Total Least Squares Identification in Gyroscopic Drift Test Scheme of the Low-angle Servo Method

Yanbo Li, He Huang, Xianfeng Xu, Meng Hui

School of Electronics and Control Engineering, Chang'an University, Xi'an 710064, Shanxi, China

Abstract
Low-angle Servo Method is one of the most important gyroscopic drift test schemes to separate some coefficients in gyroscopic drift error model such as D_F, D_O, D_S and D_{SS}. It has high precision but the error increases with test time. In order to improve the test accuracy, this paper analyzes the mechanism of the Low-angle Servo Method and derives its mathematical model. The identification error in the gyroscopic drift error model increases by the test error generating noise in the data matrix of the observation equation. Total least squares identification method is used to reduce the identification error. Compared to the traditional least squares method, the identification precision of D_O and D_S are significantly improved by total least squares identification method. The identification precision of D_S and D_{SS} are similar and the precision increases. The research results can improve test accuracy of gyroscope in application.
Key words: LOW-ANGLE SERVO METHOD; TOTAL LEAST SQUARES; GYROSCOPIC DRIFT TEST

Introduction

Gyroscope is a key component affecting the accuracy of the inertial navigation system. Gyroscopic drift test has become an important issue of improving the accuracy of inertial navigation system [1]. Generally, the test uses force feedback method with advantages of simple operation, high speed and low cost [2]. However, torque is nonlinear and the test accuracy is not high enough. In the low-precision gyroscopes, the method of research and application has been matured. Table Servo method (referred servo method) need not to measure the drift angular velocity directly but with the time integration indirectly. When the time is long enough to allow observation, it can measure very low angular velocity drift accurately. So it has the advantage of high precision. Servo test method is closer to the state of the working gyroscope, with better guidance for practical applications [3, 4].

In recent years, the gyroscope's machining already approaches the limits of precision. Only by increasing the time stability of inertial meter factor and using high-precision test technology to execute error separation compensation, the precision of instruments is further improved [5]. Currently, the force feedback method cannot meet the requirements of the test accuracy for fluid floated gyroscope applying in high-precision liquid strategic weapons [6]. Therefore, the study of servo is imperative to be carried out. However, this method calls for high precision turntable [7], so the design and debug of the servo system is difficult. And thus to carry out the corresponding servo experiment is also more difficult.

The Low-angle servo test method is used to separate the model coefficients D_F, D_O, D_S, D_{SS} in gyro drift error model [8]. However, few researches conducted in similar literature.

But it exists methodological error, test accuracy is very low [9]. In the paper, the study of the error mechanism shows that it causes data matrix of observed equations to generate noise. The Least squares identification algorithm has been used in the Low-angle servo test. It only considers the test data's noise, and do not consider the impact of data matrix noise on the identification results. Thus, error of identification is large.

In 1980, Golub and Van Loan [10, 11] raised total least squares method. The literature [12] proposed a method to reduce the computational complexity. The algorithm of fast convergence was given in the literature [13]. So in the paper we use the Total Least Squares to suppression of data matrix's noise.

This paper establishes the mathematical model of the error in the Low-angle Servo method and analyzes the impact of the error on the test results In the course of the study. The servo test of the high-precision liquid floating gyro is finished by Harbin Institute of Technology and a research institute. In order to reduce the impact of the error in the Low-angle Servo method on the test results, the paper proposes an identification method by the Total Least Squares, and the validity of the algorithm is simulated by the experiments.

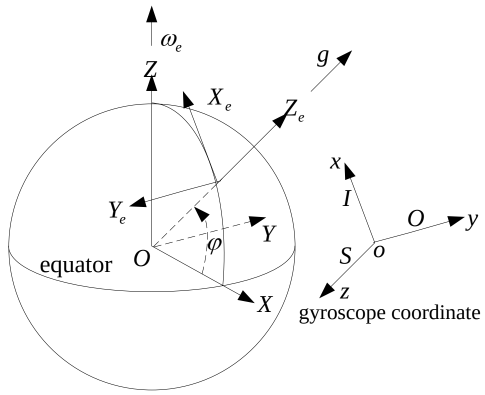


Figure 1. Direction of gyroscope in the first position

Low-angle servo scheme

The outer axis of three-axis turntable is in East or West direction, so the input axis of gyroscope by Low-angle Servo method has two direction including pointing to the level of the North or the South. No matter which direction it points to, g_I is always 0. So we will ignore the related items about g_I and reserve the Significant model items D_F, D_O, D_S and D_{SS}. After simplifying the gyro drift error model[8]

$$\dot{\theta} = \omega_{eI} + D_F + D_O g_O + D_S g_S + D_{SS} g_S^2 + \varepsilon \qquad (1)$$

The low-angle servo test can be realized by 8 installation methods [8]. Figure 1 shows the direction of the low-angle servo test in the position 1.

