A Self-Adaptive Prediction Model for Dynamic Pricing and Inventory Control

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Abstract
Based on the building of the consumer marginal utility model and the consumer utility model, and combining the dynamic pricing and inventory control of products, the optimal sales price of products and the optimal purchase amount of products in every period can be analyzed from the qualitative and quantitative perspective. The model is a relatively ideal system, because factors influencing the demands of products have been given in every period or factors, including consumers, purchase price and so on, in every period have been clarified. The analysis strategy of this paper is to first introduce the market prediction content and relevant prediction models, and then integrate the traditional prediction algorithm and the intelligent algorithm into the static optimization system featuring the combination of the dynamic pricing and the inventory control. At last, through numerical analysis of the model, characteristics of the optimization system featuring the combination of the dynamic pricing and the inventory control of various prediction models are pointed out. The application scope of relevant models is given, thus laying a theoretical basis for the commodity price of enterprises and merchants.
Keywords: DYNAMIC PRICIN, INVENTORY MANAGEMENT, ADAPTIVE PREDICTION MODEL

1. Introduction

Based on a full understanding of the qualitative and quantitative prediction model, this paper analyzes the optimization results of the traditional prediction algorithm and the intelligent prediction algorithm. Therefore, in order to analyze differences between the prediction algorithm and the practical situations, it is necessary to unify relevant model analysis parameters. Since the gray system model is applicable to the situation when data shows the index variation trend; the time series model is applicable to data with certain variation trend and periodical changes; and the neural networks and the Support Vector Machine model has none requirements of the data, this paper analyzes these models through the following steps:

Step 1: According to characteristics of various models, relevant simulated parameters are unified. The practical data for the analysis of the gray prediction model are shown in Table 1; while the data for the analysis of the time series model, the neural networks model and the Support Vector Machine model are shown in Table 2. Besides, the model adopts the specific power demand function.

Step 2: Unify the first eight periods of data as the input value and training value of various prediction models, and the latter four periods of data as the prediction value.

Step 3: Adopt the actual value of the 12 periods of data as the input data of the dynamic pricing and inventory control joint model, and obtain the optimization results of practical data in every period.

Step 4: Adopt the latter four periods of various market indexes and data obtained by various prediction models and the former eight periods of actual value as the input data for the dynamic pricing and inventory control joint model. Work out the optimization results of various prediction data in every period.

Step 5: Analyze and compare the optimization results of various prediction models and practical data; evaluate the advantages and disadvantages of various prediction systems.

Table 1. Simulated analysis of practical data based on the gray prediction model

Period (t)	Consumers' income (m)	Interest rate (r)	Product ordering price(p_{0t})	Inventory cost (C)
1	9,700	3.1	10.7	0.83
2	9,900	3	10.8	0.84
3	10,100	3	11	0.85
4	10,150	2.95	11.2	0.86
5	10,200	2.9	11.3	0.87
6	10,250	2.83	11.28	0.91
7	10,300	2.8	11.4	0.92
8	10,350	2.78	11.43	0.91
9	10,380	2.73	11.47	0.93
10	10,430	2.71	11.5	0.91
11	10,500	2.65	11.6	0.93
12	10,600	2.55	11.76	0.95

Table 2. Simulated analysis of practical data based on the time series model, the neural networks model and the Support Vector Machine model

Period (t)	Consumers' income (m)	Interest rate (r)	Product ordering price (p_{0t})	Inventory cost (C)
1	9,700	3.1	10.7	0.83
2	9,900	3.08	10.8	0.84
3	9,800	3.05	10.9	0.85
4	9,450	2.95	10.5	0.84
5	9,600	2.9	10.3	0.82
6	9,850	2.93	10.38	0.85
7	9,630	3.04	10.4	0.86
8	9,550	3.08	10.53	0.82
9	9,480	3.13	10.6	0.81
10	9,630	3.02	10.5	0.82
11	9,600	3.05	10.6	0.81
12	9,500	3.05	10.62	0.82

2. Prediction of the market self-adaptive optimization based on the gray prediction model

Under the condition when the historical data are hard to collect, modeling based on few data can better reflect the role of the recent data [1,2]. Besides, the market indexes are subject to the influence of multiple unpredictable factors, thus showing huge fluctuations. It is hard to directly employ the original data for modeling. The gray system modeling uses the gray modules generated through accumulation instead of the original data. Thus, it can weaken the randomness of data to a large extent, while enhancing the data's regularity. At the same time, the gray system theory combines the qualitative analysis with the quantitative analysis. In other words, during the modeling process, the target system turns from a gray one into a white one through determination of the system scope to decomposition of the target system to confirmation of system elements and behaviors and relationship During the process, not only are mathematical models of the modern control theory employed, but also knowledge of experience-based judgment. Qualitative analysis and quantitative analysis are combined to supplement each other, and to gradually obtain various relationship between different factors in the system and to form a modeling process from the qualitative to the quantitative, from the rough to the refined, from the gray to the white. In other words, based on the "generated series" obtained through the accumulated processing of a group of time series information, fitting modeling is conducted of the "generated series" whose randomness obtained through the accumulative processing is weakened while regularity is enhanced.

This paper adopts the gray prediction model to study the changing rend of the market demand indexes based on the following hypotheses: 1) The data required by the gray prediction model is few; 2) The changing trend of most future index market data is progressive; 3) The gray prediction model has an edge over other prediction models while being applied to short-term land demand prediction.

GM (1. 1) for the gray prediction is based on the random original time series. The regularity of the new time series formed by the time accumulation can use the solution of the linear first-order differential equation to draw near. The imminent curve can be regarded as the model. At last, conduct an inverse accumulated generating operation of the model's predicted value, and predict the system. The establishment process of GM (1. 1) is shown below:

(1) Data pre-treatment

Assume the original data series to be:

$$x^{(0)} = \left\{ x^{(0)}(i) \middle| i = 1,2,\ldots,n \right\} \tag{1}$$

The series generated after accumulation:

$$x^{(1)}(i) = \left\{ \sum_{j=1}^{i} x^{(0)}(j) \middle| i = 1,2,\ldots,n \right\} \tag{2}$$

In order to revert the accumulated series into the original series, it is necessary to conduct the subtraction-based generation, which refers to the subtraction between the former series and the latter series. See Eq. (3) below:

$$\Delta x^{(1)}(i) = x^{(1)}(i) - x^{(1)}(i-1) = x^{(0)}(i) \tag{3}$$

Where, $i = 1,2,\ldots,n,\ x^{(0)}(0) = 0$. The generated series weakens the randomness and instability of the original series and enhances the regularity. Conduct the index regularity test and the smoothness test of $x^{(0)}$ and $x^{(1)}$:

Step ratio: $$\sigma(i) = \frac{x^{(1)}(i)}{x^{(1)}(i-1)} \tag{4}$$

Smoothness ratio: $$\rho(i) = \frac{x^{(0)}(i)}{x^{(1)}(i-1)} \tag{5}$$

When $i > 3$, $\rho(i) < 0.5$ and $\sigma(i) < 2$, the data meet the smoothness conditions and the index rules. Conduct $GM(1,1)$ modeling of $x^{(1)}$.

