Theoretical foundations of extension of ARMA (AutoRegressive with Moving Average) model with the usage of connectionist technologies (Brain-inspired Systems)

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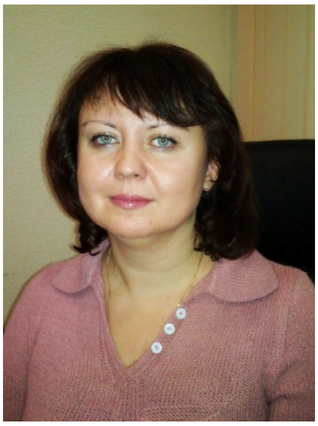

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Abstract
Results of development of synthesis method of reliably stable Box-Jenkins models applied in the controlling systems of nonlinear dynamic objects are given. Developed ARMABiS (AutoRegressive with Moving Average Brain-inspired Systems) models have adaptive properties thanks to worked out procedure of adjustment of weight coefficients of regressive lag, and condition of stability is provided by corresponding adjustment of weight coefficient of criteria lag.
Key words: ADAPTIVE CONTROL SYSTEM, NEURAL NETWORKS, AUTOREGRESSIVE SYSTEMS, CONTROLLER, ARMA

Intelligence control systems, which have adaptive properties, became of wide spread occurrence in the field of modern automatization. First ideas and methods of adaptation appeared at the beginning of the previous century and already till 80s there was formed theory of adaptive control systems. Main results of this period are based on the determination of the structure and analytical researches of stiffness of basic structures of adaptive control of linear and nonlinear objects

It should be marked that significant quantity of modern scientific achievements are obtained in result of development and further researches of adaptive systems of control of functional either at ideal conditions or with objects of control, which allow linearization of system characteristics in the artificially determined limits of work stationary point.

Fulfilled analysis of construction principles and results of functioning of traditional adaptive control system [1] allowed to draw the conclusion that synthesis of classical adaptive structures is connected with the following problems: violation of conditions of coordination of control and fluctuation, non-determined on the stage of synthesis, object parameters; at high degree of differential equation, with the help of which technological processes are described, there arise problems connected with inaccessibility of direct method of measurements necessary for control of coordinates of object state vector; facilitating mechanism of set quality indexes of process of automatic control, while changing of circuit-performance conditions of the system, is absent.
Herewith, fulfilled analysis of characteristics and quality indexes of intelligence systems of automatic control, based on the usage of the theory of fuzzy sets and traditional connectionist structures, allowed to draw a conclusion about the necessity to develop means connected with improvement of adaptive properties of intellectual regulators in the systems of automatic control and emulators of direct and inverse controlled element dynamics.

This article covers theoretical bases of synthesis and analysis, adaptive asymptotically stable systems of control on the base of parameter-oriented model of Box-Jenkins - autoregressive moving average model (ARMA), where due to usage of connectionist approaches, one succeeded to provide combination of solutions of tasks to provide stiffness and minimization of functional of control quality.

Basic conditions of discrete-continuous filters theory

Work of digital computing systems in the structures of control systems there suggested [2] to consider from the point of view of projection of discrete-continuous filters properties (1) on the base of replacement of continuous argument:

$$u(t)=\sum_{k=0}^{N}\beta_k y(t-kT)-\sum_{k=1}^{N}\alpha_k u(t-kT), \qquad (1)$$

where $y(t)$ – signal input, $u(t)$ – exit signal, $\beta_k (k=0,1,...,N)$ and $\alpha_k (k=1,2...,N)$ - weight coefficients of input and output variables.

$$u(mT)=\sum_{k=0}^{N}\beta_k y\left[(m-k)T\right]-\sum_{k=1}^{N}\alpha_k u\left[(m-k)T\right] \qquad (2)$$