The selected method of geographic coordinates is invariant and we finish the low-angle servo test in the position 2-8. Figure 2 shows the direction in the gyroscope coordinates.

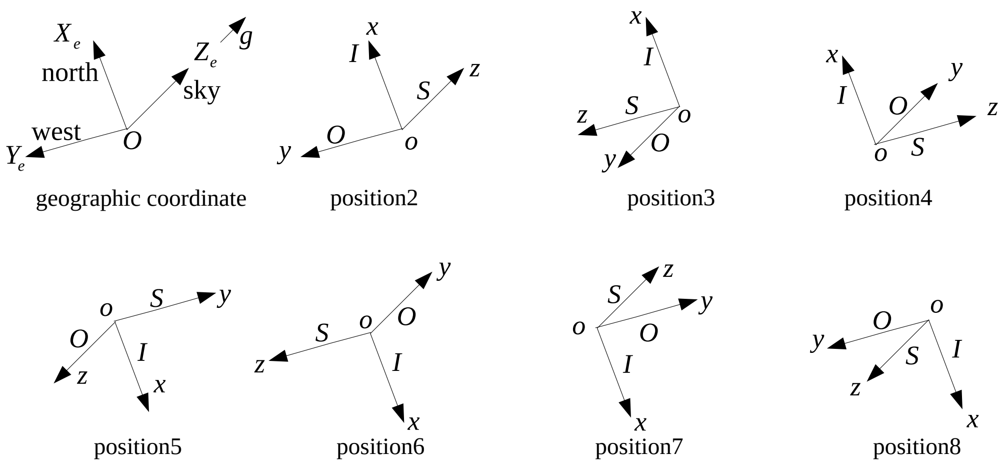


Figure 2. Direction of gyroscope in the position 2-8

Position 1: Input axis I point to the level of the North. Input axis O point to the level of the East and Rotor axis S point to the Earth.

Position 2: Input axis I point to the level of the North. Input axis O point to the level of the West and Rotor axis S point to the Sky.

Position 3: Input axis I point to the level of the North. Input axis O point to the earth and Rotor axis S point to the West.

Position 4: Input axis I point to the level of the North. Input axis O point to the Sky and Rotor axis S point to the East.

Position 5: Input axis I point to the level of the South. Input axis O point to the Earth and Rotor axis S point to the East.

Position 6: Input axis I point to the level of the South. Input axis O point to the Sky and Rotor axis S point to the West.

Position 7: Input axis I point to the level of the South. Input axis O point to the level of the East and Rotor axis S point to the Sky.

Position 8: Input axis I point to the level of the South. Input axis O point to the level of the West and Rotor axis S point to the Earth.

The observation equation in the Low-angle servo test is

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \dot{\theta}_4\\ \dot{\theta}_5\\ \dot{\theta}_6\\ \dot{\theta}_7\\ \dot{\theta}_8\end{bmatrix}=\begin{bmatrix}1&1&0&-1&1\\1&1&0&1&1\\1&1&-1&0&0\\1&1&1&0&0\\-1&1&-1&0&0\\-1&1&1&0&0\\-1&1&0&1&1\\-1&1&0&-1&1\end{bmatrix}\begin{bmatrix}\omega_e\cos\varphi\\ D_F\\ D_O g\\ D_S g\\ D_{SS} g^2\end{bmatrix} \quad (2)$$

where $\dot{\theta}_i (i=1,2,\ldots,8)$ in Equation (2) is the average angular velocity in T sampling time. The identification method is Least Squares. Reference [8] points that Equation (2) is considered to be correct only with low rotation angle of table. The Low-angle servo test works under the condition of the rotation angle of table being less than $1°$ from the initial position in the scheme. The test time is limited in 6 minutes. And according to angle measuring system we can obtain the table error contrast to the random error level of the gyroscope and find that the former is relatively large. So the paper models the test error cause by in the error in Low-angle Servo Method in order to improve the scheme.

Error in low-angle servo method

Analysis and modeling of error mechanism in Low-angle Servo Method

We describe the error in Low-angle Servo Method by taking position 1 for example. Spin of gyroscope coordinate system in position 1 is shown in Figure 3. At the beginning of the test, the gyroscope coordinate is Oxyz. It becomes $Ox'y'z'$ after the reverse rotation angle of the table around the input angular velocity of gyroscope becomes θ_1.

Figure 3 shows that the rotation of turntable table will not affect the point of the input axis in Gyroscopic Drift Test. But it will change the angle among the acceleration of gravity g, the output axis Oy, and Rotor axis Oz in the gyroscope.

