

Underground railway rolling stock rational operation mode determination

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Abstract

For the first time, it was developed the mathematical and logarithmical support for rolling stock rational operation mode determination with allowance for installation of capacitive storage of given energy capacity on its border. The criterion for rational operation mode determination was suggested. The rolling stock rational haul operation mode was determined on the basis of algorithm worked out for given conditions.

Key words: UNDERGROUND RAILWAY ROLLING STOCK, DYNAMIC PROGRAMMING METHOD, CAPACITIVE ENERGY STORAGE

Introduction

One of the main and urgent underground railway problems is reduction of power consumption and improvement of transportation quality. These problems may be solved by traffic control automation [1-4]. Partial or integrated automation of traffic process may be performed by using of automatic train operation system. These systems allow not only reduction of power consumption but also traffic safety increasing, the use of line capacity improvement and drivers working conditions facilitating. Power consumption reduction effect is reached by more accurate traffic schedule fulfillment comparing with manual operation.

The considerable number of published papers is devoted to such problems solving [1-8]. Usually, they are focused on optimum train operation mode determination by criterion of predetermined traffic schedule providing and the minimum consumption from the network [1-4]. First of all, this is due to practical utility inasmuch as energy-efficient control allows reduction of electric energy consumption from the network without significant expenditures and resources.

At the present time, the regenerative braking and energy storage units are introduced for increasing of underground railway operational energy efficiency. At that, the use of capacitive energy storages (CES) in underground railway is long-range [9-11]. The problem of rolling stock rational operation mode determination considering above-mentioned implementations is urgent and understudied [2, 12-14]. Consequently, the necessity of rolling stock rational operation mode determination arises. It is fulfilled by criterion of providing of predetermined traffic time and minimum energy consumption from the network considering the CES recoverable electric energy accumulation.

Work objective: underground railway rolling stock rational operation mode determination considering the energy-exchanged processes between the network and capacitive energy storage installed on its border.

The researches materials and results

The rolling stock rational operation mode determination comes down to objective function extremum seeking, having determined controlled variables values, which cause it. By now, a

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great deal of rationalization problems solving methods has been accumulated in science [2, 15, 16].

The discrete variant of dynamic programming method has been used for rolling stock rational operation mode determination. The use of this method makes sense when solutions finding, if the controlled object is given implicitly by means of equations describing the operations in time. The main benefit of this method is the ability of step-by-step process rationalization but considering all the previous steps. This method principle is locating on each control step where the electric energy costs along the whole haul will be minimum when predetermined travel time observing [2, 17, 18].

The mathematical model for this problem solving has been developed. It is introduced in the form of differential equation system describing the rolling stock motion along the given track profile and power interchange between network and CES:

$$\left. \begin{aligned}
 &F = m \frac{dV}{dt} (1 + \gamma) + W_{\text{main}} \pm W_i; \\
 &W_{\text{main}} = aV^2 + bV + c; \\
 &W_i = mgi; \\
 &A_{\text{net}} = A_{\text{tot}} - A_{\text{stor}}; \\
 &A_{\text{net}} = \frac{U_{\text{cstrac}} [U_{\text{csbrac}}] \cdot \int_0^t I_{\text{trac}} [I_{\text{brak}}] dt}{3600}; \\
 &A_{\text{stor}} = \frac{U_{\text{CES}} \cdot \eta_{\text{CES}} \cdot \int_0^t I_{\text{trac}} [I_{\text{brak}}] dt}{3600}; \\
 &I_{\text{trac}} = \frac{FV}{U_{\text{cstrac}} [U_{\text{CES}}] \eta_{\text{inv}} \eta_{\text{TM}} \eta_{\text{red}}}; \quad I_{\text{brak}} = \frac{FV \eta_{\text{inv}} \eta_{\text{TM}} \eta_{\text{red}}}{U_{\text{csbrac}} [U_{\text{CES}}]}; \\
 &\eta_{\text{TM}} = f(V),
 \end{aligned} \right\} (1)$$

where F – rolling stock traction (braking) power; m – rolling stock mass; V – rolling stock motion speed; t – current time; $(1 + \gamma)$ – rotating mass inertia coefficient; W_{main} – the main resistance to motion; W_i – the additional resistance to motion from effective grade; g – gravitational acceleration; a, b, c – coefficients depending on rolling stock design; i – effective grade; A_{net} – the amount of consumed (recoverable) electric energy from the network (to the network); A_{tot} – total amount of consumed (recoverable) electric energy;

A_{stor} – the amount of consumed electric energy from the storage (recoverable to the storage); U_{cstrac} – contact system voltage in traction mode; U_{csbrac} – contact system voltage in braking mode; U_{CES} – CES voltage; I_{trac} – current consumption in traction mode; I_{brak} – recoverable current in braking mode; η_{inv} – inverter efficiency coefficient; η_{TM} – traction motor efficiency coefficient; η_{red} – reduction gear efficiency coefficient; η_{CES} – CES efficiency coefficient.

