Design and Analysis of Time-Resistance Conversion Circuit

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Abstract
This paper discusses time measurement using memristor based on its characteristics, brings up single memristor time measurement circuit and double memristor differential time measurement circuit, through analysis and simulation, this paper analyzes the linearity and sensitivity of the circuit. The analysis and simulation show that, compared with single memristor measurement circuit, double memristor differential time measurement circuit has better linearity and its sensitivity can be doubled. Double memristor differential time measurement circuit not only can be applied in time measurement, but also can improve circuit performance in data storage or other applications.
Key words: MEMRISTOR, TIME, RESISTER, DIFFERENCE

1. Introduction

Time is a fundamental physical quantity, the accurate measurement of time is of great significance in industry and daily life. In many fields, time measuring is the medium of other physical quantity measurement. For instance, based on the fact that speed of light is a constant under certain conditions, we can measure the relationship of optical path and time by using a laser range finder. High precision time measurement technology, especially the time measurement in scale of picosecond or subpicosecond is of vital importance in many frontier science fields, such as astronomical observation experiment and particle physics research. With the development of electronic technology, the accuracy of time measurement has reached a very high level. High technology also provides more effective ways for time measurement. Since 1971 when scientist Shaotang Cai brought up memristor theory [1], research on memristor has achieved huge development through researchers' efforts. HP Lab made the first real memristor in 2008 [2]. Researches on memristor are always in the ascendant, at present the studies mainly focus on storage, logic operation, chaotic circuit, etc [3]-[7]. Memristor is a physical property of the material in the nanoscale size. Memristor has the characteristic of small volume, low power consumption, easy integration, etc [8]. This paper will discuss time measurement by memristor based on its characteristics, namely the design of memristor constituted time - resistance conversion circuit.

2. Memristor theory and principle analysis tandem time - resistance conversion circuit

HP Lab first discovered the TiO2 two-terminal memristor, according to this physical model, researchers established HP model. HP model is the earliest memristor and widely applied. It is one of the most accepted models by the industry at present. The physical model as shown in figure 1.

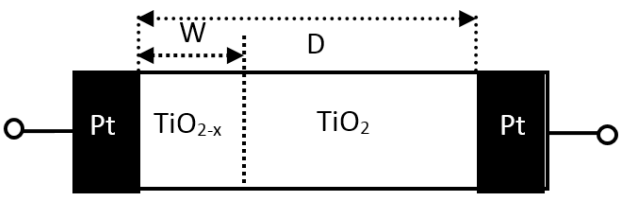

Figure 1. HP physical model

Memristor is composed of a two-layer nanoscale titanium dioxide thin film, the width is expressed in D, the two layers of titanium dioxide are sandwiched between two platinum slices. One layer is short of some oxygen atoms because of aerobic room doping. So this part is more conducive and acts like a semi-

conductor. We call this part the doped area. The other layer without doping is normal TiO2 shows high resistance and can be regarded as an insulator. The total resistance of memristor is the sum of resistance from doping area and no-doping area. We use w to represent the width of the doping, when there is a current flowing from doped area to no- doping area, w value will increase gradually, no-doping area narrows gradually, the whole resistance of memristor is reduced, when w = D, memristor's resistance reaches to minimum, which is expressed as RON. When electric current flows from no- doping area to doped area, w value will decrease gradually, doped area narrowed, the whole resistance of memristor is increased, when w = 0, memristor's resistance reaches to maximum, which is expressed as ROFF. In simple terms, this model is simulated the dopant migration between doping area and no-doping area, which shows the characteristics of memristor. Based on this model, the resistance of memristor can be defined as formula (1).

$$M(w) = \frac{w}{D} R_{ON} + (1 - \frac{w}{D}) R_{OFF} \quad (1)$$

In formula (1), resistance is expressed as *M (w)*, the unit is ohm.When voltage on both sides of memristor, the width of doping area in memristor is given, electron's velocity v_D in its internal migration is constant, the value is determined by formula (2) [9].