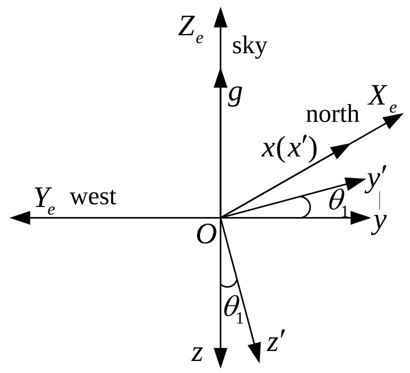


Figure 3. Spin of gyroscope coordinate system in the first position

The components of the acceleration of gravity projected to every axis of the gyroscope.

$$\begin{cases} g_I = 0 \\ g_O = g\sin\theta_1 \\ g_S = -g\cos\theta_1 \end{cases} \quad (3)$$

The components of the acceleration of gravity projected to every axis of the gyroscope in observation Equation(2) are approximated.

$$\begin{cases} g_I = 0 \\ g_O = 0 \\ g_S = -g \end{cases} \quad (4)$$

Comparison between Equations (3) and (4), we can find that the approximated error is too small to ignore with low rotation angle. But as time increases, the approximated error and separation coefficient error become bigger with the rotation angle increasing. They are named Low-angle Servo test error leading to the test can't work indefinitely.

Error in Low-angle servo method has integration effect and increases with the rotation angle of the table. Under the condition of meeting the test accuracy requirements, the table can the angle.

Figure 3 shows that at the t time the rotation angle of the table to the initial position becomes θ_1. The components of the acceleration of gravity projected to every axis of the gyroscope are shown in Equation (5) and their observation equation is shown in Equation (6).

$$\begin{cases} g_I = 0 \\ g_O = g\sin\theta_1(t) \\ g_S = -g\cos\theta_1(t) \end{cases} \tag{5}$$

$$\dot{\theta}_1(t) = \omega_e \cos\varphi + D_F + D_O g\sin\theta_1(t) - D_S g\cos\theta_1(t) + D_{SS} g^2 \cos^2\theta_1(t) \tag{6}$$

Spin of gyroscope geographic coordinate in the position 2-8. Is shown in Figure 4. (At the beginning of the test, the gyroscope coordinate is Oxyz. It becomes after the rotation of the table around the input angular velocity of gyroscope.)

