=[\tilde{\mu}_A^1(y),\tilde{\mu}_A^2(y),\tilde{\mu}_A^3(y)]$ and $\tilde{v}_A=[\tilde{v}_A^1(y),\tilde{v}_A^2(y),\tilde{v}_A^3(y)]$ are the triangular fuzzy number on $[0,1]$, which express the membership degree and non-membership degree of the element y of Y belong to A, and satisfy $0\le\tilde{\mu}_A^3(y)+\tilde{v}_A^3(y)\le 1$, $\tilde{\mu}_A^1(y)\ge 0$, $\tilde{v}_A^1(y)\ge 0,\ \forall y\in Y$.

According to the definitions of intuitionistic fuzzy number and interval intuitionistic fuzzy number, $([\tilde{\mu}_A^1(y),\tilde{\mu}_A^2(y),\tilde{\mu}_A^3(y)],\ [\tilde{v}_A^1(y),\tilde{v}_A^2(y),\ \tilde{v}_A^3(y)])$ is called triangular fuzzy number intuitionistic fuzzy number, simple as $([a^L,b^M,c^U],\ [l^L,\ m^M,n^U])$。 Let Ω be the set of all triangular fuzzy number intuitionistic fuzzy number. Now give the algorithm of triangular fuzzy number intuitionistic fuzzy number:

Definition 3 let $\tilde{\beta}_1=([a_1,b_1,c_1],[l_1,m_1,n_1])$ and $\tilde{\beta}_2=([a_2,b_2,c_2],\ [l_2,m_2,n_2])$ be two triangular fuzzy number intuitionistic fuzzy numbers, rule algorithm:

$$\tilde{\beta}_1\oplus\tilde{\beta}_2=([a_1+a_2-a_1a_2,b_1+b_2-b_1b_2,\ c_1+c_2-c_1c_2],[l_1l_2,p_1p_2,q_1q_2]$$

$$\tilde{\beta}_1\otimes\tilde{\beta}_2=([a_1a_2,b_1b_2,c_1c_2],[l_1+l_2-l_1l_2,\ p_1+p_2-p_1p_2,q_1+q_2-q_1q_2])$$

$$\lambda\tilde{\beta}=([1-(1-l_1)^\lambda,1-(1-p_1)^\lambda,\ 1-(1-q_1)^\lambda],[(a_1)^\lambda,(b_1)^\lambda,(c_1)^\lambda]),\lambda\ge 0$$

$$\tilde{\beta}^\lambda=([(a_1)^\lambda,(b_1)^\lambda,(c_1)^\lambda],\ [1-(1-l_1^L)^\lambda,\ 1-(1-p_1)^\lambda,1-(1-q_1)^\lambda]),\lambda\ge 0$$

The result of algorithm is still triangular fuzzy number intuitionistic fuzzy number, satisfying the law of algorithm.

$$\tilde{\beta}_1\oplus\tilde{\beta}_2=\tilde{\beta}_2\oplus\tilde{\beta}_1$$

$$\tilde{\beta}_1\otimes\tilde{\beta}_2=\tilde{\beta}_2\otimes\tilde{\beta}_1$$

$$(\tilde{\beta}_1\otimes\tilde{\beta}_2)^\lambda=\tilde{\beta}_1^\lambda\otimes\tilde{\beta}_2^\lambda,\quad \lambda\ge 0$$

$$\tilde{\beta}_1^{\lambda_1}\otimes\tilde{\beta}_1^{\lambda_2}=\tilde{\beta}_1^{\lambda_1+\lambda_2},\quad \lambda_1,\lambda_2\ge 0$$

The comparisons of triangular fuzzy number intuitionistic fuzzy number

Definition 4 Let $\beta=[a,b,c]$ be a triangular fuzzy number and $a\le b\le c$, then the expect value of β is

$$E^\theta(\beta)=\frac{(1-\theta)a+b+\theta c}{2}.$$

In the equation, θ is the optimistic coefficient, if $0\le\theta\le 0.5$, this indicates the pessimistic attitude of the decision-maker. If $0.5\le\theta\le 1$, this indicates the optimistic attitude of the decision-maker. If $\theta=0.5$, this indicates the neutral attitude of the decision-maker, and then the expect value is

$E(\beta)=\dfrac{a+2b+c}{4}$.