(2) Modeling principle

Provide the observation data series:

$$x^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n) \right\} \tag{6}$$

After one accumulation, the following equation can be obtained:

tion $$x^{(1)} = \left\{ x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n) \right\} \tag{7}$$

Assume that $x^{(1)}$ conforms to the first order differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u \tag{8}$$

In the equation, a stands for the constant coefficient, and the development gray number; u stands for the internal control gray number, which is a specific input of the system; the first "1" is the number of orders; the second "1" stands for the number of variables. The equation meets the initial conditions.

When $t = t_0$, the solution of $x^{(1)}(t) = x^{(1)}(t_0)$ is:

$$x^{(1)}(t) = \left[x^{(1)}(t_0) - \frac{u}{a} \right] e^{-a(t-t_0)} + \frac{u}{a} \tag{9}$$

The discrete value ($t_0 = 1$) sampled at an equal interval is:

$$x^{(1)}(k+1) = \left[x^{(1)}(1) - \frac{u}{a} \right] e^{-ak} + \frac{u}{a} \tag{10}$$

The gray modeling is to obtain an accumulated series after one accumulation. Use the least square method to calculate the constant a and u in Eq. (8).

Adopt $x^{(1)}(1)$ as the initial value. Put $x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)$ into Eq. (8) respectively. Use the difference to replace the differential. Based on equal-interval sampling, $\Delta t = (t+1) - t = 1$, then:

$$\frac{\Delta x^{(1)}(2)}{\Delta t} = \Delta x^{(1)}(2) = x^{(1)}(2) - x^{(1)}(1) = x^{(0)}(2) \tag{11}$$

Similarly,

$$\frac{\Delta x^{(1)}(2)}{\Delta t} = x^{(0)}(2), \ldots, \frac{\Delta x^{(1)}(n)}{\Delta t} = x^{(0)}(n) \tag{12}$$

Based on Eq. (5-8), then:

$$\begin{cases} x^{(0)}(2) + ax^{(1)}(2) = u \\ x^{(0)}(3) + ax^{(1)}(3) = u \\ \cdots \\ x^{(0)}(n) + ax^{(1)}(n) = u \end{cases} \tag{13}$$

Move $ax^{(1)}(i)$ to the right and write it into the dot product form of the vector:

$$\begin{cases} x^{(0)}(2) = \left[-x^{(1)}(2),1\right]\begin{bmatrix} a \\ u \end{bmatrix} \\ x^{(0)}(3) = \left[-x^{(1)}(3),1\right]\begin{bmatrix} a \\ u \end{bmatrix} \\ \cdots \\ x^{(0)}(n) = \left[-x^{(1)}(n),1\right]\begin{bmatrix} a \\ u \end{bmatrix} \end{cases} \quad (14)$$

Since $\frac{\Delta x^{(1)}}{\Delta t}$ refers to the value accumulated to $x^{(1)}(1)$ on two moments, it is more feasible for $x^{(i)}(i)$ to use the average value of the two moments. Replace $x^{(i)}(i)$ with $\frac{1}{2}\left[x^{(i)}(i)+x^{(i)}(i-1)\right],(i=2,3,\ldots,n)$.

Rewrite Eq. (14) into the matrix expression:

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(2)+x^{(1)}(1)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(3)+x^{(1)}(2)\right] & 1 \\ \vdots & 1 \\ -\frac{1}{2}\left[x^{(1)}(n)+x^{(1)}(n-1)\right] & 1 \end{bmatrix} \begin{bmatrix} a \\ u \end{bmatrix} \quad (15)$$

Make $y=\left(x^{(0)}(2),x^{(0)}(3),\ldots,x^{(0)}(n)\right)^T$,

$$B = \begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(2)+x^{(1)}(1)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(3)+x^{(1)}(2)\right] & 1 \\ \vdots & 1 \\ -\frac{1}{2}\left[x^{(1)}(n)+x^{(1)}(n-1)\right] & 1 \end{bmatrix}, \quad U = \begin{bmatrix} a \\ u \end{bmatrix}$$

Then, the expression of Eq. (15) is shown below:

$$y = BU \quad (16)$$

Therefore, the least square estimation of Eq. (16) is:

$$\hat{U} = \begin{bmatrix} \hat{a} \\ \hat{u} \end{bmatrix} = \left(B^T B\right)^{-1} B^T y \quad (17)$$

Put the estimated value, $\hat{a}$ and $\hat{u}$, into Eq. (16) to obtain the time response equation:

$$\hat{x}^{(1)}(k+1) = \left[x^{(1)}(1) - \frac{\hat{u}}{\hat{a}}\right]e^{-\hat{a}k} + \frac{\hat{u}}{\hat{a}} \quad (18)$$

When $k=1,2,\ldots,n-1$, $\hat{x}^{(1)}(k+1)$ obtained through Eq. (18) is a fitted value. When $k \ge n$, $\hat{x}^{(1)}(k+1)$ is a predicted value. $\hat{x}^{(1)}(k+1)$ is the fitted value of $x^{(1)}$. Subtract the former with the latter to restore $x^{(1)}$. When $k=1,2,\ldots,n-1$, the fitted value, $\hat{x}^{(0)}(k+1)$, of the original series, $x^{(0)}$, can be obtained; when $k \ge n$, the predicted value, $\hat{x}^{(0)}(k+1)$, of the original series, $x^{(0)}$, can be obtained.

Based on the above calculation method, the predicted value at "$k+1$" can be obtained:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \quad (19)$$

The model's precision and reliability can be tested through residual error, relational degree and posterior error. After the test, if the model is not precise enough, calibration and optimization test can be conducted.

(3) Precision test

a. The major calculation method of the residual error test is shown below:

Residual error:

$$\varepsilon^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k), k=2,3,\ldots,n \quad (20)$$

Relative residual error:

$$e(k) = \left[x^{(0)}(k) - \hat{x}^{(0)}(k)\right] / x^{(0)}(k), k=2,3,\ldots,n \quad (21)$$

Absolute error of the corresponding percentage:

$$MAPE = \frac{1}{n-1}\sum_{k=2}^{n}\left|\frac{\varepsilon^{(0)}(k)}{x^{(0)}(k)}\right| \quad (22)$$

Generally speaking, if $MAPE \le 10\%$ and the error of the original point is less than 2%, it is apt to say that the model meets the precision requirement.

b. The major calculation method of the relational degree test is shown below:

$$\eta(i) = \frac{\min\left\{\varepsilon^{(0)}(k)\right\} + \xi \max\left\{\varepsilon^{(0)}(k)\right\}}{\varepsilon^{(0)}(k) + \rho \max\left\{\varepsilon^{(0)}(k)\right\}} \quad (23)$$

Where, ξ stands for the resolution ratio, which is usually "0" and the relational degree is $r=\frac{1}{n}\sum_{i=1}^{n}\eta(i)$. When $\xi=0.5$, $r>0.6$ meets the predicted precision.

c. The major calculation method of the posterior error test is shown below:

Mean of $x^{(0)}$: $\overline{X} = \frac{1}{n}\sum_{k=1}^{n}x^{(0)}(k)$ (24)

Variance of $x^{(0)}$: $S_1 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}\left(x^{(0)}(k)-\overline{X}\right)}$ (25)

Mean value of the residual error:

$$\overline{E} = \frac{1}{n-1}\sum_{k=2}^{n}\varepsilon^{(0)}(k) \quad (26)$$

Variance of the residual error:

$$S_2 = \sqrt{\frac{1}{n-1}\sum_{k=2}^{n}\left[\varepsilon^{(0)}(k) - \overline{E}\right]} \quad (27)$$

Specific value of the posterior error: $C = \frac{S_2}{S_1}$ (28)

Probability of the small error:

$$P = P\left\{\left|\varepsilon^{(0)}(k) - \overline{E}\right| < 0.6745 S_1\right\} \quad (29)$$

Generally speaking, in order to diminish the error range of the predicted value, the value of C should be small enough, even if the original data have no rules to follow. The model prediction results can be judged according to the value of α, C and P. The higher the value of α and C is, the smaller the value of P is. Generally speaking, the corresponding precision of the value of P and C is shown in Table 3. After the model is built, it is tested. If the test results are unqualified, the model can be improved so as to efficiently improve the precision.

Table 3. Precision test grade reference table

Index / Precision grade	Relative error α	Specific value of the posterior error C	Probability of minor error P
Grade 1	0.01	0.35	0.95
Grade 2	0.05	0.5	0.8
Grade 3	0.1	0.65	0.7
Grade 4	0.2	0.8	0.6

Based on the above discussion, the major modeling steps of GM(1,1) can be boiled down as below:

Step 1: Assume the original data series to constitute $x^{(0)}$ [See Eq. (6)] and conduct an accumulation series calculation to obtain $x^{(1)}$ [See Eq. (7)];

Step 2: Build the matrix form as like Eq. (15) and obtain the corresponding B and y;

Step 3: Work out the inverse matrix, $\left(B^T B\right)^{-1}$;

Step 4: Work out the estimated value, $\hat{a}$ and $\hat{u}$, according to Eq. (17);

Step 5: Work out $\hat{x}^{(1)}(i)$ according to Eq. (18) and adopt the subtraction of the former with the latter to achieve restoration, namely:

$$x^{(0)}(i) = x(i) - \hat{x}^{(1)}(i-1), i = 2,3,\dots,n$$

Step 6: Precision test and prediction:

Put the data in Table 1 into GM (1, 1). Predict the future market indexes. The prediction results, the prediction precision indexes and the comparison results are shown in Table 4, Table 5 and Fig. 1 below:

Table 4. Prediction results by the gray model

Period (t)	Consumers' income (m)	Interest rate (r)	Product ordering price(p_{0t})	Inventory cost (C)
9	10,444.69	2.729	11.60	0.938
10	10,512.53	2.690	11.70	0.954
11	10,580.81	2.651	11.81	0.969
12	10,649.53	2.613	11.91	0.985

Table 5. Precision results of various indexes based on GM(1,1)

Index	Development coefficient (a)	Gray actuating quantity (u)	Standard deviation error ©	Mean relative error (MAPE)	Relational degree (r)
Consumers' income	-0.006474	-1527166	0.1248	0.0029	0.751
Interest rate	0.014550	212.31	0.0708	0.0045	0.563
Product ordering price	-0.008789	-1224.9	0.1766	0.0044	0.554
Stock-holding cost unit price	-0.016203	-50.458	0.1717	0.0083	0.536

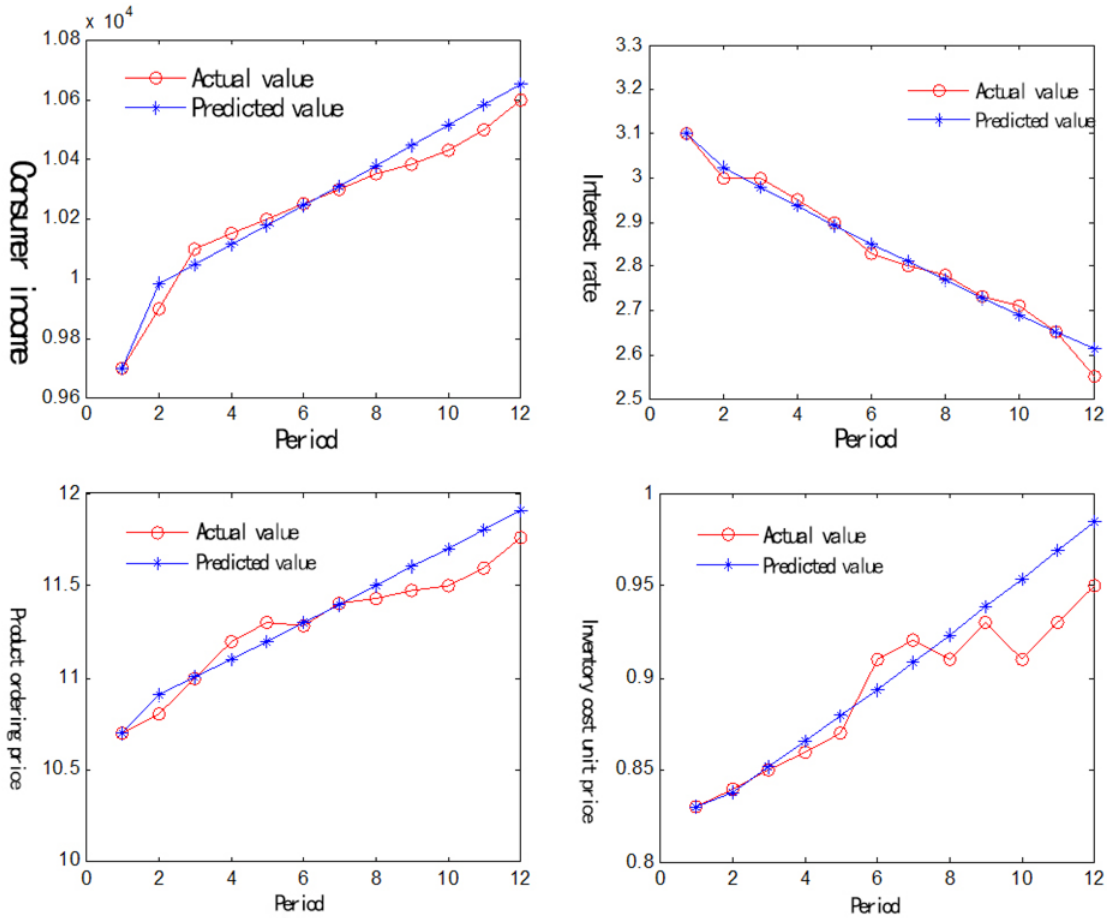


Figure 1. Actual value and predicted value of GM(1,1)