Taking into account (2) there suggested [3] general view of the model *ADL(p,q)* (autoregressive distributed lag) with one independent variable:

$$y_i=\alpha+\sum_{k=1}^{p}\beta_k y_{i-k}+\sum_{j=0}^{q}\gamma_j x_{i-j}+\varepsilon \;. \qquad (3)$$

Special cases of general model *ADL(p,q)*:
Model *ADL(1,1)*:

$$y_t=\alpha+\beta_1 y_{t-1}+\gamma_0 x_0+\gamma_1 x_{t-1}+\varepsilon \;. \qquad (4)$$

Model of apportioned lag *ADL(0,q)* or *AR(0,q)* – model, where the lag of criterion variable is absent.
Model of linear filter:

$$x_t=\varepsilon_t+\Psi_t\varepsilon_{t-1}+\dots . \qquad (5)$$

AR-model is obtained [3] from general *ADL* structure by means of constraint satisfaction $\gamma_i=0$ or *AR(p)=ADL(p,0)*:

$$y_t=\alpha+\sum_{k=1}^{p}\beta_k y_{t-k}+\varepsilon_t \;. \qquad (6)$$

MA(q) model (reduced model of linear filter):

$$x_t=\varepsilon_t-\sum_{i=1}^{q}\theta_i\varepsilon_{t-i}\;, \qquad (7)$$

or in matrix form:

$$x_t=\theta(B)\varepsilon_t B\varepsilon_t=\varepsilon_{t-1}, \qquad (8)$$

where *B* – backward shift operator.
Characteristic function of process is written in the form of:

$$\theta(B)=1-\sum_{i=1}^{q}\theta_i B^i \;. \qquad (9)$$

Herein the process *MA(q)* is always stationary and reciprocal, if all the roots of the characteristic equation $\theta(B)=0$ fulfil the condition $|B_i|>1$.

ARI(p,d) and *ARIMA(p,d,q)* models look as follows:

$$\varepsilon_t=\overline{\varphi}(B)x_t=\varphi(B)(1-B)^d x_t . \qquad (10)$$

$$\overline{\varphi}(B)x_t=\theta(B)\varepsilon_t . \qquad (11)$$

Theoretical bases of extension of ARMA model

Realization of adaptive properties of the model of autoregressive moving average is possible by means of adoption of lag operator (*B*):

$$B^\tau y_i=y_{i-\tau} . \qquad (12)$$

Using multinomial of lag

$$f(B) = a_n B^n + ... + a_1 B + a_0, \quad (13)$$

to the decomposition of variable *x*, we will obtain:

$$f(B)y_i = \left(\sum_{j=0}^{n} a_j B^j \right) y_i = \sum_{j=0}^{n} a_j (B^j y_i) = \sum_{j=0}^{n} a_j y_{i-j} \quad (14)$$

Herein the first finite difference looks as follows:

$$\Delta y_i = 1 - B = y_i - y_{i-1}, \quad (15)$$

the second finite difference:

$$\Delta^2 y_i = 1 - 2B + B^2 = y_i - 2y_{i-1} + y_{i-2}. \quad (16)$$

Having applied lag operator (12) to the *ADL*-model (4) we will obtain *ADL(p, q)* –model in operator form:

$$y_i = \alpha + Bf(B)y_i + g(B)x_i + \varepsilon_i, \quad (17)$$

where *f(B)* and *g(B)* are multinomials.

Herein, difference equation (17) characterizes autoregressive processes (*AR(p)*-model with apportioned lag of criteria parameter) with moving average (*MA(q)*-model with apportioned lag of regressor).