$$v_D = \eta \frac{u_D R_{ON}}{D} i(t) \quad (2)$$

In formula (2), η=±1, which corresponds to the migration direction of electrons when direction of voltage is different, u_D represents the electronic voltage under unit velocity, i (t) represents current value at t.For:

$$v_D = \frac{dw}{dt} \quad (3)$$

Plug formula (3) into formula (2), and we can get formula (4) by integral on both sides:

$$w(t) = w_0 + \eta \frac{Dq(t)}{Q_0} \quad (4)$$

In formula (4),

$$Q_0 = \frac{D^2}{u_D R_{ON}},$$

Q0 is in the total quantity of electric charge that the memristor can migrate.

Plug formula (4) into formula (1), and we can get formula (5), $\Delta R = R_{OFF} - R_{ON}$ in formula (5).

$$M(q) = R_0 - \eta \frac{\Delta R q}{Q_0} \quad (5)$$

Based on the ohm theorem, u (t) = M (q) i (t), we can get formula (6) by formula (5):

$$u(t) = (R_0 - \eta \frac{\Delta R q}{Q_0}) i(t) = (R_0 - \eta \frac{\Delta R q}{Q_0}) \frac{dq}{dt} = \frac{d}{dt}(R_0 q - \eta \frac{\Delta R q^2}{Q_0}) \quad (6)$$

When q(0)=0, we can get formula (7) by solving the differential equation

$$M(t) = u(t)/i(t) = R_0 \sqrt{1 - 2\eta \Delta R \Phi(t) / Q_0 R_0^2} \quad (7)$$

From formula (7) we know that the value of memristor only associated with time t at the moment.

It is assumed that the measured time is transformed into a pulse signal with corresponding time length, which means the measurement transformation of time can be equivalent to the corresponding pulse width measurement. In formula (7), when signal on both ends of memristor is DC signal, namely constant U,

For $\int_0^t u(\tau) d\tau = U \cdot t$, formula (7) can be transformed to:

$$M(t) = u/i = R_0 \sqrt{1 - 2\eta \Delta R U t / Q_0 R_0^2} \quad (8)$$

Whenη=-1,η=1, formula (8) can be transformed to formula (9) and (10):

$$M(t) = R_0 (1 + \frac{2\Delta R U}{Q_0 R_0^2} t)^{0.5} \quad (9)$$

$$M(t) = R_0 (1 - \frac{2\Delta R U}{Q_0 R_0^2} t)^{0.5} \quad (10)$$

From formula (9) and (10), memristor is a dependent variable of pulse duration time t, when parameter of memristor is given, the two variables are in a functional relationship. Therefore, we can measure pulse width using this relationship.

3. Circuit design, performance analysis and simulation of single memristor

3.1. Circuit design of single memristor

In simple terms, a single memristor can form a time - resistance conversion circuit measuring pulse width.The circuit diagram is shown in Figure2, X1 represents memristor, U1 represents a pulse generator. In the circuit shown, as long as we measure the resistance of memristor, we can get the corresponding pulse width according to the formula (9) and (10).

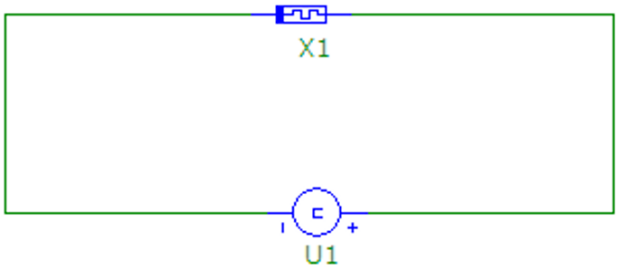

Figure 2. time - resistance conventional circuit of single memristor

3.2. performance analysis of single memristor

Sensitivity and linearity is an important indicator of measuring system, the sensitivity measures the ratio of relative change in output volume and the relative change rate of input volume [10].Its expression is shown in formula (11). Generally, for measuring system performance, the higher sensitivity coefficient can be, the easier we can get accurate measurement results.

$$k = \frac{\Delta y}{\Delta x} \tag{11}$$

In formula (11), k represents sensitivity of the system, Δy refers to the relative variation of output, Δx refers to the amount of variation of input.