The criterion for rolling stock rational operation mode determination has been suggested for this problem solving:

$$Q = \int_0^L (a_1 \cdot T_{\text{rel}} + a_2 \cdot A_{\text{rel}}) dL \rightarrow \min, \quad (2)$$

where $T_{\text{rel}} = \left(\frac{t_{\Sigma} - T}{T} \right)^2$ – relative time value; t_{Σ} – rolling stock motion time along the haul; T – predetermined time of motion along the haul; $A_{\text{rel}} = \left(\frac{A_{\text{net}}}{A_{\text{tot}}} \right)$ – relative electric energy value; L – haul length.

The underground railway rolling stock rational operation mode determination algorithm has been developed on the basis of differential equation system (1) and criterion (2) considering the CES installation on its border (Fig. 1). This algorithm implementation contains the steps, which principles are given below.

Assigned data: tractive characteristic – $F_{\text{trac}}(V)$; braking characteristic – $F_{\text{brak}}(V)$; a, b, c, T ; a_1, a_2 ; $\eta_{\text{TM}}(V)$; η_{inv} ; η_{red} ; m ; $(1 + \gamma)$; A_{CES} ; U_{CES} ; η_{CES} ; U_{cstrac} ; U_{csbrac} .

Control actions possible conditions are assigned: traction force control – α , braking force control – β .

Assigned haul speed limitation – $V_{\text{max1...M}}$.

The subprogram of track profile is used (Fig. 2). The track profile rectification technique is described in more detail in the papers [19, 20].

The variation steps are assigned considering that the track profile and allowable speed should not be changed on each of its control steps.

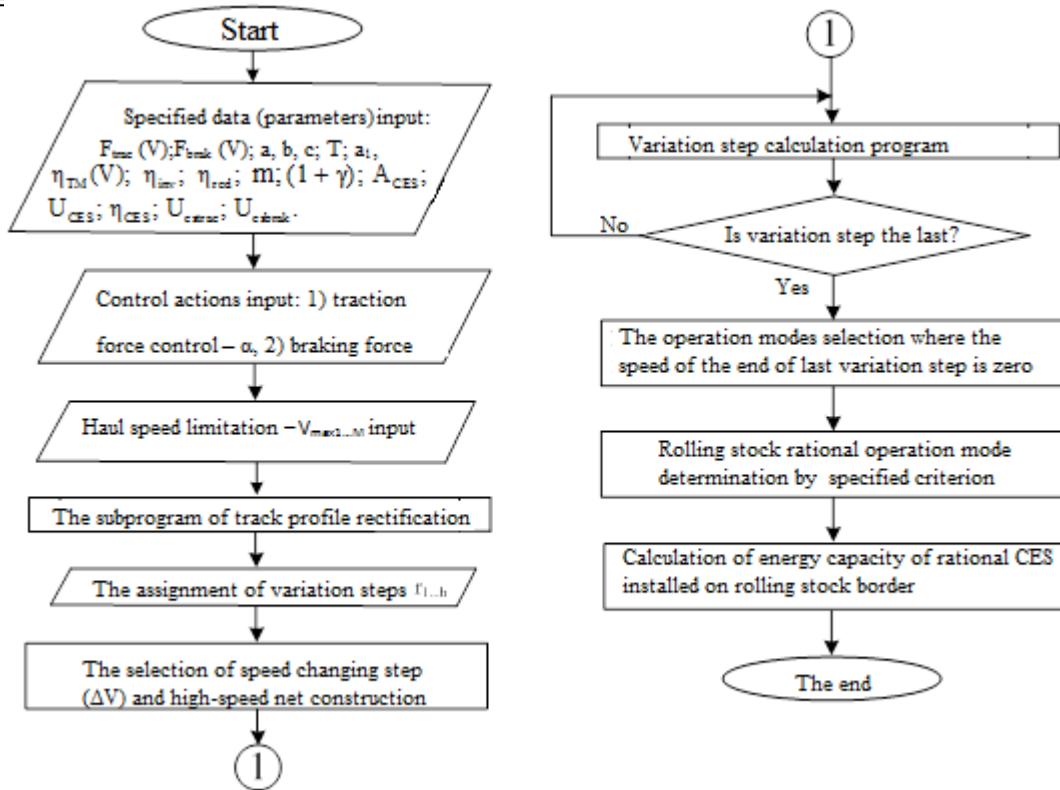


Figure 1. Control flow chart of underground railway rolling stock rational operation mode determination