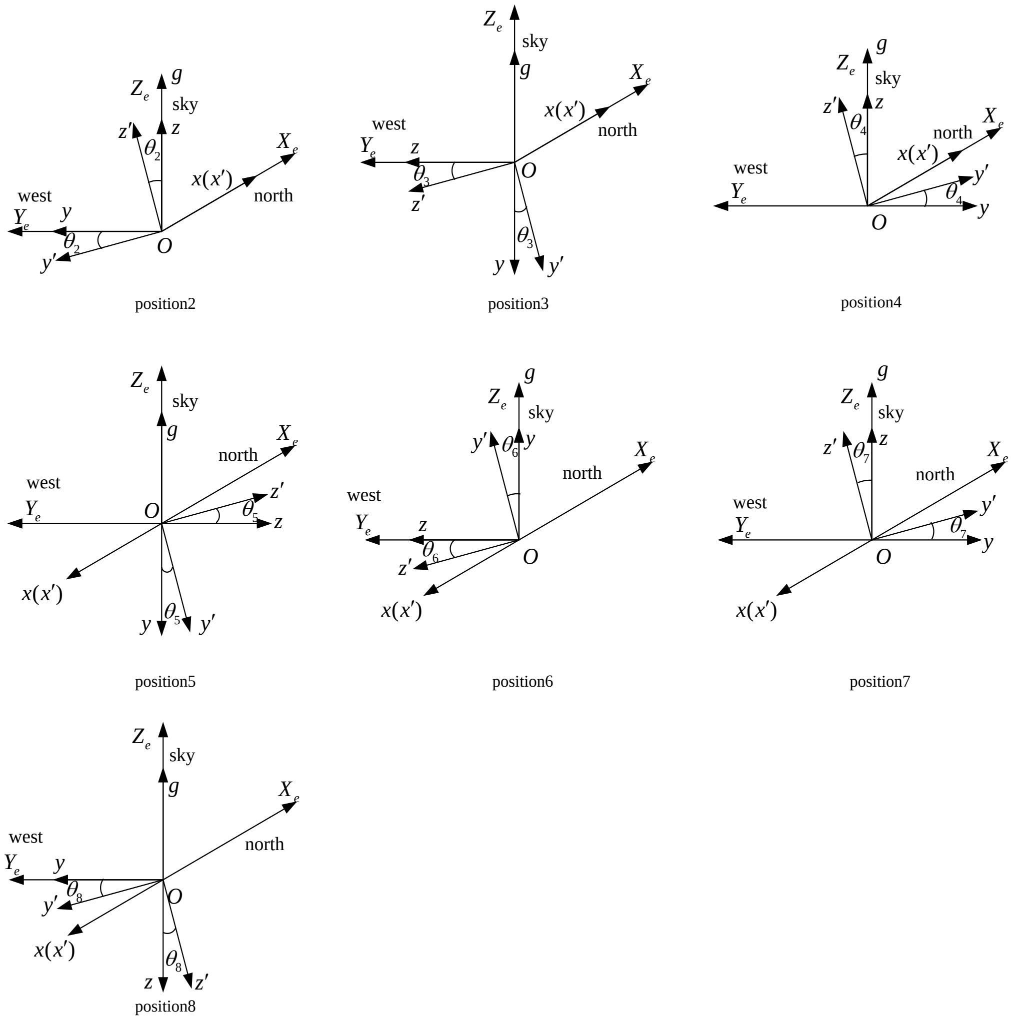


Figure 4. Spin of gyroscope coordinate system in the position 2-8

Figure 4 shows that the instant expressions of the components of the acceleration of gravity projected to every axis of the gyroscope at t time for the test position 2-8, just like the position 1.

$$\dot{\boldsymbol{\theta}}(t)=\begin{bmatrix}\dot{\theta}_1(t)\\ \dot{\theta}_2(t)\\ \dot{\theta}_3(t)\\ \dot{\theta}_4(t)\\ \dot{\theta}_5(t)\\ \dot{\theta}_6(t)\\ \dot{\theta}_7(t)\\ \dot{\theta}_8(t)\end{bmatrix}=\begin{bmatrix}\omega_e\cos\varphi+D_F+D_O g\sin\theta_1(t)-D_S g\cos\theta_1(t)+D_{SS}g^2\cos^2\theta_1(t)\\ \omega_e\cos\varphi+D_F-D_O g\sin\theta_2(t)+D_S g\cos\theta_2(t)+D_{SS}g^2\cos^2\theta_2(t)\\ \omega_e\cos\varphi+D_F-D_O g\cos\theta_3(t)-D_S g\sin\theta_3(t)+D_{SS}g^2\sin^2\theta_3(t)\\ \omega_e\cos\varphi+D_F+D_O g\cos\theta_4(t)+D_S g\sin\theta_4(t)+D_{SS}g^2\sin^2\theta_4(t)\\ -\omega_e\cos\varphi+D_F-D_O g\cos\theta_5(t)-D_S g\sin\theta_5(t)+D_{SS}g^2\sin^2\theta_5(t)\\ -\omega_e\cos\varphi+D_F+D_O g\cos\theta_6(t)+D_S g\sin\theta_6(t)+D_{SS}g^2\sin^2\theta_6(t)\\ -\omega_e\cos\varphi+D_F-D_O g\sin\theta_7(t)+D_S g\cos\theta_7(t)+D_{SS}g^2\cos^2\theta_7(t)\\ -\omega_e\cos\varphi+D_F+D_O g\sin\theta_8(t)-D_S g\cos\theta_8(t)+D_{SS}g^2\cos^2\theta_8(t)\end{bmatrix} \quad (7)$$

In the case of known $\dot{\theta}_i(t)$ and $\theta_i(t)$, we can obtain the expression of $\theta_i(t+1)$ as follow.

$$\theta_i(t+1)=\dot{\theta}_i(t)+\theta_i(t) \qquad i=1,2,...,8 \quad (8)$$

Similarly, in sampling time T the average article velocity $\dot{\theta}_i(i=1,2,...,8)$ is to be used as observation value. The expression of $\dot{\theta}_i$ in sampling time T is

$$\dot{\theta}_i=\frac{\theta_i}{T}=\int_0^T\dot{\theta}_i(t)dt/T \qquad i=1,2,...,8 \quad (9)$$

The integral Mathematical model of every Low-angle Servo test is above mentioned. We can describe the motion State at any time and any position.

Simulation analysis in error model of low-angle servo test

The advantage of the servo method is to extend time and to improve accuracy in the test. But the error increases with Low-angle Servo test time. The simulation research the relationship between the parameter Identification error and test rime. We will analyze how the error impact on the error by the result.

In the simulation, we only consider the error in Low-angle Servo test without noise. Then the law that the parameter identification error changes with the time τ is found. The identification method is the Least Squares.

(1) Every parameter given. The initial value $\theta_i(0)=0(i=1,2,\ldots,8)$ and the observation time τ is set up.

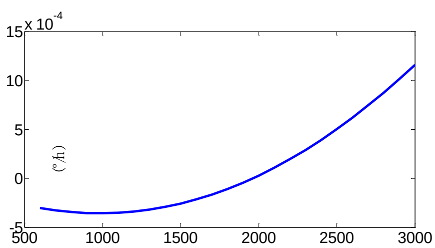
(2) To obtain $\dot{\theta}_i(0)$ by substituting the $\theta_i(0)(i=1,2,\ldots,8)$ into the Equations (7);

(3) To obtain $\theta_i(1)$ by Equation (8);

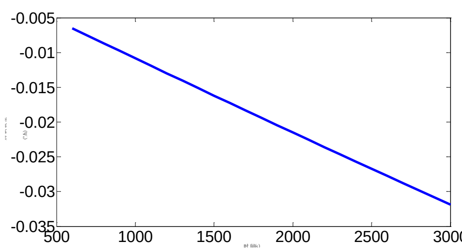
(4) $\theta_i(1)$ is substituted into step b and step b and c are iterated repeatedly to obtain $\theta_i(\tau)$. The result and τ are substituted into Equation (9) to obtain the observation value $\dot{\theta}_i$;

(5) To obtain the identification result by the Least Squares method.