Let us write discrete *ARMA*-model with apportioned lag of sequence *(0,q)* (*MA(q)*) for reference value (*desired*) $y^*(t)$ of output coordinate of control object $y(t)$:

$$y^*[i] = \alpha + \sum_{j=0}^{q} \gamma_j x[i-j] + \varepsilon[i] \cdot \quad (18)$$

Let us write finite differences of the first three periods:

$$\Delta y[i] = \mu(1-B) = \mu\left(y^*[i] - y[i-1]\right), \quad (19)$$

$$\Delta^2 y[i] = \mu(1-B)^2 = \mu\left(y^*[i] - 2y[i-1] + y[i-2]\right) \quad (20)$$

$$\Delta^3 y[i] = \mu(1-B)^3 = \mu\left(y^*[i] - 3y[i-1] + 3y[i-2] - y[i-3]\right) \quad (21)$$

Finite difference of *m* period looks as follows:

$$\Delta^m y[i] = \mu(1-B)^m = \mu\left(\Delta^{m-1} y^*[i] - \Delta^{m-1} y[i-1]\right),\ i = 0...n-m \quad (22)$$

where μ - regularization coefficient of *ARMA*-model.

Having excluded from (19-22) the reference value, we will obtain discrete adaptive structures of *ADL* models, which characterize *ARMA*-processes:

adaptive structure of *ADL(1,1)* model:

$$y[i] = \mu\gamma_0 x[i] + \mu\gamma_1 x[i-1] + (1-\mu)y[i-1] + \mu\varepsilon[i] \quad (23)$$

adaptive structure of *ADL(2,2)* model:

$$y[i] = \mu\gamma_0 x[i] + \mu\gamma_1 x[i-1] + \mu\gamma_2 x[i-2] + \\ +2(1-\mu)y[i-1] - (1-\mu)y[i-2] + \mu\varepsilon[i], \quad (24)$$

adaptive structure of *ADL(3,3)* model:

$$y[i] = \mu\gamma_0 x[i] + \mu\gamma_1 x[i-1] + \mu\gamma_2 x[i-2] + \mu\gamma_3 x[i-3] + \\ +3(1-\mu)y[i-1] - 3(1-\mu)y[i-2] + (1-\mu)y[i-3] + \mu\varepsilon[i] \quad (25)$$

Taking into account synthesized structures (23-25) general view of adoptive model *ADL (p, q)* will look as follows:

$$y[i] = \mu\sum_{j=0}^{q} \gamma_j x[i-j] + (1-\mu)\cdot\left(\left[\sum_{j=1}^{p-1} (-1)^{j+1}\cdot p\cdot y[i-j]\right] + (-1)^{p+1}\cdot y[i-p]\right) + \mu\varepsilon[i]. \quad (26)$$

Having applied modified gradient method of quadratic functional minimization

$$J(\varepsilon_u) = 0{,}5\varepsilon_u^T \varepsilon_u, \quad (27)$$

with the aim to develop an algorithm of weight coefficients adjustment $\gamma_0, \gamma_1, \gamma_2 ... \gamma_n$ (26), it is possible to give to *ADL*-models adaptive properties.

As there absent explicit dependence of vector ε_u and function $J(\varepsilon_u)$ on the weight coefficients $\gamma_0, \gamma_1, \gamma_2 ... \gamma_n$, failure ε_u while model (26) adaptation may be represented in a format of generalized failures $\delta^{(\ell)}$, which directly depend on the values $\gamma_0 ... \gamma_\ell$. Herein adaptation of weight coefficients of the model (26) in the step *[i+1]* will be performed according to the following pattern:

$$\gamma_j[i+1] = \gamma_j[i] - hq^{(i-j)}[i]\Lambda^{(j)}[i], \quad (28)$$

where $q^{(i-j)}$ - apportioned lag of regressor, *h* - tuning rate

$$\Lambda^{(j)}[i] = col\left(\frac{\partial J}{\partial q_1^{(i-j)}}, ..., \frac{\partial J}{\partial q_{n_j}^{(i-j)}}\right) = -\varepsilon_u[i], \quad (29)$$

consequently,

$$\gamma_j[i] = \gamma_j[i-1] + h\cdot\varepsilon_u[i]\cdot q[i-j-1], \\ j = 0,1...\ell, \lambda > 0. \quad (30)$$

Failure of adaptation $\varepsilon_u[i]$ may be defined as difference of reference and actual value of model output at the i^{th} iteration.