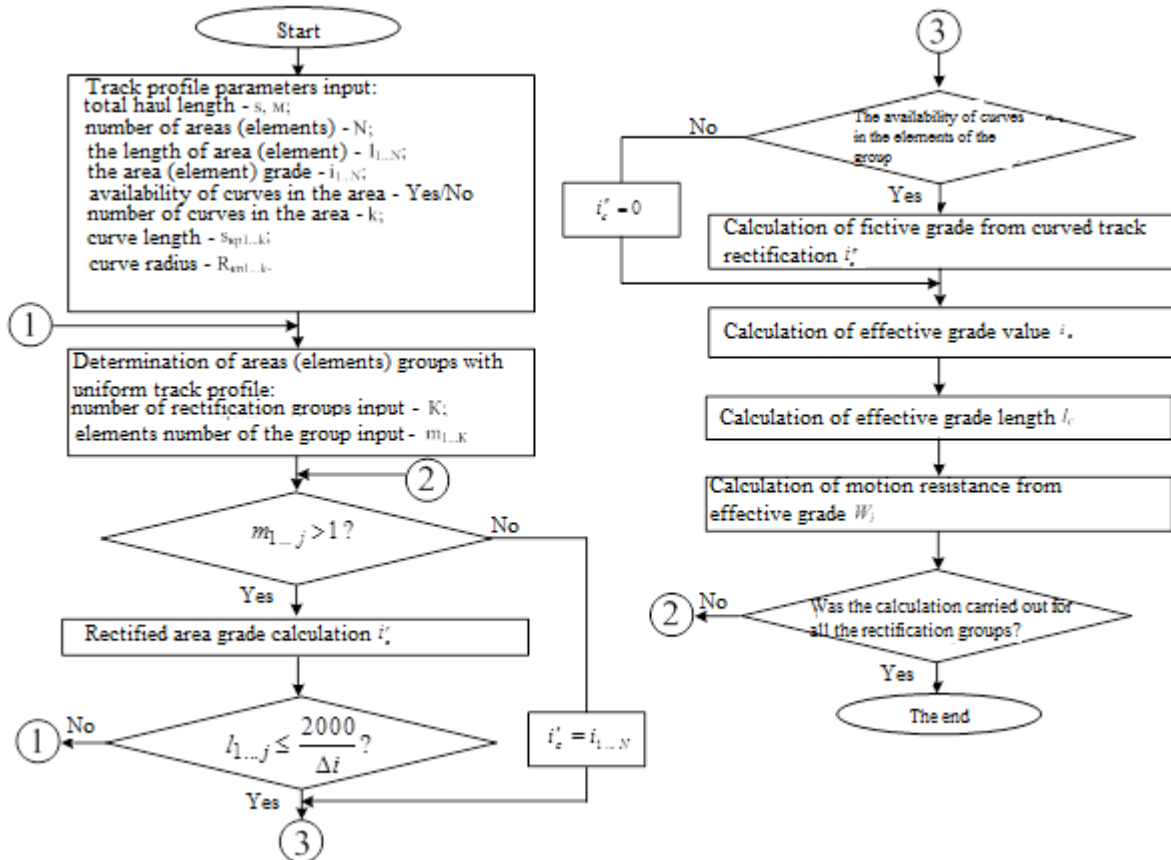


Figure 2. The subprogram of rectified track profile

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The step of speed change (ΔV) is selected and the speed net for various control actions and rectified track profile grades is formed. The formation of speed net means the calculation of distance covered, motion time and consumed (recoverable) electric energy under speed change ΔV varying from 0 to V_{max} .

The distance covered and motion time are determined by the method of rolling stock motion equation integration [19–22]. The distance covered:

traction mode

$$\Delta s = \int_0^s ds = \frac{m(1+\gamma)}{\zeta'(F_{trac} - W_{main} \pm W_i)} \int_{V_1}^{V_2} V dV; \quad (3)$$

stopping regime

$$\Delta s = \int_0^s ds = \frac{m(1+\gamma)}{\zeta'(-W_{main} \pm W_i)} \int_{V_1}^{V_2} V dV = \frac{m(1+\gamma)}{\zeta'(W_{main} \pm W_i)} \int_{V_2}^{V_1} V dV; \quad (4)$$