Linearity of measurement system measures the degree of deviation between output - input calibration curve (or average calibration curve) and the selected linear fitting line. In static state, if not considering the hysteresis and creep effect, input/output relationship of measurement system can be approximated by the following polynomia.

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots + a_n x^n \tag{12}$$

In formula (12) X refers to input signal, y refers to output signal, a_0 refers to zero output, coefficient a_1, a_2……a_n, are called nonlinear coefficient except a_1. When the nonlinear system is zero, input/output relation of the system can be expressed in a straight line, when this system has a good linearity. Taking the following formula (9) for example, the sensitivity and linearity of the circuit with single memristor can be analyzed as follows.

When $t=t_0$, output of memristor is as follows:

$$M(t_0) = R_0(1 + \frac{2\Delta RU}{Q_0 R_0^2} t_0)^{0.5} \tag{13}$$

When $t=t_{0+}\Delta t$, output of memristor is as follows:

$$M(t_0 + \Delta t) = R_0[1 + \frac{2\Delta RU}{Q_0 R_0^2}(t_0 + \Delta t)]^{0.5} \tag{14}$$

At this point, the relative variation of time measurement circuit output are as follows:

$$\Delta y = M(t_0 + \Delta t) - M(t_0) \tag{15}$$

Plug formula (9), (14) and (15) into formula (15) respectively, we can get formula (16) by simplification.

$$\Delta y = R_0[1 + \frac{2\Delta RU}{Q_0 R_0^{\,2}}(t_0 + \Delta t)]^{0.5} - R_0(1 + \frac{2\Delta RU}{Q_0 R_0^{\,2}} t_0)^{0.5} \tag{16}$$

To the power function part in formula (16), through the power series expansion, representing

$$\frac{2\Delta RU}{Q_0 R_0}$$

with m, formula (16) can be transformed to formula (17).

$$\Delta y = 0.5 m\Delta t - \frac{1}{8}(m\Delta t)^2 + \frac{1}{16}(m\Delta t)^3 + \cdots + \frac{\frac{1}{2}(\frac{1}{2}-1)\cdots(\frac{1}{2}-n+1)}{n!}(m\Delta t)^n \tag{17}$$

As we can see, formula (17) contains rich nonlinear term, Δt <<1, thus we can get approximate linear relation formula (18) omitting nonlinear term.

$$\Delta y = 0.5 m\Delta t \tag{18}$$

The sensitivity of measured circuit is:

$$k = \frac{\Delta y}{\Delta x} = 0.5m \tag{19}$$

As $m = \frac{2\Delta RU}{Q_0 R_0}$,we can get the sensitivity of single memristor is proportional to input voltage U and ΔR, is inversely proportional to Q_0 and R_0.

3.3. Circuit simulation of single memristor

Time t in the formula (9) and (10) are power function of memristor, they are nonlinear related. We simulate circuit diagram assshown in figure 2.1 using SPICE model. We define parameters of memristor as following:η＝－1,R_0＝1K, ΔR＝99K, the pulse signal amplitude is 5 v, D =10 nm. The simulation results are shown in figure 2.2, abscissa diagram represents time t, y coordinate represents memristor value M. As the simulation result shows, nonlinear characteristics exists in circuit.