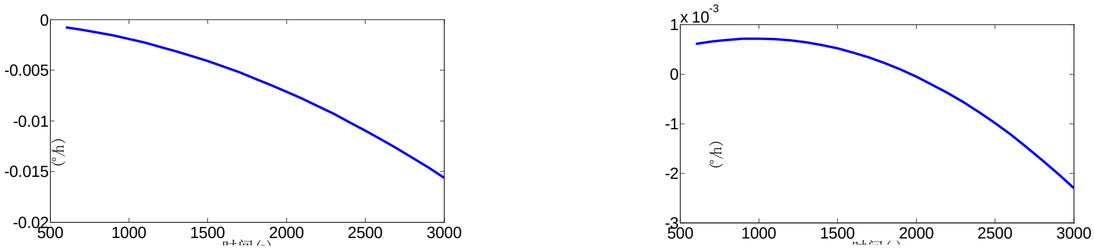
The Low-angle Servo test is in Matlab software, the latitude angle φ is $39^{\circ}54'44''$, and the ground speed $\omega_e=15.04107°/\mathrm{h}$. The true values in table 1 refere to the parameter in the existing error model of Low-angle Servo test. The error curve increasing with the time τ is shown in Figure 5. In the picture: abscissa represents time(s); ordinate is identification error($°/\mathrm{h}$).



a) Curve of Identification error(D_F)



b) Curve of Identification error(D_O)



c) Curve of Identification error(D_S) d) Curve of Identification error(D_{SS})

Figure 5. Spin of Curve of coefficient identification error

Every identification parameter results in some observation time points are given in table 1.

Table 1. The values of time in the mill drum first section

Parameter	Dimension	Truth-value	Test time τ (s)			
			600	900	1200	1500
D_F	°/h	-1.5	-1.5003	-1.5003	-1.5003	-1.5003
D_O	°/h/g	-0.1	-0.1065	-0.1097	-0.1129	-0.1161
D_S	°/h/g	3	2.9992	2.9984	2.9973	2.9958
D_{SS}	°/h/g^2	0.4	0.4006	0.4007	0.4007	0.4005

We can have the following conclusions by analyzing the Figure 5 and the table 1. The absolute error of the identification parameter D_F is no more than 3×10^{-4} °/h. The absolute error of the identification parameter D_O, the identification parameter D_S and the identification parameter D_{SS} is respectively -0.0129 °/h, 0.0027 °/h and 7×10^{-4} °/h. So the Low-angle Servo test has weak influence on the identification parameter D_F and D_{SS} and it has large impact of the identification parameter D_O and D_S.

The identification of the low-angle servo test

The theory of least squares identification

The principle of least squares identification is described hereinafter. If a system has the observation equations as follows:

$$Y = AK + E \qquad (10)$$

where $Y=\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$ ——Observation value of system ;

$A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ ——State matrix of system ;

$X=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ ——State vector of system ;

$E=\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix}$ ——Error vector of system。

The equation is over determined usually, and the least squares solution X_{LS} is with the minimum sum of squares of the error.

$$X_{LS} = (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}Y \qquad (11)$$

Total Least Squares Identification Theory

Total least squares identification theory in the last century, in the eighties, by Golub and Van Loan from the point of view of numerical analysis for the first time this approach for the overall analysis, and officially called the total least squares. In mathematical statistics, this method is called orthogonal regression error or variable regression. In system identification, total least squares method called feature vectors or Koopmans-Levin method. Now, the total least squares method has been widely used in engineering fields.

Total Least Squares (TLS) is in solving the matrix equation, considering the state of the matrix A and an identification method observables Y disturbances exist.

In total least squares, the consideration is solving the following matrix equation

$$Y + \Delta Y = (A + \Delta A)X \quad (12)$$

Equation can be rewritten as

$$([-Y, A] + [-\Delta Y, \Delta A])\begin{bmatrix} 1 \\ X \end{bmatrix} = 0 \quad (13)$$

The above equation may be equivalent to:

$$(B + D)Z = 0 \quad (14)$$

where $B = [-Y, A] \in R^{m\times(n+1)}$

$D = [-\Delta Y, \Delta A] \in R^{m\times(n+1)}$

$Z = \begin{bmatrix} 1 \\ X \end{bmatrix} \in R^{(n+1)\times 1}$

Solving the homogeneous Equation (14) can be expressed as total least squares method constrained optimization problem:

$$\min \|D\|_F^2 \quad (15)$$

Constraint is:

$$(Y + \Delta Y) \in \text{Range}(A + \Delta A) \quad (16)$$

where $\|D\|_F$ is the Frobenius norm of the matrix D:

$$\|D\|_F = (\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^2)^{1/2} = \sqrt{tr(D^T D)} \quad (17)$$

Constraint means: if $(Y + \Delta Y) \in R^{m\times 1}$, will be able to find a $X \in R^{n\times 1}$, make $(Y + \Delta Y) = (A + \Delta A)X$.