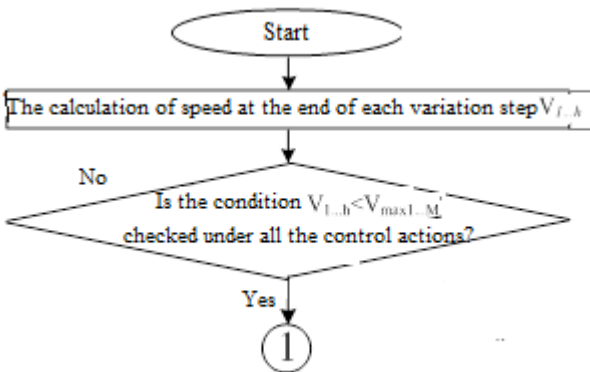
braking mode

$$\Delta s = \int_0^s ds = \frac{m(1+\gamma)}{\zeta'(F_{brak} + W_{main} \pm W_i)} \int_{V_2}^{V_1} V dV, \quad (5)$$

where $\zeta' = 12,96$ – unit conversion coefficient;

V_1, V_2 – rolling stock initial and final speed respectively.

Motion time:
traction mode



$$\Delta t = \int_0^t dt = \frac{m(1+\gamma)}{\zeta(F_{trac} - W_{main} \pm W_i)} \int_{V_1}^{V_2} dV; \quad (6)$$

stopping regime

$$\Delta t = \int_0^t dt = \frac{m(1+\gamma)}{\zeta(-W_{main} \pm W_i)} \int_{V_1}^{V_2} dV = \frac{m(1+\gamma)}{\zeta(W_{main} \pm W_i)} \int_{V_2}^{V_1} dV; \quad (7)$$

braking mode

$$\Delta t = \int_0^t dt = \frac{m(1+\gamma)}{\zeta(F_{brak} + W_{main} \pm W_i)} \int_{V_2}^{V_1} dV, \quad (8)$$

where $\zeta' = 3,6$ – unit conversion coefficient.

Amount of electric energy [19, 21, 23]:

traction mode

$$\Delta A = \frac{U_{cstrac} \cdot \int_0^t I_{trac} dt}{3600 \cdot 1000}, \quad (9)$$

$$\text{where } I_{trac} = \frac{F_{trac} \cdot V}{U_{trac} \eta_{inv} \eta_{TM} \eta_{red}}; \quad (10)$$

braking mode

$$\Delta A = \frac{U_{csbrak} \cdot \int_0^t I_{brak} dt}{3600 \cdot 1000}, \quad (11)$$

$$\text{where } I_{brak} = \frac{\eta_{inv} \eta_{TM} \eta_{red} \cdot F_{brak} \cdot V}{U_{brak}}. \quad (12)$$

The variation step calculation subprogram is fulfilled (Fig. 3).

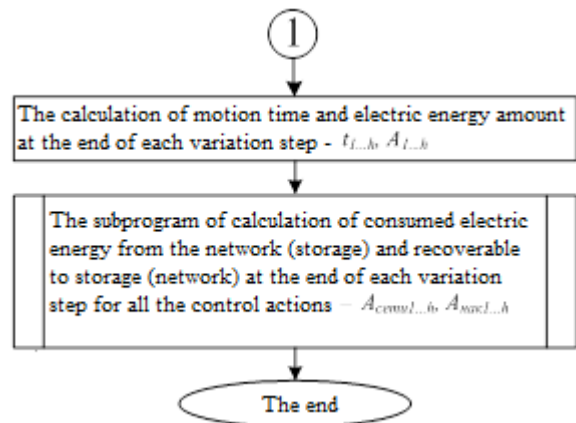


Figure 3. The variation step calculation subprogram

The speed at the end of variation step is determined by the formed speed net for given control actions.

After the speed at the end of variation step determination the condition is repeated: “Isn’t the speed at the end of variation step higher than allowable speed on this area?”. If the condition is fulfilled, the motion time and consumed (recoverable) electric energy is determined by formed speed net for given variation step.

Otherwise, the control actions, under

which the conditions are not fulfilled, are excluded in this step.

The subprogram of calculation of consumed electric energy from the network (storage) and recoverable to the storage (network) at the end of variation step for the other control actions (Fig. 4).

For all the other control action the condition is checked: “Traction mode?”. If the condition is not fulfilled, another condition is checked: “Braking mode?”. If the above mentioned

conditions are not fulfilled, the calculations for stopping regime are carried out.

If the condition “Traction mode?” is fulfilled, the following condition is checked:

“(A_{cons}-A_{stor} · η_{CES}) ≥ 0?”. If the condition is fulfilled: (A_{cons}-A_{stor h-1} · η_{CES}) ≥ 0: A_{net}=A_{cons}-A_{stor h-1} · η_{CES}; A_{stor}=0. Otherwise: A_{net}=0; A_{stor}=A_{cons h-1}-A_{cons}/η_{CES}.