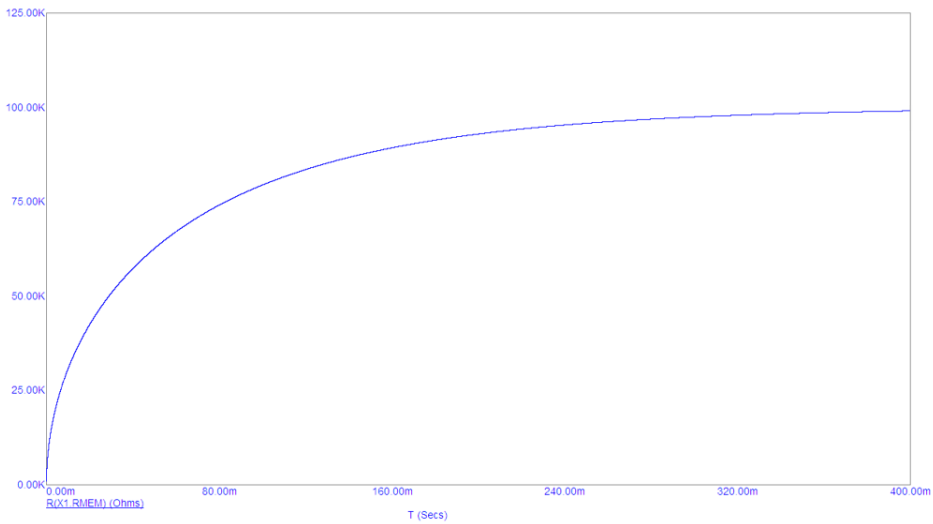

Figure 3. Time – resistance of single memristor time - resistance conversion circuit

4. Design, performance analysis and simulation of differential time measuring circuit with double memristor

4.1. Design of differential time measuring circuit with double memristor

We can see the time measurement circuit diagram with differential memristor Double memristor in Figure 3.1.Memristor X1 and X2 in circuits are parallel installed, with the opposite polarity.Take the value difference between memristor X1 and X2 as circuit output.

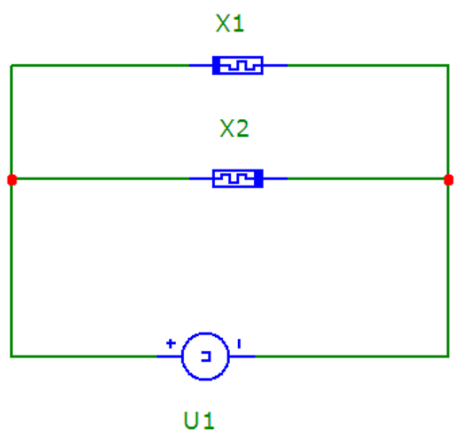

Figure 4. Time measurement circuit diagram with differential memristor

4.2. Circuit analysis of differential time measuring circuit with double memristor

By formula (9) and (10), when t = t0, the output with X_1, X_2 as memristor output are respectively:

$$M(t_0) = R_0(1+\frac{2\Delta RU}{Q_0R_0^2}t_0)^{0.5} \quad (20)$$

$$M(t_0) = R_0(1-\frac{2\Delta RU}{Q_0R_0^2}t_0)^{0.5} \quad (21)$$

When $t=t_{0+}\Delta t$, output of memristor X1 and X2 are respectively:

$$M_1(t_0+\Delta t) = R_0[1+\frac{2\Delta RU}{Q_0R_0^2}(t_0+\Delta t)]^{0.5} \quad (22)$$

$$M_2(t_0+\Delta t) = R_0[1-\frac{2\Delta RU}{Q_0R_0^2}(t_0+\Delta t)]^{0.5} \quad (23)$$

At this point, the relative variation of time measurement circuit output is as follows:

$$\Delta y = [M_1(t_0+\Delta t)\text{-}M_2(t_0+\Delta t)]-[M_1(t_0)-M_2(t_0)] \quad (24)$$

Plug formula (9), (14) and (15) into formula (24), representing $\frac{2\Delta RU}{Q_0R_0}$ with m, we can get formula (25).

$$\Delta y = [1+\frac{2\Delta RU}{Q_0R_0^{\ 2}}(t_0+\Delta t)]^{0.5}-[1-\frac{2\Delta RU}{Q_0R_0^{\ 2}}(t_0+\Delta t)]^{0.5}-[R_0(1+\frac{2\Delta RU}{Q_0R_0^2}t_0)^{0.5}-R_0(1-\frac{2\Delta RU}{Q_0R_0^{\ 2}}t_0)^{0.5}] \quad (25)$$