The main difference between the total least squares method with the least squares method is to consider the presence of noise in the data matrix error.

Total Least Squares Solution

This paper considers only under observation for over determined Equation (14) conditions, total least squares problem solving. When $m \geq n+1$, the two cases to discuss the solution of the Equations [13-14].

Case 1: The matrix B is only a minimum singular value

Assuming total least squares solution is a unit vector norm, by the Equation (14) can be written as:

$$BZ = R = -DZ \quad (18)$$

Suppose Z is a vector of total least squares solutions of a unit norm, by the Equation (14) can be written as:

$$\min \|BZ\|_2^2 = \min \|R\|_2^2$$

(19)

Constraint is

$$Z^T Z = 1 \quad (20)$$

where R can be seen as an error vector of the matrix $BZ = 0$ total least squares solution Z, namely the total least squares solution Z is to make the sum squared error $\|R\|_2^2$ for a minimum of least squares solution. The constrained least squares problem is easily solved by Lagrange multiplier method. The objective function is defined as

$$J = \|BZ\|_2^2 + \lambda(1 - Z^T Z) \quad (21)$$

where λ ——Lagrange multiplier.

Standard solutions of the objective function is

$$B^T BZ = \lambda Z \quad (22)$$

The equation shows, you should choose λ the smallest eigenvalues of matrix $B^T B$, The total least squares solution "Z" is the right singular vectors and matrices "B" corresponding to the smallest singular value.

Making the augmented matrix singular value decomposed as:

$$B = U\begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \ddots & \\ 0 & & & \sigma_{n+1} \end{bmatrix} V^T \quad (23)$$

And the singular values sorted by $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{n+1}$, the right singular vectors and singular values corresponding to $v_1, v_2, \cdots, v_{n+1}$. Thus, based on the analyzed before, the total least squares solution for $Z = v_{n+1}$. It means, Overall original matrix equation $Y = AX$ least squares solution is given by:

$$X_{TLS} = \frac{1}{v(1, n+1)}\begin{bmatrix} v(2, n+1) \\ \vdots \\ v(n+1, n+1) \end{bmatrix} \quad (24)$$

where $v(i, n+1), i = 1, 2, \cdots, n+1$ is the i element of the $n+1$ column belongs to V .

Case 2 : There is multiple minimum singular value of matrix **B** (the smallest number of singular values are the same or very close)

Make $\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_p > \sigma_{p+1} \approx \cdots = \sigma_{n+1}$ (25)

And v_i is any column vector of subspace $S = Span\{v_{p+1}, v_{p+2}, \cdots, v_{n+1}\}$, so any right singular vectors v_i are given a set of overall least-squares solution

$$\mathbf{K} = \frac{y_i}{\alpha_i}, \quad i = p+1, p+2, \cdots, n+1 \qquad (26)$$

where α_i is the first element of the vector v_i, The other elements of the vector $\mathbf{y}_i$, which also means $\mathbf{v}_i = \begin{bmatrix} \alpha_i & \mathbf{y}_i \end{bmatrix}^{\mathrm{T}}$. So, There will be $n+1-p$ total least squares solutions. But, we can find out the total least squares solution only in a certain sense. There are two only possible solutions:

(1) The minimum norm solution

The minimum norm solution of n arguments overall least-squares solution. Solving the minimum norm solution of the total least squares algorithm proposed by Golub Van Loan.

By Equation (26) can be derived the number of main singular value p, make

$$\mathbf{V}_1 = [v_{p+1}, v_{p+2}, \cdots, v_{n+1}] \qquad (27)$$

where $\mathbf{V}_1$ —— $\mathbf{V}$ in the form of a column block.

And calculate the Householder transformation matrix $\mathbf{Q}$ makes

$$\mathbf{V_1 Q} = \begin{bmatrix} \alpha & \vdots & 0 & \cdots & 0 \\ \cdots & \vdots & \cdots & \cdots & \cdots \\ \mathbf{y} & \vdots & & \times & \end{bmatrix} \qquad (28)$$

where α ——a scalar;

$\mathbf{y}$ ——a vector;

$\times$ ——data block we can ignore in the calculation, if $\alpha \ne 0$, we can get the total least squares solution, else if $\alpha = 0$, decrease p and compute them repeatedly until getting the solution.