Combine the original data and the predicted data obtained by the gray prediction model, namely the actual data in Table 1 and the predicted data in Table 4. Put them into the optimized model of the specific power demand function respectively to find a solution. The optimization results of the actual data and the predicted data are shown in Table 6 and Table 7 below:

Table 6. Practical optimization results of the specific power demand function model

T	I_t	D_t	S_t	AI_t	p_{1t}	Q_t	R_t
1	200	144	144	134	17.264	6	2291.2
2	62	126	126	76	18.774	77	3754.6
3	13	98	98	50	21.153	86	4858.9
4	1	104	104	106	20.744	157	5252.9
5	54	91	91	129.5	21.659	121	5810.8
6	84	103	103	62.5	21.1	30	7578.9
7	11	97	97	103.5	22.362	141	7956
8	55	89	89	195.5	22.567	185	8088.9
9	151	83	83	128.5	22.94	19	9553.5
10	87	86	86	46	22.99	2	11474
11	3	100	100	99	21.27	146	11978
12	49	118	118	61	19.686	71	13452

Table 7. Gray predicted optimization results of the specific power demand function model

T	I_t	D_t	S_t	AI_t	p_{1t}	Q_t	R_t
1	200	129	129	182.5	18.612	47	1687.2

2	118	111	111	90.5	20.485	28	3563.6
3	35	121	121	85.122	18.377	86	4808.4
4	0	106	106	207	20.46	260	4033.8
5	154	132	132	210	16.99	122	4771.1
6	144	95	95	126.5	22.279	30	6408.8
7	79	100	100	51	21.974	22	8309.5
8	1	100	100	126	20.889	175	8422.2
9	76	93	92	264.5	21.302	16	10188
10	0	155	155	128.5	15.551	206	10279
11	51	99	99	122.5	21.404	121	11016
12	73	119	119	153.92	19.55	46	12773

Based on solution results of the above models, it can be seen that the overall prediction performance of GM (1, 1) is better. The precision indexes of both models reach Grade 1, so the precision is relatively high. Optimize the market indexes obtained through the predicted results and find a solution. The results obtained by the predicted model and the results obtained by the practical model differ not greatly from each other. Except that in the 10th period the product sales price is relatively low and the product demand volume is relatively large, the overall predicted value of the model is good. The final target benefits obtained through the optimization show no significance difference. Therefore, when the market demand variation trend features an index variation, the dynamic pricing based on the gray prediction model and the inventory control optimization model have no practical guiding significance for the product sales planning.

3. Self-adaptive market optimization and prediction based on the time series model

When various index data on the market cannot meet the exponential changes, the error of the gray prediction model is relatively huge. Besides, during the variation process of the practical market indexes, they usually fluctuated up and down. Therefore, it is necessary to predict the data with periodical fluctuations. This problem can be well solved through the time series model.

The time series is to arrange different values of certain index on different time nodes in accordance with the time sequence [3, 4].Among the time series models, Auto Regressive Moving Average (ARMA) is one with a full consideration and a high feasibility. The model contains the characteristics of the auto regression model and the moving average model. However, since the time series shows certain trend or characteristics under most conditions, it cannot meet the stability requirement of the model [5, 6]. Therefore, ARMA model cannot be directly used. However, in the 1970s, due to limits of the ARMA model, Box and Jenkins put forward a new time series model, namely ARIMA, in 1970 [7]. The model obtains a stable time series after d -order gradual differentiation of non-stable time series, y_t. Based on the differentiated time series, ARMA model can be built. At last, the built-up model obtains its original sequence through the inverse transformation. The basic mathematic description of ARIMA model is shown in Eq. 30:

$$\Delta^d y_t = \theta_0 + \sum_{i=1}^{p} \phi_i \Delta^d y_{t-1} + \varepsilon_t + \sum_{i=1}^{q} \phi_j \varepsilon_{t-j} \quad (30)$$

Where, $\Delta^d y_t$ stands for the series transformed by d -order differentiation of the series, y_t; ε_t stands for the immediate error item at the moment of t, which is also the white noise series in line with the normal distribution whose mean is "0" and whose variance is a constant; ϕ_i and ϕ_j are the estimated parameters of the model; p and q are the number of order of the model. From Eq. 30, it can be seen that, if the series $\Delta^d y_t$ meets the modeling requirements of ARMA, then the series, y_t, meets the modeling requirements of ARIMA.

Based on the analysis of the above ARIMA model, the calculation steps of ARIMA are shown below:

Step 1: Test the series' stability and stabilize it: Conduct the stability test of the original series data; if the original series is stable, ARMA can be directly used; if the original series is not a stable one, the original series should be differentiated until the stability of the series is met.

Step 2: Confirm the number of orders of the model: The number of orders of the model, namely p and q, can be confirmed through the coefficient of autocorrelation and partial autocorrelation coefficient;

Step 3: Estimate and test the model parameters: Judge whether the model is feasible through the estimation of parameters and the test of significance and residual error test of parameters;

Step 4: Predict the original series based on the confirmed model and the model parameters.

Put the data from Table 2 into the above ARIMA, and predict the future market indexes. The prediction results, the prediction precision indexes and the comparison results of the model are shown in Table 8, Table 9 and Fig. 2:

Table 8. Time series prediction results

Period (t)	Consumers' income (m)	Interest rate (r)	Product ordering price(p_{0t})	Inventory cost (C)
9	9,558.64	3.03	10.55	0.835
10	9,611.24	2.92	10.54	0.862
11	9,597.23	2.89	10.53	0.829
12	9,570.04	2.96	10.52	0.825

Table 9. Precision results of various indexes based on the time series model

Indexes	Development coefficient (a)	Gray actuating quantity (u)	Standard deviation error ©
Consumers' income	1.33772	0.0165	0.434
Interest rate	0.79031	0.0142	0.608
Product ordering price	0.81717	0.0110	0.530
Stock-holding cost unit price	1.03307	0.0131	0.651