Through power series expansion reduction with power function in foumula(25) representing $\frac{2\Delta RU}{Q_0R_0}$, we can get formula(26).

$$\Delta y = m\Delta t-\frac{1}{8}[(m(t_0+\Delta t)^3-mt_0^{\ 3}]+\cdots+\frac{\frac{1}{2}(\frac{1}{2}-1)\cdots(\frac{1}{2}-n+1)}{n!}[m(t_0+\Delta t)^n-mt_0^{\ n}] \quad (26)$$

Compare formula (26) with formula (17), the nonlinear term in circuit output expression is greatly reduced, only strange power of nonlinear term is left, in formula (17) $\Delta t << 1$, omitting nonlinear term we can get approximate linear relation formula (27).

$$\Delta y = m\Delta t \tag{27}$$

The sensitivity of measured circuit is:

$$k = \frac{\Delta y}{\Delta x} = m \tag{28}$$

For $m = \frac{2\Delta RU}{Q_0 R_0}$,we can get sensitivity of time measuring circuit with single memristor is proportional to the input voltage U, inversely proportional to Q0. Compare with single memristor time measurement circuit, sensitivity of testing circuit doubled.

4.3. Circuit simulation of differential time measuring circuit with double memristor

Simulation are conducted through PSPICE, the parameters of memristor are as follows: X1 = 1, = 100 k , = 99 k, the pulse signal amplitude is 5 v, D = 10 nm; X2 = －1,＝1K,＝99K, the pulse signal amplitude is 5 v, D = 10 nm. Circuit simulation results are shown in figure 3.2.

Figure 3.2 shows function of value differential between memristor X1and X2 and time t. As the figure shows, difference value greatly increases linearity of the curve, and is beneficial to the improvement of accuracy in time measurement.

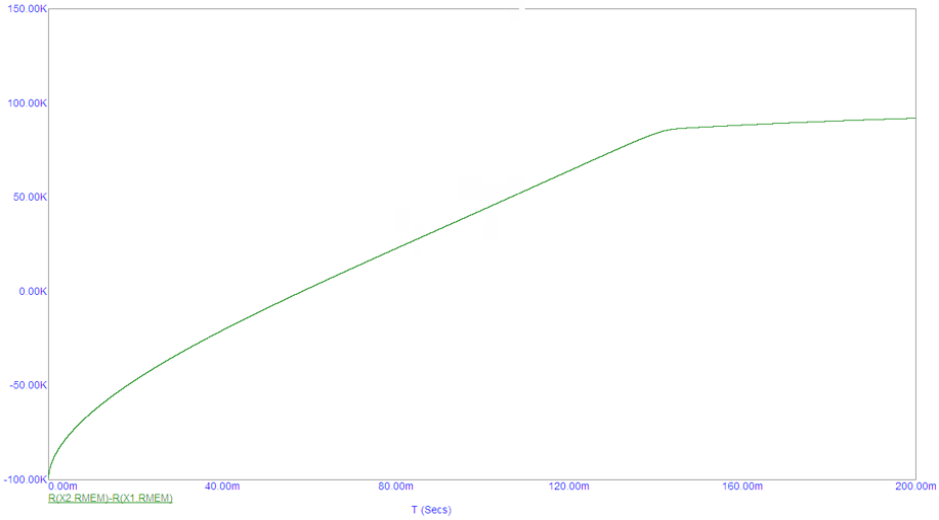

Figure 4. Simulation results of differential memristor measurement circuit

5. Conclusions

Through analysis and simulation, this paper analyzes the linearity and sensitivity of single memristor time measurement circuit and double memristor differential time measurement circuit. We can proved that compared with single memristor measurement circuit, double memristor differential time measurement circuit has better linearity and sensitivity, the specific sensitivity can be doubled. Double memristor differential time measurement circuit not only can be applied in time measurement, but also can improve circuit performance in data storage or other applications.

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