Considering adaptive properties of the model (26), which are provided by neuromorphic adjustment of weight coefficients $\gamma_0, \gamma_1, \gamma_2 ... \gamma_n$, discrete adaptive structures of *ADL(p, q)* models characterizing adaptive *ARMA*-processes, we will denote in abbreviated form as **ARMABiS** (AutoRegressive with Moving Average Brain-inspired Systems).

Characteristic equation of ARMABiS-models in ACS

Finite differences (15) and (16) in general case may be represented as:

$$a_p y[i] = d_o \Delta^q \zeta[i] + d_1 \Delta^{q-1} \zeta[i] + ... + d_q \zeta[i] - a_1 \Delta^{p-1} y[i] - ... - a_0 \Delta^p y[i] + \nu\varepsilon[i] \quad (33)$$

$$c_0 y[i+p] = b_o \zeta[i+q] + b_1 \zeta[i+q-1] + ..b_q \zeta[i] - c_1 y[i+p-1] - ..c_p y[i] + \nu\varepsilon[i] \quad (34)$$

or while *n* discrete restriction:

$$c_0 y[i] = b_o \zeta[i+q-p] + b_1 \zeta[i+q-1-p] + .. + b_m \zeta[i-p] - \\ -c_1 y[i-1] - .. - c_n y[i-p] + \nu\varepsilon[i] \quad (35)$$

General pattern of difference equation (35) allows to write equation for *ARMABiS* model *ADL(p,q)*, where output coordinate of control object (*x*) is regressor:

$$y[i] - (1-\mu)\left(\sum_{j=1}^{p-1} (-1)^{j+1} \cdot p \cdot y[i-j]\right) - (1-\mu)\cdot(-1)^{p+1}\cdot y[i-p] = \\ = \mu \sum_{j=0}^{q} \gamma_j x[i-j] + \mu\varepsilon[i] \quad (36)$$

Herein characteristic equation of *ARMABiS* model *ADL(p,q)* looks as follows:

$$y[i] - (1-\mu)\left(\sum_{j=1}^{p-1} (-1)^{j+1} \cdot p \cdot y[i-j]\right) - (1-\mu)\cdot(-1)^{p+1}\cdot y[i-p] = 0 \quad (37)$$

General solution of non-homogeneous difference equation (36) let us represent as the sum of transitional and forced components. Transitional component is determined on the base of general formula:

$$y(i) = C_1 \lambda_1^i + C_2 \lambda_2^i + ... + C_n \lambda_n^i, \quad (38)$$

where $\lambda_v (v = 1, 2, ..., n)$ - ordinal roots of characteristic equation (38), C_v - free components.

According to the equation of transitional component (38), condition of dying-away of system motion (condition of stability), which is described by difference equation (36) is of the common form [2]:

$$|\lambda_v| < 1 \qquad (v = 1, 2, ..., n) \quad (39)$$

In such a way, there developed *ARMABiS*-model on the base of *ADL(p,q)* structure having adaptive properties due to adjustment of weight coefficients of regressive lag and condition of stability is provided by adjustment of weight coefficients of criteria lag.

Results of simulation modeling. Let us consider realization of *ADL(2,2)* (23) model, taking into account developed methodology of neuromorphic adaptation.

$$y[i] = b_0 x[i] + b_1 x[i-1] + b_2 x[i-2] - c_1 y[i-1] - c_2 y[i-2] + \mu\varepsilon[i]. \quad (40)$$

As basic structure there taken adoptive control system with reference model (figure 1). The structure of *ARMABiS*-regulator corresponds to the equation (26). Digitalization time 0.01 s., regularization coefficient of *ARMABiS*-structure $\mu = 0,8$, tuning rate of weight coefficients $\gamma, h = 0,05$. Non-stationary second-order object with variable parameters is taken as control object (table 1).