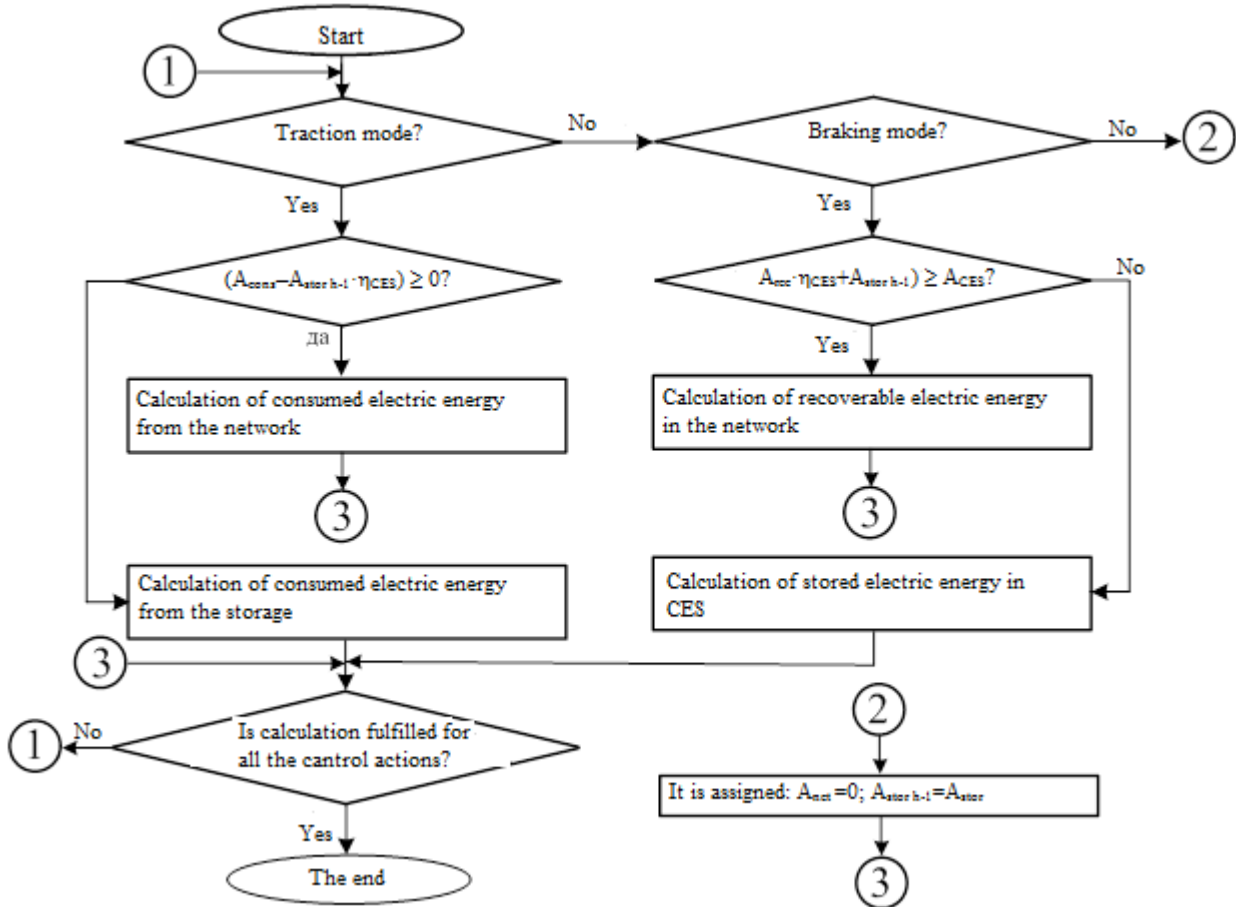


Figure 4. The subprogram of calculation of consumed electric energy from the network (storage) and recoverable to the storage (network)

If the condition “Braking mode?” is fulfilled, the following condition is checked: “A_{rec} · η_{CES} + A_{stor h-1} ≥ A_{CES}?”. If the condition is fulfilled: (A_{rec} · η_{CES} + A_{stor h-1}) ≥ A_{CES}: A_{stor}=A_{CES}; A_{net}=A_{rec}+A_{stor h-1} · η_{CES}-A_{CES} · η_{CES}. Otherwise: (A_{rec} · η_{CES} + A_{stor h-1} < A_{CES}: A_{stor}=A_{CES}; A_{net}=A_{rec}+A_{stor h-1} · η_{CES}-A_{CES} · η_{CES}.