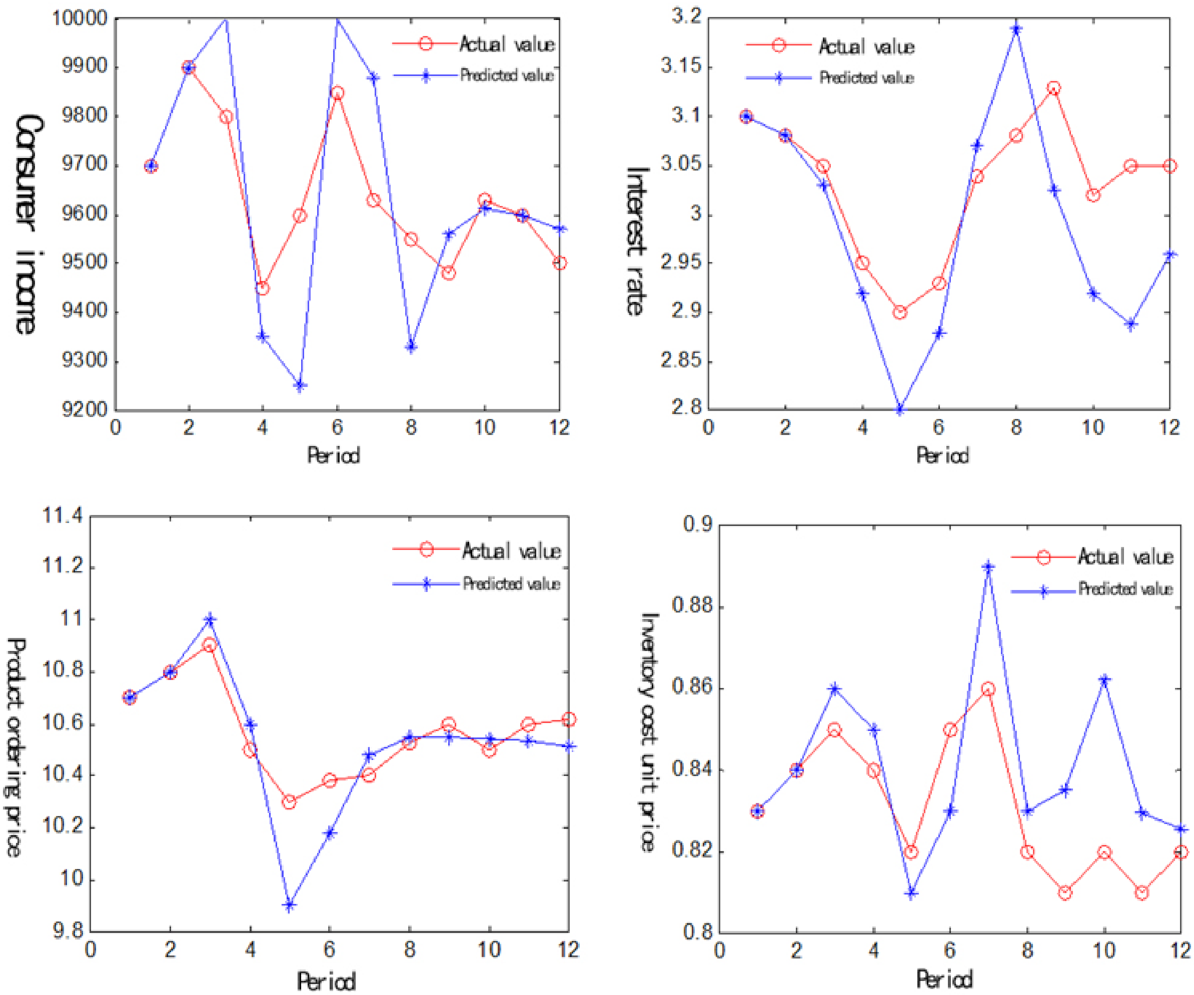


Figure 2. Results of the actual value and the predicted value of the time series model

Put the actual data in Table 2 and the predicted data in Table 8 obtained through the time series prediction model into the optimized model of the specific power demand function to find a solution. The optimization results of the actual data and the predicted data are shown in Table 10 and Table 11:

Table 10. Practical optimization results of the specific power demand function model

T	I_t	D_t	S_t	AI_t	p_{1t}	Q_t	R_t
1	200	96	96	185	21.244	33	1,532.6
2	137	100	100	140	20.923	53	2,940.4
3	90	94	94	1104.5	21.726	4	4,893.8
4	0	100	100	155	20.294	205	4,647.1
5	105	103	103	174.5	20.167	121	5,333.6
6	123	103	103	74.5	20.522	3	7,349.9
7	23	99	99	64.48	20.656	76	8,560.3
8	0	103	103	101.5	20.111	153	8,929.8
9	50	110	110	75	19.102	80	10,122
10	20	139	139	81.181	16.512	119	11,113
11	0	97	97	72.5	20.962	121	11,805
12	24	97	97	64	20.839	72	13,006

Table 11. Predicted optimization results of the specific power demand model obtained through the time series

T	I_t	D_t	S_t	AI_t	p_{1t}	Q_t	R_t
1	200	107	107	171.5	19.745	25	1,705.5
2	118	166	166	177	14.962	142	2,503.8
3	94	98	98	130	21.123	85	3,534
4	81	96	96	87	20.952	54	4,897.2
5	39	103	103	228.5	20.127	241	4,305
6	177	101	101	156.5	20.758	30	5,958.9
7	106	99	99	118.5	20.79	62	7,259.6
8	69	97	97	168.02	20.818	28	8,953.2
9	0	96	96	66	21.028	114	9,716.4
10	18	96	96	59.077	21.087	78	10,881
11	0	106	106	94	19.816	147	11,348
12	41	114	113	88.674	18.824	72	12,677

Based on the solution results of the above models, it can be seen that the overall prediction performance of the time series model is good. The evaluation of the precision indexes of various models according to the gray system also reached the Grade 1 level. The market index value obtained through the predicted results is optimized to work out the solution. The results obtained by the prediction model and the practical model differ from each other slightly. Their difference mainly lies in terms of sales price of products. However, the product ordering amount and the model's accumulated profits obtained by both models generally coincide with each other. Therefore, when the market demands fluctuate up and down, the dynamic pricing and inventory control optimization model based on the time series prediction model has no practical guiding significance for the product sales planning. Besides, most practical market indexes are fluctuating.

4. Dynamic pricing and inventory control model based on the intelligent prediction algorithm

Based on the optimization results through the above traditional prediction algorithms, it can be seen that the gray system prediction model has lots of prerequisites (for example, the gray system prediction requires the data to change in the exponential form); the predicted optimization results and the actual optimization results of the model differ from each other slightly. The cause of the difference is that, when the series is unstable, the error thus caused is relatively huge. Through the inverse transformation, the accumulated error will be accumulated. Therefore, the final optimization results and the actual optimization results of the model show a larger error. Therefore, in order to overcome shortages of the above traditional prediction models, this paper employs the BP neural networks model and the support vector basis model to predict various market indexes, compare its optimization results with those of the timer series model and the practical model and show the characteristics of the intelligent optimization algorithm.

4.1. BP neural networks mathematic model

According to the weight of the input node and the output node, the weight between the input node and the implicit node, and the weight between the implicit node and the output node, the relationship between various layers of nodes can be expressed below:

(1)The forward transmission process of signals:

The input of the i node in the implicit layer, net_i:

$$net_i = \sum_{j=1}^{M} w_{ij} x_j + \theta_i \tag{31}$$

The output of the i node in the implicit layer, y_i:

$$y_i = \phi(net_i) = \phi\left(\sum_{j=1}^{M} w_{ij} x_j + \theta_i\right) \tag{32}$$

The input of the k node in the output layer, net_k:

$$net_k = \sum_{i=1}^{q} w_{ki} y_i + a_k = \sum_{i=1}^{q} w_{ki} \phi\left(\sum_{j=1}^{M} w_{ij} x_j + \theta_i\right) + a_k \tag{33}$$

The output of the k node in the output layer, o_k:

$$o_k = \psi(net_k) = \psi\left(\sum_{i=1}^{q} w_{ki} y_i + a_k\right) = \psi\left(\sum_{i=1}^{q} w_{ki} \phi\left(\sum_{j=1}^{M} w_{ij} x_j + \theta_i\right) + a_k\right) \tag{34}$$