Table 1. Parameter variations of control object

No of interval	time, s	K	T, s	ξ
1	0-10	9	0.7	4

2	10-20	3	0.05	3
3	20-30	7	0.5	0.9
4	30-40	5	0.5	1.5
5	40-1000	3	0.05	2

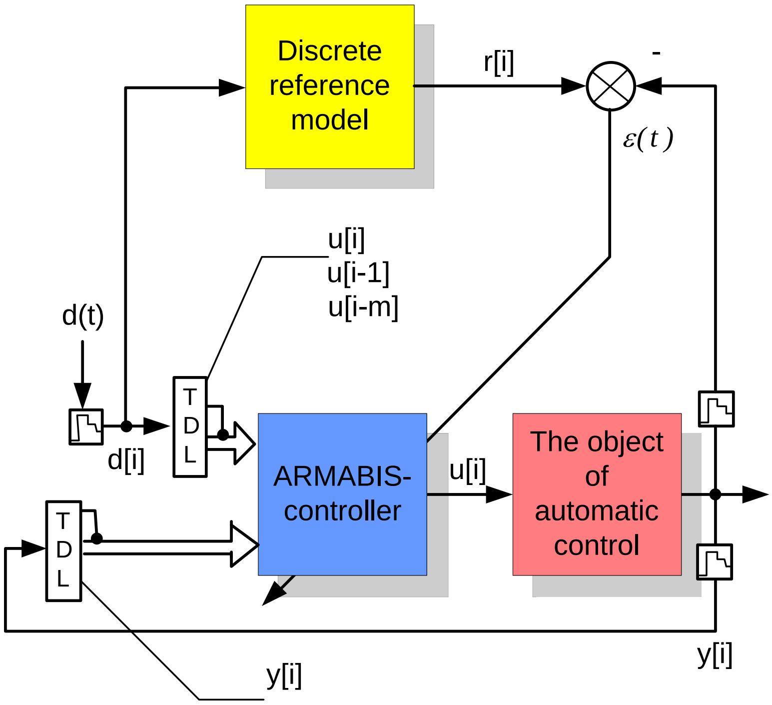

Figure 1. System architecture of model reference adaptive control

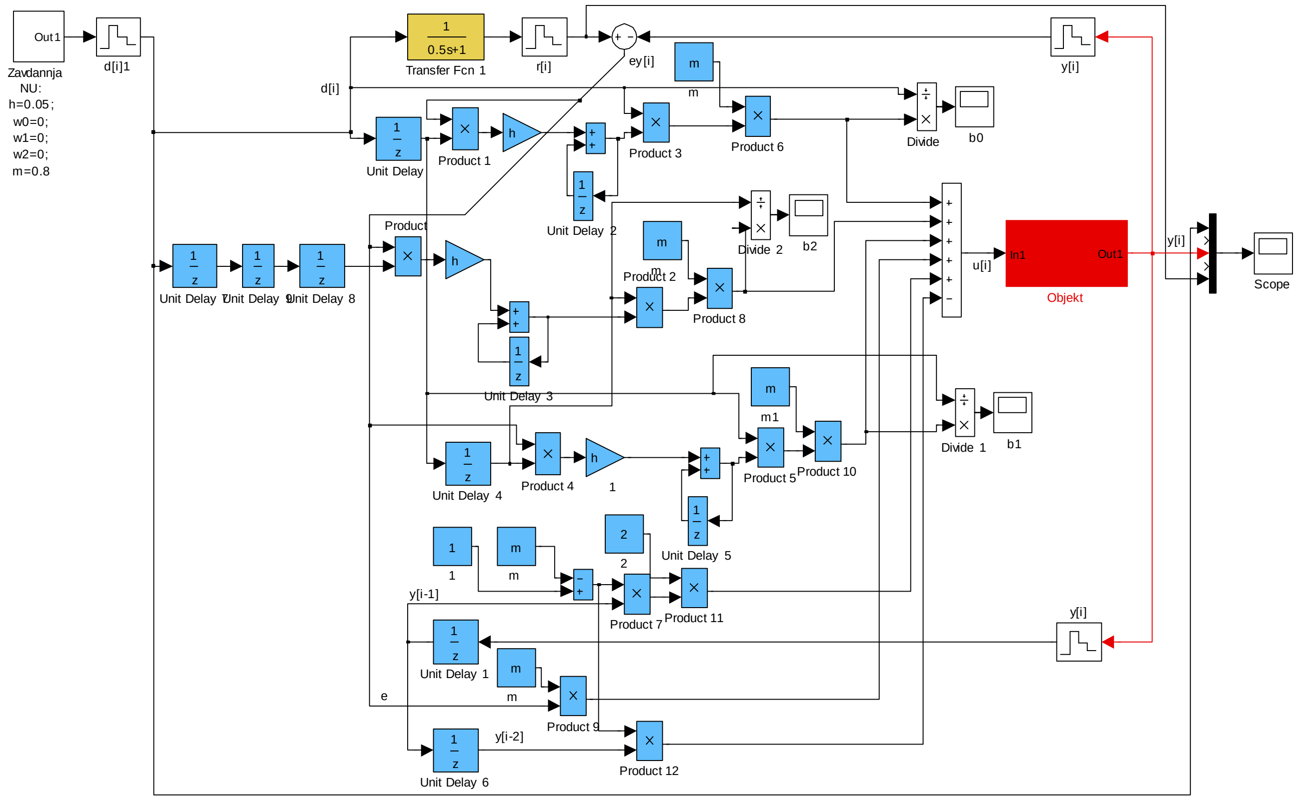

Figure 2. Matlab-model of adaptive control of non-stationary second-order object with *ARMABiS*-regulator and reference model

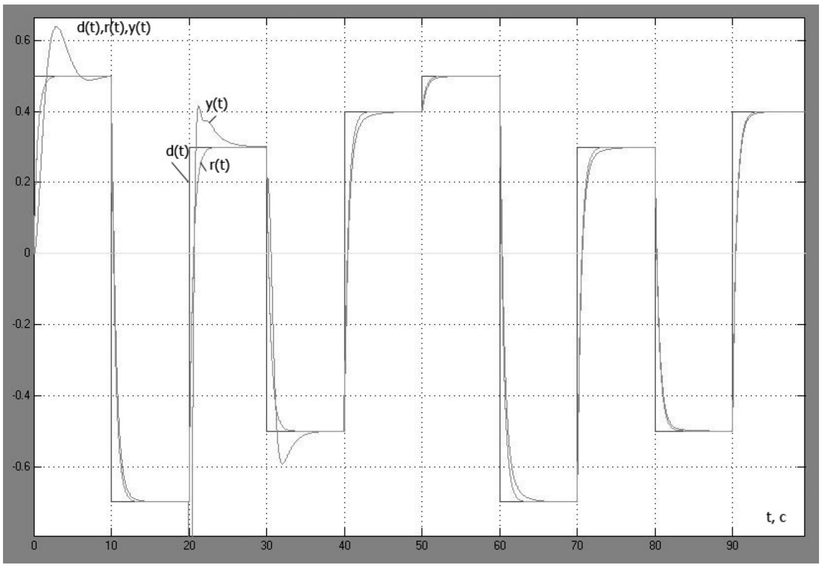

Figure 3. Transition process in adoptive control system with *ARMABiS* –regulator during the initial period of work (d(t) – task signal, r(t) – reference type of transition process, y(t) - isolated yield of control object)

In order to fulfill comparative analysis of ACS working efficiency with *ARMABiS*-regulator and traditional structure of neurocontrol, the figure 4 reflects transition processes, obtained in conditions of previous experiment during usage of traditional system of neurocontrol on the base of Adaline-elements with reference model. Structure of neuroregulator - Adaline; training algorithm- δ-rule; discreteness 0.01s; training step 0.03s.