Calculation of stopping regime: A_{net}=0; A_{stor}=A_{stor h-1}.

After each control action calculation, the condition is checked: “Is calculation fulfilled for all the control actions?”. If the condition is fulfilled, this variation step subprogram is finished. Otherwise, the calculations for the next control variation of this variation step are fulfilled.

When each variation step calculation is finished, the condition is checked: “Is variation step the last?”. If the condition is fulfilled, the selection of haul operation is carried out, where the

speed in the end of last variation step is zero. Otherwise, the calculations of the next variation step for given control actions are carried out. For chosen haul operations (the speed in the end of last variation step is zero), the amount of consumed electric energy from the network (storage) and the amount of recoverable electric energy to the storage (network) are calculated.

The amount of consumed electric energy from the storage for the traction mode is determined by formulas respectively [2, 20]:

$$A_{\text{cons(net)}} = \sum_{a=1}^h A_{\text{net}}; \tag{13}$$

$$A_{\text{cons(stor)}} = \sum_{a=1}^h A_{\text{stor}}. \tag{14}$$

The amount of recoverable electric energy to the storage (network) for the braking mode is determined by formulas respectively [2, 20]:

$$A_{rec(stor)} = \sum_{a=1}^h A_{stor}; \quad (15)$$

$$A_{rec(net)} = \sum_{a=1}^h A_{net}. \quad (16)$$

The underground rail rolling stock rational operation mode is determined by given criterion (2).

For selected rolling stock rational operation mode, the condition is checked: « $A_{rec(net)} > 0$?». If the condition is fulfilled, the author suggested determining the rational CES energy capacity by formula:

$$A_{rat} = A_{given} + A_{netmax}; \quad (17)$$

Otherwise:

$$A_{rat} = A_{stormax}. \quad (18)$$

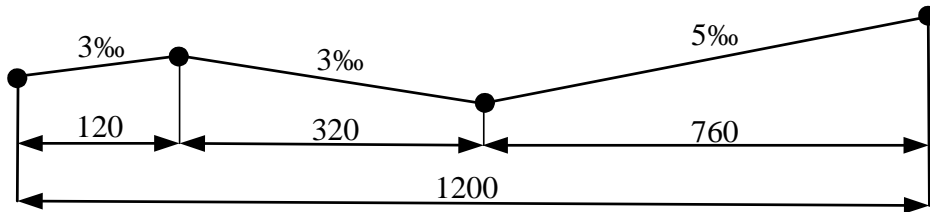


Figure 5. Given track profile

The underground rail rolling stock traction and braking characteristics and traction motor energy characteristics are defined discretely in the form of functions (Fig. 6).

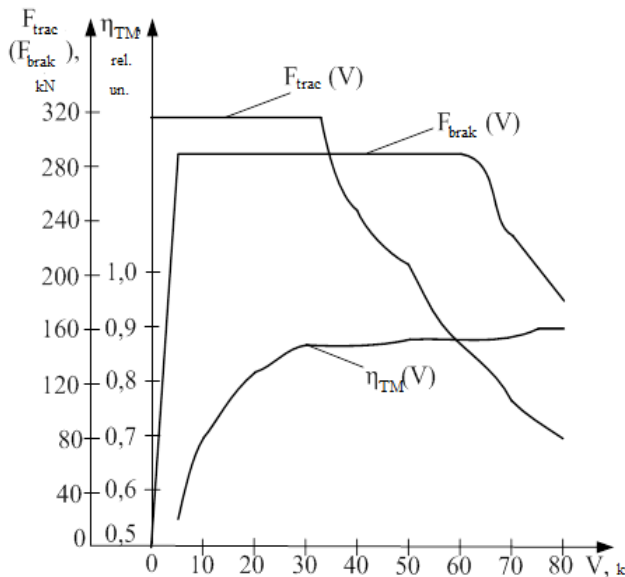


Figure 6. The underground rail rolling stock traction and braking characteristics and traction motor energy characteristics

Possible control actions are assigned:
for traction mode: $\alpha_1=1$; $\alpha_2=0,75$; $\alpha_3=0,5$;
for braking mode: $\beta_1=1$; $\beta_2=0,75$.

On the basis of this algorithm, the calculation example of rolling stock rational operation mode determination has been carried out between two KM “Kiev Metro” stations under the following input parameters: $a=0,00189$; $b=0,03942$; $c=2,22$; $m=262$ t; $\eta_{inv}=0,98$ $\eta_{red}=0,98$; $(1+\gamma)=1,06$; $a_1=1000$; $a_2=1$; $T=98$ c; $U_{CES}=865$ B; $\eta_{CES} = 0,96$; $U_{кст\text{яги}}=825$ B; $U_{csbrak}=908$ B; $A_{CES} = 5$ kW · h.