The major idea of the BP neural networks is to rectify the weight and the threshold to make the error function to descend in the gradient direction. After the step-by-step treatment of the input information in the implicit layer, the practical output can be obtained from the output layer. If the practical output and the sample output disagree with each other, the error will be sent back in a reverse direction layer by layer. Modify the weight of every layer in accordance with the fitting rules required by the algorithm. The improvement is repeated until reaching the convergence or stable state. In other words, the overall error of the practical output and the target output should reach the required minimum error. Below are the specific steps:

The quadric form error criterion function of every sample, p, is E_p:

$$E_p = \frac{1}{2}\sum_{k=1}^{L}(T_k - o_k)^2 \tag{35}$$

The overall error criterion function of “P” training samples is:

$$E = \frac{1}{2}\sum_{p=1}^{P}\sum_{k=1}^{L}(T_k^p - o_k^p)^2 \tag{36}$$

According to the error gradient descent method, the correction of the weight of the correction output layer is Δw_{ki}; the correction of the threshold of the output layer is Δa_k; the correction of the weight of the implicit layer is Δw_{ij}; and the correction of the threshold of the implicit layer is $\Delta\theta_i$.

$$\Delta w_{ki} = -\eta\frac{\partial E}{\partial w_{ki}};\ \Delta a_k = -\eta\frac{\partial E}{\partial a_k};\ \Delta w_{ij} = -\eta\frac{\partial E}{\partial w_{ij}};$$

$$\Delta\theta_i = -\eta\frac{\partial E}{\partial \theta_i} \tag{37}$$

The adjustment equation of the weight of the output layer:

$$\Delta w_{ki} = -\eta\frac{\partial E}{\partial w_{ki}} = -\eta\frac{\partial E}{\partial net_k}\frac{\partial net_k}{\partial w_{ki}} = -\eta\frac{\partial E}{\partial o_k}\frac{\partial o_k}{\partial net_k}\frac{\partial net_k}{\partial w_{ki}} \tag{38}$$

The adjustment equation of the threshold of the output layer:

$$\Delta a_k = -\eta\frac{\partial E}{\partial a_k} = -\eta\frac{\partial E}{\partial net_k}\frac{\partial net_k}{\partial a_k} = -\eta\frac{\partial E}{\partial o_k}\frac{\partial o_k}{\partial net_k}\frac{\partial net_k}{\partial a_k} \tag{39}$$

The adjustment equation of the weight of the implicit layer:

$$\Delta w_{ij} = -\eta\frac{\partial E}{\partial w_{ij}} = -\eta\frac{\partial E}{\partial net_i}\frac{\partial net_i}{\partial w_{ij}} = -\eta\frac{\partial E}{\partial y_i}\frac{\partial y_i}{\partial net_i}\frac{\partial net_i}{\partial w_{ij}} \tag{40}$$

The adjustment equation of the threshold of the implicit layer:

$$\Delta\theta_i = -\eta\frac{\partial E}{\partial \theta_i} = -\eta\frac{\partial E}{\partial net_i}\frac{\partial net_i}{\partial \theta_i} = -\eta\frac{\partial E}{\partial y_i}\frac{\partial y_i}{\partial net_i}\frac{\partial net_i}{\partial \theta_i} \tag{41}$$

Besides,

$$\frac{\partial E}{\partial o_k} = -\sum_{p=1}^{P}\sum_{k=1}^{L}(T_k^p - o_k^p) \tag{42}$$

$$\frac{\partial net_k}{\partial w_{ki}} = y_i,\ \frac{\partial net_k}{\partial a_k} = 1,$$

$$\frac{\partial net_i}{\partial w_{ij}} = x_j,\ \frac{\partial net_i}{\partial \theta_i} = 1 \tag{43}$$

$$\frac{\partial E}{\partial y_i} = -\sum_{p=1}^{P}\sum_{k=1}^{L}(T_k^p - o_k^p)\cdot\psi'(net_k)\cdot w_{ki} \tag{44}$$

$$\frac{\partial y_i}{\partial net_i} = \phi'(net_i) \tag{45}$$

$$\frac{\partial o_k}{\partial net_k} = \psi'(net_k) \tag{46}$$

At last, the following equation can be obtained:

$$\Delta w_{ki} = \eta\sum_{p=1}^{P}\sum_{k=1}^{L}(T_k^p - o_k^p)\cdot\psi'(net_k)\cdot y_i \tag{47}$$

$$\Delta a_k = \eta\sum_{p=1}^{P}\sum_{k=1}^{L}(T_k^p - o_k^p)\cdot\psi'(net_k) \tag{48}$$

$$\Delta w_{ij} = \eta \sum_{p=1}^{P} \sum_{k=1}^{L} \left(T_k^p - o_k^p\right) \cdot \psi'\left(net_k\right) \cdot w_{ki} \cdot \phi'\left(net_i\right) \cdot x_j \tag{49}$$

$$\Delta \theta_i = \eta \sum_{p=1}^{P} \sum_{k=1}^{L} \left(T_k^p - o_k^p\right) \cdot \psi'\left(net_k\right) \cdot w_{ki} \cdot \phi'\left(net_i\right) \tag{50}$$

4.2. Establishment and optimization results of the BP neural networks modeling

The network structure and relevant parameters established in this paper are shown below:

The neural networks have two layers. The number of neurons in the first layer is 5 and 1 in the second layer. The transfer function between the first layer of neurons and the second layer of neurons is tansig (tangent transfer function). The transfer function between neurons of the second layer and neurons of the output layer is purelin (linear transfer function). Besides, the training function of the network is trainlm (Levenberg-Marquardt method). The maximum iterations of the network are 1,000 times. The network fitting rate is 0.3 and the network's set error is 1×10^{-3}.

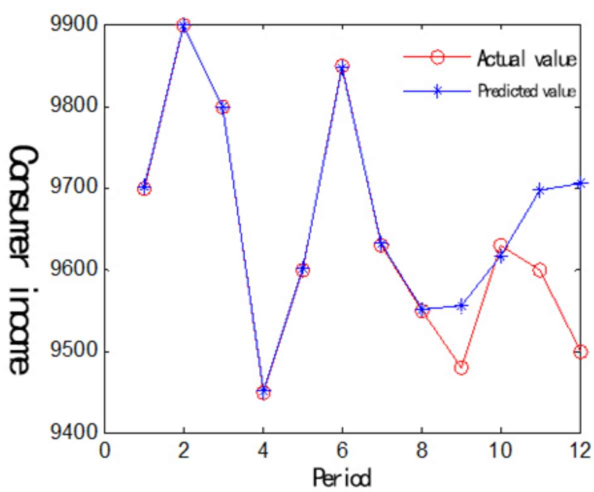
Put the data in Table 2 into the above BP neural networks model and predict the future market indexes. The prediction results, prediction precision indexes and comparison results of the models are shown in Table 12, Table 13 and Fig. 4:

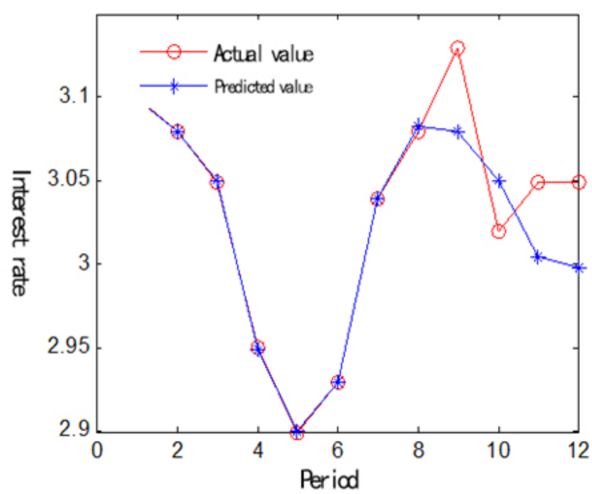
Table 12. Prediction results of BP neural networks

Period (t)	Consumers' income (m)	Interest rate (r)	Product ordering price(p_{0t})	Inventory cost (C)
9	9,554.49	3.0794	10.6077	0.81900
10	9,616.87	3.0510	10.6098	0.81899
11	9,697.33	3.0047	10.6098	0.81899
12	9,704.99	2.9989	10.6098	0.81899

Table 13. Precision results of the BP neural networks model based on various indexes

Index	Development coefficient (a)	Gray actuating quantity (u)	Standard deviation error ©
Consumers' income	0.0098	0.00011	0.53236
Interest rate	0.0128367	0.00015	0.69782
Product ordering price	0.0396336	0.00042	0.69324
Stock-holding cost unit price	0.0431116	0.00026	0.75108





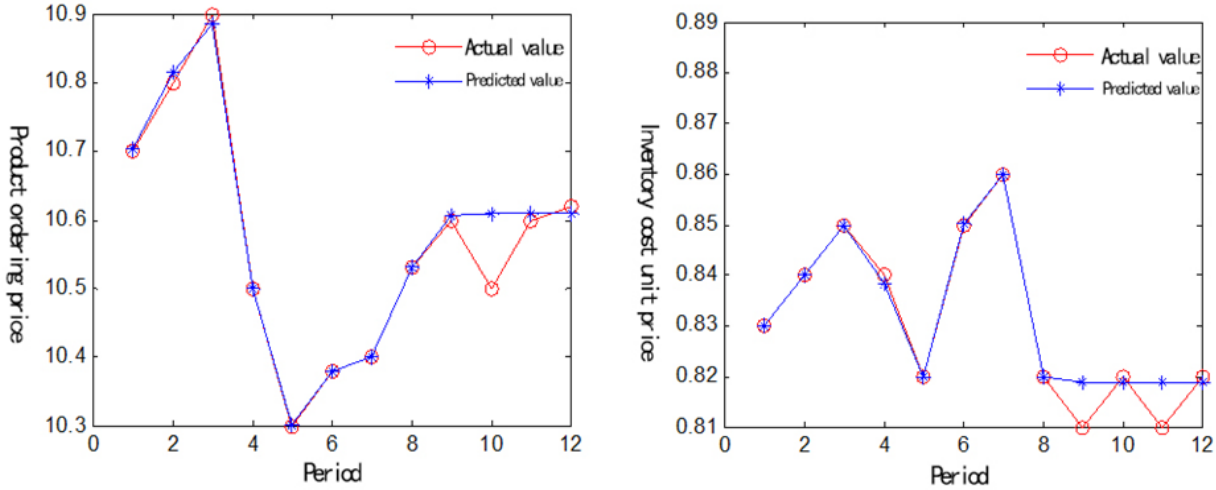


Figure 3. Predicted and evaluated results of the BP neural networks model

Put the predicted data obtained by the BP neural networks model, namely the predicted data in Table 12 into the optimized model of the specific power demand function, to get the solution. The optimization results of the predicted data are shown in Table 14.

Table 14. Optimized predicted results of the specific power demand function model

T	I_t	D_t	S_t	AI_t	p_{1t}	Q_t	R_t
1	200	116	116	212	18.777	70	1,243.9
2	154	104	104	164	20.43	62	2,558.9
3	112	94	94	131	21.691	66	3,767.7
4	84	99	99	145.5	20.481	111	4,506
5	96	100	100	100	20.599	54	5,921.1
6	50	101	101	51.5	20.717	52	7,437.6
7	1	99	99	131.5	20.741	180	7,502.3
8	82	115	115	65.5	18.594	41	9,163
9	8	97	97	80.5	20.96	121	9,835.9
10	32	98	98	123	20.906	140	10,288
11	74	97	97	71.5	21.179	46	11,784
12	23	97	97	63.123	21.182	73	13,015

Based on the above solution results obtained through the above models, it can be seen that the overall prediction results of the BP neural networks model is better than those of the time series model. The value of various precision indexes of the model also reaches Grade 1 level according to the gray system evaluation. Through optimization solution of the value of various market indexes obtained through the prediction results, it can be found that the optimization results obtained by the predicted model are in consistent with those obtained by the practical model, which is mainly reflected in terms of product sales price and product accumulated profits. Therefore, when market demands fluctuated up and down, the dynamic pricing and inventory control optimization model based on the BP neural networks prediction is closer to the practical situations.

4.3. Self-adaptive market optimization prediction based on the Support Vector Machine model

Support Vector Machine (SVM) is a new-type machine fitting algorithm and an improvement and optimization of the neural networks model. SVM is far more powerful in solving over-fitting, under-fitting and local minimum than that of the other machines [8, 9].At present, SVM model has got extensive application in the field of securities prediction, and can accurately predict the short-term market changes [10].

SVM built in this paper conducts parameter optimization through the genetic algorithm (GA). Relevant structural parameters of various indexes obtained by the prediction model are shown in Table 15 below:

Table 15. Parameter values of various indexes obtained by SVM prediction model

Index	e	C	g
Consumers' income	0.0863	15.845	4.8599
Interest rate	0.0483	1.9409	2.0790
Product ordering price	0.1002	1.1127	5.8613
Stock-holding cost unit price	0.5632	25.8389	5.1088