Definition 5 Let $\beta=([a,b,c],[l,m,n])$ be a triangular fuzzy number intuitionistic fuzzy number, then

$\tilde{S}(\tilde{\beta})=\dfrac{a+2b+c}{4}-\dfrac{l+2m+n}{4}$

is called the score function of $\tilde{\beta}$, where $\tilde{S}(\tilde{\beta})\in[-1,1]$. Obliviously, the value of $\tilde{S}(\tilde{\beta})$ is bigger, and the value of $\tilde{\beta}$ is bigger. If $\tilde{S}(\tilde{\beta})=1$,then $\tilde{\beta}$ get the maximum value $([1,1,1],\ [0,0,0])$. If $\tilde{S}(\tilde{\beta})=-1$, then $\tilde{\beta}$ get the maximum value $([0,0,0],\ [1,1,1])$。

When the value of $\tilde{S}(\tilde{\beta}_j)(j=1,2,\cdots n)$ appears equal, definite the exact function as

$\tilde{L}(\tilde{\beta})=\dfrac{a+2b+c}{4}(2-\dfrac{a+2b+c}{4}-\dfrac{l+2m+n}{4})$

where $\tilde{L}(\tilde{\beta})\in[0,1]$. The value of $\tilde{L}(\tilde{\beta})$ is bigger, and the value of $\tilde{\beta}$ is bigger.

Definition 6 Let $\tilde{\beta}_1,\tilde{\beta}_2$ be two triangular fuzzy number intuitionistic fuzzy numbers, then

(1)if $\tilde{S}(\tilde{\beta}_1)<\tilde{S}(\tilde{\beta}_2)$, then $\tilde{\beta}_1<\tilde{\beta}_2$;

(2)if $\tilde{S}(\tilde{\beta}_1)=\tilde{S}(\tilde{\beta}_2)$, then

①when $\tilde{L}(\tilde{\beta}_1)=\tilde{L}(\tilde{\beta}_2)$, $\tilde{\beta}_1=\tilde{\beta}_1$;

②when $\tilde{L}(\tilde{\beta}_1)<\tilde{L}(\tilde{\beta}_2)$ 时, $\tilde{\beta}_1<\tilde{\beta}_1$。

Fuzzy random variable

Definition 7 Call ξ as a fuzzy random variable, if ξ be a function from the probability spacer ((Ω,A,Pr) to the fuzzy variable set, and for the $Borel$ set B of R, and $Cr\{\xi(\omega)\in B\}$ is measurable function of ω.

Theorem 1 Let ξ be a fuzzy random variable, then for the $Borel$ set B of R, and the credibility measure of $Cr\{\xi(\omega)\in B\}$ is a random variable.

Theorem 2 Let ξ be a fuzzy random variable, and then if for each ξ, expect value $E(\xi(\omega))$ is limited, then $E(\xi(\omega))$ is a random variable.

For example, $\xi=(([a,b,c],[l,m,n]),\mathrm{Pr})$ is a fuzzy random variable, where triangular fuzzy number intuitionistic fuzzy number $([a,b,c],[l,m,n])$ indicate the value of i th state, and Pr indicate the probability of i th state.

The prospect theory

Most of the existing researches which are aimed at the random and multi-attribute decision problems build upon the classical expected utility theory, but the expected utility theory exist the phenomenon of Allias paradox, Ellsberg paradox and so on. Nobel Prize winners in economics, Kahneman, came up with the famous prospect theory. The theory and its latest results respectively uses the value and probability weights replace the utility and probability of the expected utility theory, in addition, the two can also represented as binary function which is affected by the results and probability, because of they all have the background dependence. Experts and scholars at home and abroad now generally considered that this theory could reflect and describe the process of practical distinguish outstanding more realistic. For example, Gregory made use of this theory to analysis the financial decision-making process of people. In the language environment, Peide Liu proposed the risk decision method based on the prospect theory. In view of the influences of risk attitude of decision to the multi-objective decision, he proposed the multi-objective gray situation decision method which is based on the prospect theory. Here are the concepts of the prospect theory:

The prospect value is decided by the value function and weight of decision making,

$V=\sum_{i=1}^{n}\pi(p_i)v(x_i)$

where $\pi(p_i)$ is the weight of decision making. It is a monotone increasing function of probability evaluation, and its weight can be get by the logarithmic function:

$\pi(p)=\dfrac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{\frac{1}{\gamma}}}$.