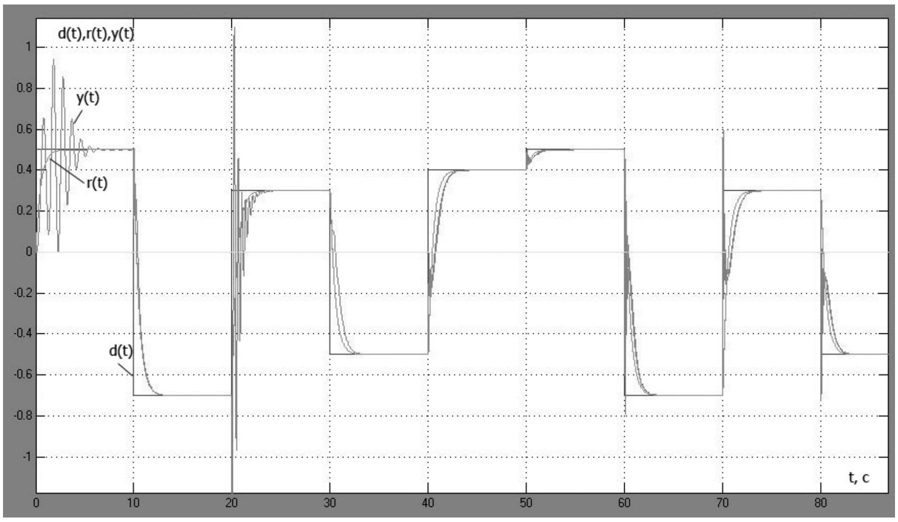

Figure 4. Transition processes in classic adaptive system of neurocontrol of non-stationary second-order object

Analysis of transition processes showed that the adjustment of *ARMABiS*-structure is of monotonous character with excessive correction not exceeding 10%, steady-state error is absent, integral error is 60% less as compared with classic system of neurocontrol.

To analyze the conditions of stability of ACS, let us determine coefficients of equation (40) $c_0 = 1, c_1 = -2(1-\mu), c_2 = (1-\mu), b_0 = \mu\gamma_0, b_1 = \mu\gamma_1, b_2 = \mu\gamma_2$.

Characteristic equation of adaptive model *ADL (2,2)* looks as follows:

$$\lambda^2 - 2(1-\mu)\lambda - (\mu - 1) = 0 \quad (41)$$

Roots of characteristic equation $\lambda^2 - 0,4\lambda + 0,2 = 0$; $\lambda_{1,2} \approx 0,2 \pm j0,4$ satisfy the condition (39), and the system is stable.