The considered track profile is shown on Fig. 5. The total haul length is 1200 meters. The haul consists of three elements without curves: the first is of 120 meters length (ascent 3 ‰), the second is of 320 meters length (descent 3 ‰) and the third is of 760 meters length (ascent 5 ‰).

The speed limitation along the whole haul area is $V_{max}=80$ km/h.

The number of variation steps: $r=17$. The lengths of variation steps: $r_1=60$ m; $r_2=60$ m; $r_3=60$ m; $r_4=130$ m; $r_5=130$ m; $r_6=300$ m; $r_7=160$ m; $r_8=50$ m; $r_9=50$ m; $r_{10}=25$ m; $r_{11}=25$ m; $r_{12}=25$ m; $r_{13}=25$ m; $r_{14}=25$ m; $r_{15}=25$ m; $r_{16}=25$ m; $r_{17}=25$ m.

The speed net is formed with step $\Delta V=0,5$ km/h for the speed range from 0 to 53 km/h, with step $\Delta V= 0,25$ km/h for the speed range from 53 to 80 km/h.

On the basis of algorithm (Fig. 1), the great amount of possible control actions, where the rolling stock stop in the end of haul is provided, is calculated.

For illustrative purposes, the underground rail rolling stock rational operation mode is determined from five modes, where the speed in the end of the haul is zero. Five operation modes (control actions) are assigned in the following form: $F_{112330000-2-2-2-2-2-2-2-2}$ (a);

$F_{112330000000-1-1-1-1-1-1}$ (b); $F_{1120000000000-1-1-1}$ (c); $F_{1120000000000-2-2-2-2}$ (d); $F_{1100000000000-1-1}$ (e),

where 1 – traction mode 100%; 2 - traction mode 75%; 3 - traction mode 50%; 0 - stopping regime; - 1 – braking mode 100%; -2 – braking mode 75%.

For control actions (a, b, c, d, e), the following data are calculated: total time of motion along the haul (t_{Σ}); consumed electric energy from the network

($A_{cons.net}$); consumed electric energy from storage ($A_{cons.stor}$); recoverable electric energy to the storage ($A_{rec.stor}$); recoverable electric energy to the

network ($A_{rec.net}$); optimum test (Q). Theoretical researches results are shown in Table 1.

Table 1. Theoretical researches results

Operation mode	t_{Σ} , s	$A_{cons.net}/ A_{rec.net}$, kW·h	$A_{cons.stor}/ A_{rec.stor}$, kW·h	Q
a	86	18,86/5,76	5/5	18949
b	87,5	14,68/2,89	5/5	14680
c	97,4	9,18/0	5/4,35	833
d	98,5	9,18/0	5/4,44	819
e	106,4	6,31/0	5/3	9498

In this way, according to criterion (2), the operation mode «d» is reasonable. The rational CES energy capacity, which is 4.44 kW·h, has been calculated by formula (18) for obtained operation mode. After theoretical determination of underground rail rolling stock rational operation mode, the experimental researches have been conducted. While these researches conducting, it was implemented six loggings with different given rolling stock haul operation modes (including the rational mode «d»).

The implemented rolling stock operation modes are introduced in the form of dependences of given tractive (braking) force from distance covered (Fig. 7) and dependences of motion speed from distance covered (Fig. 8); they are marked by 1, 2, 3, 4, 5, 6.