The weight function of decision making has the features：

(1) The weight function of decision making is not probability, π is the increasing function of p, but it not match the axioms of probability, and not explain the degree of personal expectations.

(2)When probability p is small, $\pi(p)>p$ explains that the decision maker excessively notes the very small probability event. When probability is general and very big, $\pi(p)<p$ explains that the decision maker excessively notes the very low probability event, and ignores the routine events.

$v(x_i)$ is a value function, it is the value of decision maker subjective felling. Tversky and Kahneman gave the form of value function as power function:

$$v(x)=\begin{cases}x^{\alpha}, & x\geq 0\\ -\theta(-x)^{\beta}, & x<0\end{cases}$$

where parameter α and β indicate the concave and convex degree of profit and loss region value power function, $\alpha,\beta<1$ indicates sensitivity decreases. Parameter θ indicates the steeper character of loss area than profit area. $\theta>1$ indicates loss aversion.

Definition 7 Let $\beta_1=([a_1,b_1,c_1],[l_1,m_1,n_1])$, $\beta_2=([a_2,b_2,c_2],[l_2,m_2,n_2])$ be two triangular fuzzy number intuitionistic fuzzy number, if considering β_2 as reference point, then the prospect utility value function of triangular fuzzy number intuitionistic fuzzy number β_1 is :

$$v(\beta_1)=\begin{cases}(d(\beta_1,\beta_2))^{\alpha}, \beta_1\geq\beta_2\\ -\theta(d(\beta_1,\beta_2))^{\beta}, \beta_1\leq\beta_2\end{cases}$$

where $d(\beta_1,\beta_2)$ indicates the distance of two triangular fuzzy number intuitionistic fuzzy number.

The fuzzy evaluation method of physical fitness sports subjective fatigue based on prospect theory

Let sports fatigue solution set be $X=\{x_1,x_2,\cdots,x_m\}$, and attribute set be $C=\{c_1,\cdots,c_n\}$. Because the uncertainty of environmental around the activists. Let states set be $Z=\{Z_1,\cdots,Z_k\}$, and the probability of the s th state is p_k. The solution x_j is measured according to the attribute c_i. The given value is triangular fuzzy number intuitionistic fuzzy number $\tilde{x}_{ij}^{(s)}$, so the value of solution x_j measured according to the attribute c_i under the state s is intuitionistic fuzzy random variable. So we can get the decision making matrix $D^{(s)}=(\tilde{x}_{ij}^{(s)})_{mn}$.

Step 1 Standardize status value.

In order to eliminate various properties of the influence of different physical dimension to the decision result, we need standard status values respectively according to efficiency and cost using the standard formula. So we can get k decision making matrix : $D^{(s)}=(\tilde{x}_{ij}^{(s)})_{mn}$.

Step 2 Decide the prospect decision matrix of solutions.

Consider the other solutions as reference point; calculate the prospect value under each attribute, and get the prospect value matrix. The prospect value of each attribute is $V_{ij}(\beta)=\sum_k\sum_l v(\tilde{x}_{ijl}^{(s)})\pi_{ijl}(p_k)$,

where prospect function is

$$v(\tilde{x}_{ijl}^{(s)})=\begin{cases}(d(\tilde{x}_{ij}^{(s)},\tilde{x}_{lj}^{(s)}))^{\alpha}, \tilde{x}_{ij}^{(s)}\geq\tilde{x}_{lj}^{(s)}\\ -\theta(d(\tilde{x}_{ij}^{(s)},\tilde{x}_{lj}^{(s)}))^{\beta}, \tilde{x}_{ij}^{(s)}<\tilde{x}_{lj}^{(s)}\end{cases}.$$