(2) The optimal least-squares approximation solution

The optimal least-square approximation solution vector contains only p parameters. Make $m \times (n+1)$ matrix $\hat{\mathbf{B}}$ to be the best approximation of a rank $\mathbf{B}$ of p augmented matrix , in other worlds

$$\hat{\mathbf{B}} = \mathbf{U}\hat{\mathbf{B}}_{\boldsymbol{\sigma}}\mathbf{V}^{\mathbf{T}} \qquad (29)$$

where $\hat{\mathbf{B}}_{\boldsymbol{\sigma}} = diag(\sigma_1, \sigma_2, \cdots \sigma_p, 0, \cdots, 0)$。

Make $m \times (p+1)$ matrix $\hat{\mathbf{B}}_j^{(p)}$ to be a submatrix of the optimal approximation matrix $\hat{\mathbf{B}}$, defined as: $\hat{\mathbf{B}}_j^{(p)}$: j to $p+j$ columns of submatrix consists of the $\hat{\mathbf{B}}$.

In the submatrix $\hat{\mathbf{B}}_j^{(p)}$, $j = 1, 2, \cdots, n+1-p$. If $\mathbf{a} = \begin{bmatrix} 1 \\ \mathbf{K}^{(p)} \end{bmatrix}$, the least-square solution turns into:

$$\begin{bmatrix} \hat{B}(1:p+1) \\ \hat{B}(2:p+1) \\ \vdots \\ \hat{B}(n+1-p:p+1) \end{bmatrix} a = 0 \qquad (30)$$

According to the principle of least squares, getting the least squares solution of Equation(30) can be equivalent to minimizing the cost function

$$f(a) = a^{\mathrm{T}} S^{(p)} a \qquad (31)$$

where $S^{(p)} = \sum_{i=1}^{n+1-p} [\hat{B}(i:p+1)]^{\mathrm{T}} \hat{B}(i:p+1)$

The total least squares solution:

$$K(i) = S^{-(p)}(i+1,1) / S^{-(p)}(1,1) \quad i = 1, 2, \cdots, p \qquad (32)$$

Verification of the two identification algorithm

The least squares solution of in Low-angle Servo Method is

$$X_{LS} = (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}Y \qquad (33)$$

where

$$Y = \begin{bmatrix} \dot{\theta}_1 - \omega_e \cos\varphi \\ \dot{\theta}_2 - \omega_e \cos\varphi \\ \dot{\theta}_3 - \omega_e \cos\varphi \\ \dot{\theta}_4 - \omega_e \cos\varphi \\ \dot{\theta}_5 + \omega_e \cos\varphi \\ \dot{\theta}_6 + \omega_e \cos\varphi \\ \dot{\theta}_7 + \omega_e \cos\varphi \\ \dot{\theta}_8 + \omega_e \cos\varphi \end{bmatrix}$$

$$Y = \begin{bmatrix} \dot{\theta}_1 - \omega_e \cos\varphi \\ \dot{\theta}_2 - \omega_e \cos\varphi \\ \dot{\theta}_3 - \omega_e \cos\varphi \\ \dot{\theta}_4 - \omega_e \cos\varphi \\ \dot{\theta}_5 + \omega_e \cos\varphi \\ \dot{\theta}_6 + \omega_e \cos\varphi \\ \dot{\theta}_7 + \omega_e \cos\varphi \\ \dot{\theta}_8 + \omega_e \cos\varphi \end{bmatrix}$$

The Total Least Squares solution X_{TLS} is obtained by Equation (33).

$$B=\begin{bmatrix} 1 & 0 & -1 & 1 & \dot{\theta}_1-\omega_e\cos\varphi \\ 1 & 0 & 1 & 1 & \dot{\theta}_2-\omega_e\cos\varphi \\ 1 & -1 & 0 & 0 & \dot{\theta}_3-\omega_e\cos\varphi \\ 1 & 1 & 0 & 0 & \dot{\theta}_4-\omega_e\cos\varphi \\ 1 & -1 & 0 & 0 & \dot{\theta}_5+\omega_e\cos\varphi \\ 1 & 1 & 0 & 0 & \dot{\theta}_6+\omega_e\cos\varphi \\ 1 & 0 & 1 & 1 & \dot{\theta}_7+\omega_e\cos\varphi \\ 1 & 0 & -1 & 1 & \dot{\theta}_8+\omega_e\cos\varphi \end{bmatrix}$$

Simulation and experimental validation of two identification algorithm

Two identification algorithm was applied to simulate low-angle servo method. Where Latitude φ=39°54′44″, earth rate $\omega_e = 15.04107^{\circ}/\text{h}$. Added rotary table space pointing error and the central shaft angle error, simulation has been made 20 times, we get each coefficient identification results such as table 2:

Table 2. Identification results

Parameter	Dimension	Truth-value	Least Squares		Total Least Squares	
			Identification value	Standard deviation	Identification value	Standard deviation
D_F	°/h	-1.5	-1.4988	0.0031	-1.4090	0.0031
D_O	°/h/g	-0.1	-0.0975	0.0027	-0.0991	0.0021
D_S	°/h/g	3	2.9978	0.0039	2.9989	0.0023
D_{SS}	°/h/g^2	0.4	0.3982	0.0031	0.3985	0.0028

From table 2 we know, Precision difference of the total least squares identification method and the least squares identification method about D_F and D_{SS} is similar, relatively speaking, identification accuracy of the identification values and standard deviation about D_O and D_S has been improved. Therefore, Effect of the total least squares identification will be better.