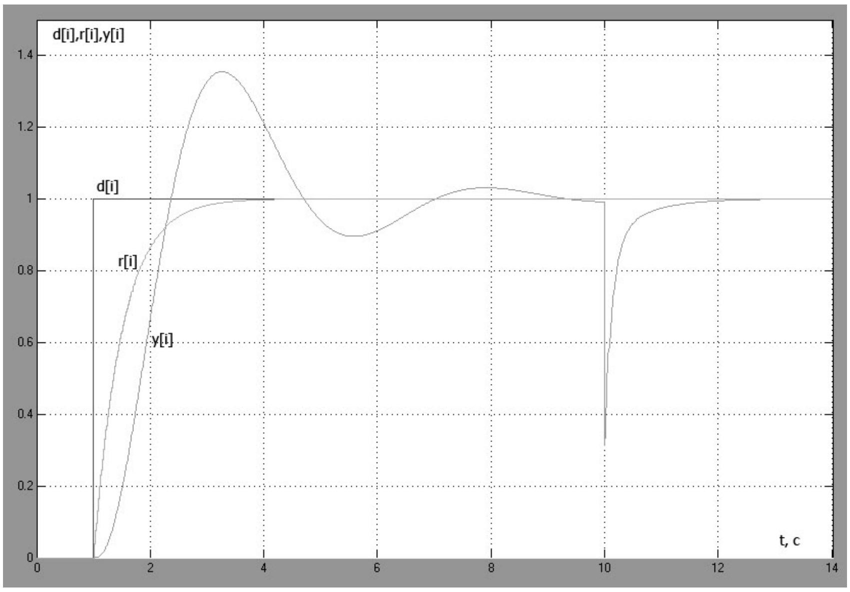

Figure 5. Transition processes along the output coordinate *y[i]* of control object, task signal *d[i]* and reference model *r[i]* at regularization coefficient of *ARMABiS*-structure $\mu = 0,8$

Let us repeat experiments with new regularization coefficient of *ARMABiS*-structure $\mu = 11,5$. Herein the roots of characteristic equation $\lambda^2 + 21\lambda - 10,5 = 0$, $\lambda_1 \approx -21, \lambda_2 \approx 0,5$ do not satisfy the condition (39), so the system should break stability limit, which is proved by the result of modeling (figure 6).

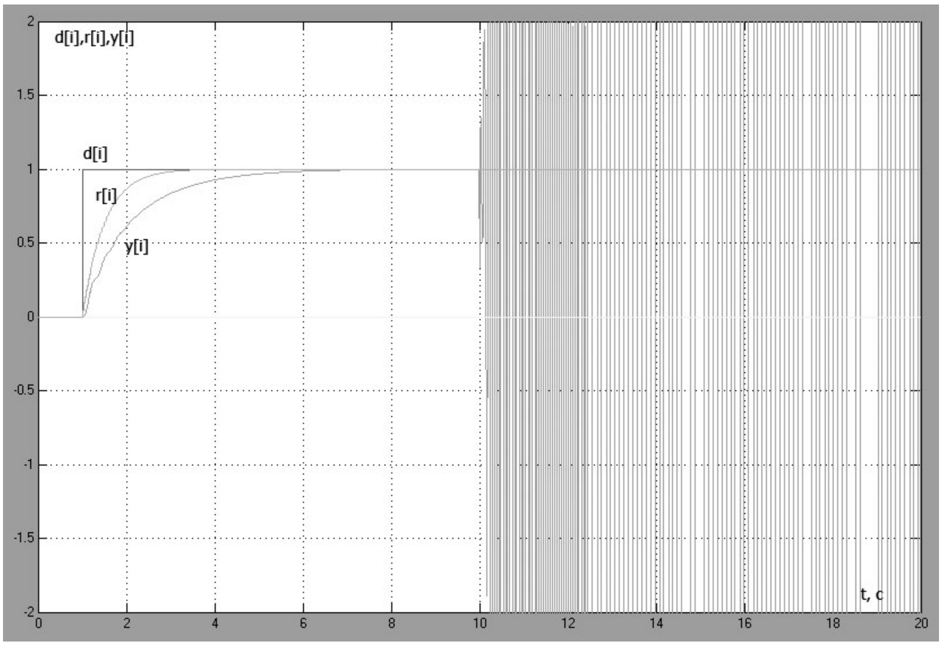

Figure 6. Transition processes along the output coordinate *y[i]* of control object, task signal *d[i]* and reference model *r[i]* at regularization coefficient of *ARMABiS*-structure $\mu = 11,5$

Conclusions

There developed method of parametric synthesis of asymptotically stable adoptive systems of objects control, which are under the influence of random disturbances, uniting solution of the task for providing system stability due to choice of regularization coefficient $\mu^{AR(p)}$ of *AR(p)*-part developed *ARMABiS*-structure and task of minimization of quadratic functional by means of neuromorphic adaptation of weight coefficients of regressive lag *MA (q)*.

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