According to the decision makers' attitude facing the profit and loss, the weight of prospect function is

$$\pi(p_s)=\begin{cases}\dfrac{p_s^{\gamma}}{(p_s^{\gamma}+(1-p_s)^{\gamma})^{\frac{1}{\gamma}}}, \tilde{x}_{ij}^{(s)}\geq\tilde{x}_{lj}^{(s)}\\ \dfrac{p_s^{\delta}}{(p_s^{\delta}+(1-p_s)^{\delta})^{\frac{1}{\delta}}}, \tilde{x}_{ij}^{(s)}<\tilde{x}_{lj}^{(s)}\end{cases}.$$

The parameter γ and δ control the curvature of prospect weighting function curve, and respectively indicates the value facing profit and loss. So get the prospect decision making matrix

$R=(V_{ij}(\beta))$.

Step 3 Calculate the weights and rankings.

The merits of the solutions are only distinguished under the unified standard. So the comprehensive prospect value must be from the same criteria weight vector. Therefore, by adopting the idea of maximizing deviation, structure optimization modal.

For prospect decision matrix, the deviation of decision solution x_i to other all solutions:

$$L_i=\sum\sum d(V_{ij}^{(s)},V_{kj}^{(s)})\omega_j.$$

By the idea of maximizing deviations, structure the optimization modal:

$$\max Z=\sum L_i$$

$$s.t\begin{cases}\sum_{j=1}^{n}\omega_j=1\\ \omega\in Z\end{cases}.$$

We can get the optimal solution ω^*, and then calculate the comprehensive prospect value:

$$V_i(\beta)=\sum_{j=1}^{n}V_{ij}(\beta)\omega_j^*。$$

Conclusion

F or physical fitness sports fatigue of subjective evaluation information which is intervened by the fuzziness and randomness may result in the misjudgment of experts, the author proposes an evaluation methodology which is based on the prospect theory. This method uses the triangular fuzzy number, intuitionist fuzzy numbers and random variable to present the self-evaluation information of campaign fatigue. Furthermore, it uses the value function and weighting function of prospect theory to replace the utility function and probability of expected utility theory,this reflects the characteristics of psychological behaviors of self-evaluators' exercise fatigue, so that the result of decision analysis is more consistent with the subjective perception of campaigners.

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Reference

1. Ahumad A M, Cervera A (1989) Cardoso G Effects of aerobic physical conditioning on the profile of plasma lipoproteins in a group of healthy volunteers. *Archieve Instance of Cardiol Mex,* 59(1), p.p.43-50.
2. Eisenmann J. (2002) Blood lipids and lipoproteins in child and adolescent athletes. *Sports Medicine*, 32(5): 297-307.
3. Borsheim E, Knardahl S, Hostmark A. (1999) Short-term effects of exercise on plasma very low density lipoproteins (VLDL) and fatty acids. *Medical Science Sports Exercise*, 31(41), p.p.522-530.
4. Yu L, Lai K K. (2011) A distance-based group decision-making methodology for multi-person multi-criteria emergency decision support. *Decision Support Systems,* 51(2), p.p. 307-315.
5. Peng Y, Zhang Y, Tang Y, et al. (2011)An incident information management framework based on data integration. data mining, and multi-criteria decision making. *Decision Support Systems,* 51(2), p.p. 316-327.
6. Kahneman D, Tversky A. Prospect theory: An analysis of decision under risk. Econometrica, 1979, 47(2), p.p. 263-291.
7. Tversky A, Kahneman D. (1992) Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty,* 5(4), p.p. 297-323.
8. Langer T, Weber M. (2001) Prospect theory, mental accounting, and differences in aggregated and segregated evaluation of lottery portfolios. *Management Science,* 47(5): 716-733.
9. Leichrodt H, Schmidt U, Zank H. (2009) Additive utility in prospect theory. *Management Science* , 55(5), p.p. 863-873.