Applying the total least squares identification method and the least squares identification to the test data processing low-angle servo experiment. We can get identification results about model coefficient D_F, D_O, D_S, D_{SS} such as table 3(due to the normalization processing, table data is dimensionless):

Table 3. Experimental results

Parameter	Least Squares		Total Least Squares	
	Identification value	Standard deviation	Identification value	Standard deviation
D_F	-0.2379	0.0013	-0.2382	0.0012
D_O	-0.0162	0.0014	-0.0151	0.0010
D_S	0.4477	0.0016	0.4492	0.0011
D_{SS}	0.0670	0.0020	0.0671	0.0019

From the simulation result of table 2 and table 3, we can draw the following conclusion: the total least squares identification result of confidence level is better and closer to the true value.

Conclusions

In this paper, established the model for the error and studied the total least squares and least squares identification algorithm simulated and compared the results of two identification methods of identification for the issue that the coefficient's identification error was larger. The simulation results showed that the identification accuracy of D_O and D_S coefficient was significantly improved and D_F and D_{SS} were similar in two methods using total least squares algorithm. The total least squares algorithm was applied to identify the test data and the recognition accuracy was improved.

Acknowledgments

This research is supported by the Postdoctoral Science Foundation of China (No. 2013M542310) and the National Nature Science Foundation of China (No. 51407012, No. 61201407 and No. 61402052).

References

1. Zhang S. F., Yang H. B., Cai H. (2009) *Analysis and evaluation of the accuracy of inertial guidance weapon.* National University of Defense Technology Press: Changsha.

2. America government. (1964) Gyro Laboratory Manual. p.p.87-112.
3. Su Z., Fu M. Y., Li Q. (2013) Research on bell-shaped vibratory angular rate gyro's character of resonator. *Sensors*, 13(4), p.p.4724–4741.
4. Gallacher B. (2012) Principles of a Micro-Rate Integrating Ring Gyroscope. *IEEE Transactions on Aerospace and Electronic Systems*, 48(1), p.p.658–672.
5. Kim D., M'Closkey R.T. (2013) Spectral Analysis of Vibratory Gyro Noise. *IEEE Sensors Journal*, 13(11), p.p.4361-4374.
6. Hidalgo J., Poulakis P., Köhler J. (2012) Improving planetary rover attitude estimation via MEMS sensor characterization. *Sensors*, 12(2), p.p.2219–2235.
7. Saraswathy S.G., Geen J., Chang J. (2012) High performance gyro with fast startup time, high range, wide bandwidth, low noise and excellent vibration immunity. *IEEE/ION. Position Location and Navigation Symposium* (*PLANS*), Myrtle Beach, SC, p.p.20-23.
8. Hu H. Z. (1981) *Gyroscope drift measurement principle and experimental technology*. National Defence Industry Press: Beijing.
9. Tanenhaus M., Carhoun D., Geis T. (2012) Miniature IMU/INS with optimally fused low drift MEMS gyro and accelerometers for applications in GPS-denied environments. *IEEE/ION. Position Location and Navigation Symposium* (*PLANS*), Myrtle Beach, SC, p.p.259-264.
10. Golub G. H., Van C. F. (1980) An Analysis of the Total Least Squares Problem. *Society for Industrial and Applied Mathematics*, 17(1), p.p.883-893.
11. Van H. S., Vandewalle J. (1989) On the Accuracy of Total Least Squares and Least Squares Problem Ax=b When Some or All Columns in a Are Subject to Error. *Journal on Matrix Analysis and Applications*, 10(1), p.p.294-315.
12. Arablouei R., Dogancay K. (2012) Linearly-Constrained Recursive Total Least-Squares Algorithm. *IEEE, Signal Processing Letters*, 19(12), p.p.821-824.
13. Rhode S., Usevich K., Markovsky I. (2014) A Recursive Restricted Total Least-Squares Algorithm. *IEEE Transactions on Signal Processing*, 62(21), p.p.5652-5662.
14. Dasgupta K., Soman, S.A. (2013) Line parameter estimation using phasor measurements by the total least squares approach. *Power and Energy Society General Meeting*, Vancouver, BC, p.p.1-5.