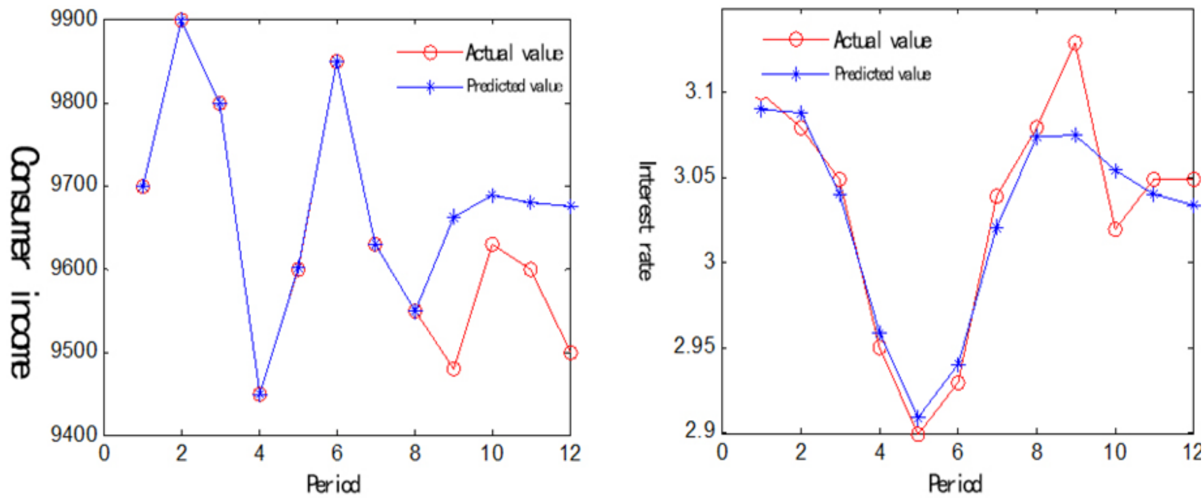
Put data in Table 2 into the SVM model already established above, and predict the future market indexes. The prediction results, prediction precision indexes and comparison results of the model are shown in Table -16, Table 17 and Fig. 6:

Table 16. Prediction results of the SVM model

Period (t)	Consumers' income (m)	Interest rate (r)	Product ordering price(p_{0t})	Inventory cost (C)
9	9,661.65	3.0758	10.6061	0.80964
10	9,687.55	3.0552	10.5913	0.82494
11	9,678.33	3.0403	10.5826	0.83233
12	9,675.61	3.0347	10.5818	0.83342

Table 17. Precision results of various indexes obtained by the SVM model

Index	Development coefficient (a)	Gray actuating quantity (u)	Standard deviation error ©
Consumers' income	0.000615	0.0000007	0.50554
Interest rate	0.146374	0.003330	0.47897
Product ordering price	0.000588	0.000011	0.52574
Stock-holding cost unit price	0.000565	0.000008	0.47218



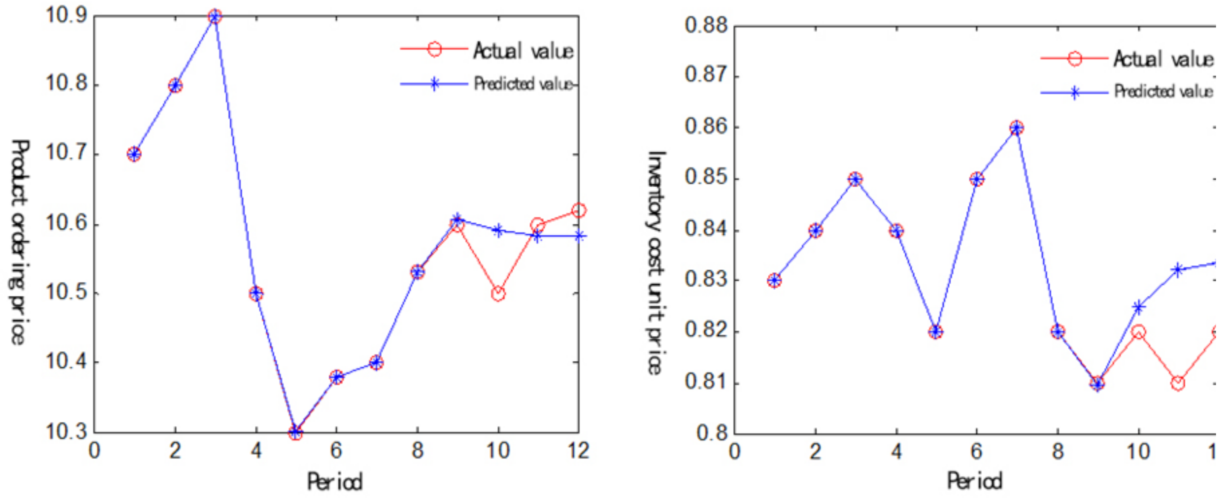


Figure 4. Results of the actual value and the predicted value of the SVM model

Put the predicted data obtained by the SVM model, namely the predicted data into Table 16, into the optimization model of the specific power demand function to get a solution. The optimization results of the prediction data are shown in Table 18:

Table 18. Prediction optimization results of the SVM model of the specific power demand function

T	I_t	D_t	S_t	AI_t	p_{1t}	Q_t	R
1	200	134	134	133	17.042	0	2166.3
2	66	110	110	67	19.63	56	3670.7
3	12	96	96	54.857	21.378	84	4768.2
4	0	101	101	100.5	20.262	151	5135.5
5	50	106	106	108	19.766	111	6000
6	55	101	101	159.5	20.747	155	6354.4
7	109	99	99	135.5	20.772	76	7495.8
8	86	96	96	65	21.046	27	9175.3
9	17	96	96	54	21.204	85	10263
10	6	97	97	114.5	21.158	157	10547
11	66	96	96	68	21.164	50	12000
12	20	96	96	49	21.159	77	13183

Based on the solution results obtained through the above models, it can be seen that the overall prediction results of the SVM model are better than those of the time series model. Value of various precision indexes also reaches the Grade 1 level according to the evaluation of the gray system. At last, based on the market index value obtained through the prediction results, the product's dynamic pricing and inventory control are optimized together. The optimization results obtained by the prediction model coincide with those obtained by the practical model, which is mainly reflected as the product's accumulated profits and product pricing. However, the product price of 10^{th} and 11^{th} period obtained by the BP neural networks and the SVM model is both slightly higher than the actual value. This is because, during the model prediction process, time is the only input information. In other words, it is caused by few index dimensions. However, generally speaking, it will not influence the final prediction results.

Based on the optimization results of the above traditional prediction models and the intelligent prediction models, it can be seen that the fluctuation of various indexes is not simply index changes, but periodical random fluctuations. Such fluctuations have their internal variation rules. Therefore, the combination of the intelligent prediction model and the dynamic pricing and inventory control joint optimization model can better adapt to changes of various market indexes.

5. Conclusions

This paper conducts optimization of the dynamic pricing and inventory control joint model, and focuses on the problem that the mode cannot be applied to the prediction of the practical market situations. Based on the simulation of the traditional prediction algorithms and the intelligent prediction model algorithms, this paper comes to the following conclusions: 1) When market indexes show exponential changes, the gray model can be easily combined with the dynamic pricing and inventory control joint optimization model to predict and control the future market indexes; 2) When market indexes fluctuate up and down irregularly, the time series model can be combined with the optimization model to conduct prediction, but the prediction results deviate greatly from the practical results; 3) BP neural networks model and the SVM model can be used to predict and optimize the future market situations. All these findings suggest that: the prediction results obtained by the dynamic pricing and inventory control joint model based on the intelligent prediction algorithm are closest to the practical optimization results. Therefore, the intelligent prediction algorithm can be used to predict various future market indexes, and can further obtain the optimal sales price and optimal order quantity by combining with the dynamic pricing and inventory control joint model.

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