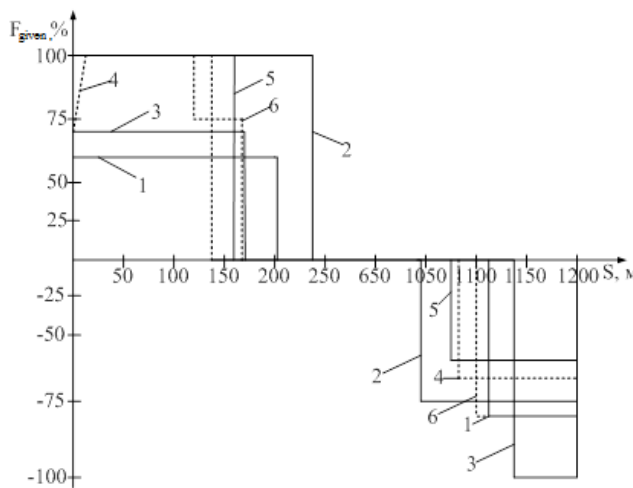


Figure 7. Dependences of given tractive (braking) force from distance covered

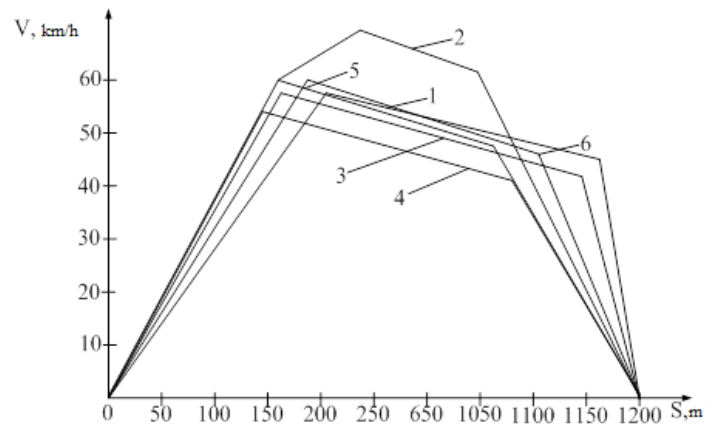


Figure 8. Dependences of motion speed from distance covered

Operation mode 1. Acceleration with given traction force of 60% to the speed of 59km/h, stopping distance to the speed of 45km/h, the braking with the speed of 45km/h to the complete stop with given braking force of 80%.

Operation mode 2. Acceleration with given traction force of 100% to the speed of 69km/h, stopping distance to the speed of 58km/h, the braking with the speed of 58km/h to the complete stop with given braking force of 75%.

Operation mode 3. Acceleration with given traction force of 70% to the speed of 57km/h, stopping distance to the speed of 41km/h, the braking with the speed of 41km/h to the complete stop with given braking force of 100%.

Operation mode 4. Acceleration with given traction force of 70% to the speed of 10km/h, acceleration with given traction force of 100% to the speed of 54km/h, stopping distance to the speed of 41km/h, the braking with the speed of 41km/h to the complete stop with given braking force of 60%.

Operation mode 5. Acceleration with given traction force of 70% to the speed of

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60km/h, stopping distance to the speed of 48km/h, the braking with the speed of 48km/h to the complete stop with given braking force of 65%.

Operation mode 6. Acceleration with given traction force of 100% to the speed of 53km/h, acceleration with given traction force of 75% to the speed of 60km/h, stopping distance to the speed of 60km/h, the braking with the speed of

46km/h to the complete stop with given braking force of 75%.

This investigation is about determination of haul motion time, the amount of consumed and recoverable electric energy under different underground railway rolling stock haul operation modes (including rational one). The results of above mentioned parameters determination are shown in Table 2.

Table 2. Experimental researches results

Operation mode	t_{Σ} , s	$A_{\text{cons.net}}/ A_{\text{rec.net}}$, kW·h	$A_{\text{cons.stor}}/ A_{\text{rec.stor}}$, kW·h	Q
1	101	8,04/0	5/4,6	1865
2	89	13,37/2,52	5/5	10994
3	100	7,51/0	5/2,25	1220
4	106,5	4,34/0	5/3,05	9700
5	98,5	9,1/0	5/4,59	806
6	98	9,1/0	5/4,55	774

The theoretical and experimental researches results comparative analysis for underground rail rolling

stock rational operation mode along the given haul considering CES installation on its border is shown in Table 3.

Table 3. The results comparative analysis

Investigation	t_{Σ} , s	$A_{\text{tot.cons}}$, kW·h	$A_{\text{tot.rec}}$, kW·h
Theoretical	98,5	14,18	4,44
Experimental	98	14,1	4,55
Percentage ratio, %	0,5	0,6	2,4

The results obtained from theoretical and experimental researches differ no more than by 2.4 % under rolling stock rational operation mode, which proves the reproducibility of results.

Conclusion

It was developed the mathematical and logarithmical support for rolling stock rational operation mode determination considering the energy-exchanged processes between the network and CES installed on its border.

It was suggested the criterion of underground rail rolling stock rational operation mode determination considering the energy-exchanged processes between the network and CES where given motion time along the haul and electric energy minimum consumption from the network are provided.

On the basis of algorithm worked out for given conditions (the haul, motion time, rolling stock mass), the underground rail rolling stock rational operation mode is determined, where consumed electric energy from the network is 9.18

kW·h and the rational energy capacity of CES installed on its border is 4.44 kW·h.

The experimental researches have been conducted; their results prove the correspondence of theoretical researches results (imprecision is < 5 %).

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