The numerical analysis method of fuzzy-logic control systems stability of agglomerate sintering process

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Abstract
In the article the method of numerical stability analysis of fuzzy-logic control systems of technological objects of sinter plants, the so-called method of multiple equations is proposed. The distinctive features of the developed method, compared to the traditionally used imitation modeling, are the reduced amount of calculations required for the analysis, and the ability to determine the obtained accuracy estimates of the stability regions in the system parameter space.
Key word: STABILITY, MULTIPLE EQUATIONS, AGGLOMERATIVE PROCESS, FUZZY LOGICAL CONTROL SYSTEMS

In the field of modern automation the intelligent control systems with adaptive properties are widely used. The first ideas and methods of adaptation appeared in the beginning of the last century, and by the 80s - theory of adaptive control systems was formed. The main results of this period are based on definition of structure and analytical researches of basic structures stability of adaptive control by linear and nonlinear objects.

It should be noted that a substantial number of modern scientific results obtained by the development and follow-up study of adaptive control systems, which functioning either under ideal conditions, or with a control objects admitting the system characteristics linearization in an artificially defined limits of the stationary working point.

Analytical methods for the study of control systems allow to obtain the connection between the stability, as well as indicators of quality system and its parameters expressed in analytical form.

At this time, the interest in analytical research methods of intelligent control systems is largely weakened due to the inability to use them to obtain the satisfactory results for many practical problems.

However, the analytical study allows to argumentatively assert about the application effectiveness of the studied systems variants, as well as to avoid the gross errors, which can occur at the stage of numerical analysis.

There are two approaches to determine the stability of systems. The first – is analytical one, in this case the verification of stability is reduced to verifying the conditions, which were composed in a special way. The second approach - is imitation one, in this case the stability of the system is verified by numerical solutions describing its differential or difference equations [1].

Each of these approaches has its advantages and disadvantages. The disadvantage is the inability of the analytical approach to determine the entire stability region, but only a small part, in the majority of cases occurring in practice

This fact has led to that the analytical methods are used relatively rarely. The disadvantage of simulation approach is the very high computational costs required to find a solution, which results in the inability to check the stability in all operating modes of the system.

The proposed method for studying the stability, which, in some degree, intermediate between the analytical and simulation approaches is described below. And it appears free from their inherent disadvantages.

Problem statement

Subsystems of the lower level of process automatic control system (PACS) of agglomerate sintering comprise the following automatic control systems (ACS): ACS of mixture loading on the sintering belt, which focused on the maintaining of a given level of material in the hopper, considering the sintering belt performance and appropriate correction of the feeder plate performance; ACS of the mixture bed height on the sintering belt, whose functions include the monitoring and appropriate control of sintering temperature, sintering belt speed considering the gas permeability (negative pressure) of the mixture sintered layer; ACS of “mixture-water” ratio, which is in accordance with the actual value of the weight of mixture, which supplied to the drum pelletizer is automatically maintaines the process water flow rate for pelletizing at a certain level with correction for humidity in the intermediate hopper (mixture-storing bin) considering the inertia of the process equipment; ACS of the mixture temperature at the outlet from pelletizing drum supports a given level of the mixture preheating temperature above the “dew point” (60-65°C), with the aim of increasing the initial gas permeability of mixture layer on sintering belt by avoiding overwetting; ACS of “gas-air” ratio indrum pelletizer, provides complete combustion of the gas upon condition of technological ratio of gas and air; ACS of mixture surface temperature under the ignition furnace, which automatically maintains the optimal fill mode of solid fuel (coke) due to supplying a predetermined amount of the mixed gas, and thereby providing the desired thermal conditions for the mixture upper layers ignition; ACS of “gas-air” ratio in the furnace, designed to ensure complete combustion of gas by controlling the fan air flow; ACS of mixture sintering on sintering belt, provides the required vertical speed of mixture sintering considering matching with the speed of sintering belt movement.

As the main indirect indicator of the agglomerate sintering process completion the complex estimation of indirect indicators of the sintering process dynamic state is used [2].

In the case of complex estimation deviations from the technological tasks the effective channels of sintering process control will be determined algorithmically, taking into account the dynamic state of the mixture preparation technological process: “mixture-water” ratio correction; the temperature change of the mixture upper layers ignition, which is on the sintering belt; sintering belt speed correction.

It should be noted, that the equations describing the dynamics of the agglomerate sintering process by different control channels can be reduced to the form:

$$\vec{v}_{i+1} = f(\vec{v}_i), \qquad (1)$$

which have a single stationary solution (equilibrium) $\vec{v}_i = \vec{v}_{i+1} = \vec{0}$, where $\vec{0}$ – vector with zero components.

Let’s select some simply connected region, including a zero vector of $\vec{0} \in D$, wherein by equation (1) for any $\vec{v}_i \in D$, the relation $\vec{v}_{i+1} \in D$ will be performed.

This region D for the physically realizable systems always exists due to the limited phase coordinates of the system. The trivial solution $\vec{v}_i = v_{i+1} = \vec{0}$ stable in the region D in the Lyapunov sense,if for any $\varepsilon > 0$,exists a $\lambda(\varepsilon) > 0$, that for all $i > 0$ from $\vec{v}_0 \in D$ and $\|\vec{v}_0\| \le \varepsilon$ by equation (1) follows $\|\vec{v}_i\| < \lambda(\varepsilon)$. If, instead of the last inequality the relation

$$\lim_{i \to \infty} \|\vec{v}_i\| = 0 \qquad (2)$$

is satisfied than the asymptotic stability of the system takes place [3].

Further, let's consider only the asymptotic stability in the region. According to the method of contracting maps for the asymptotic stability of the equation (1) is sufficient to satisfy the inequality [3]

$$\|\vec{v}_i\| > \|f(\vec{v}_i)\|, \qquad (3)$$

for any $\vec{v}_i \in D$, $\vec{v}_i \neq \vec{0}$.

It is required to develop a numerical method for the stability analysis of the automatic control system over a channel "mixture ignition temperature - the temperature in the vacuum chamber 24", described by equation (1).

Method multiple equations

Let's introduce into consideration the difference equation of the system (1) of k_u multiplicity:

$$\vec{v}_{i+1} = \varphi_{k_u}(\vec{v}_i) = \underbrace{f(\ldots f(f}_{k_u}(\vec{v}_i))\ldots). \qquad (4)$$

In accordance with the method of contracting maps for the asymptotic stability of the equation (4) it is sufficient to satisfy the inequality [3]:

$$\|\vec{v}_i\| > \|\varphi_{k_u}(\vec{v}_i)\|, \qquad (5)$$

for any $\vec{v}_i \in D$, $\vec{v}_i \neq \vec{0}$.

Let's formulate a series of theorems concerning the entered multiple equation (4).

Theorem 1. The asymptotic stability (instability) in the region of difference equation (1) is equivalent to asymptotic stability (instability) in the region of equation (4).

Proof. Substitution of the difference equations (1) and (4) in the definition of stability (2) leads to the same result:

$$\left\| \underbrace{f\ldots(f(f}_{i \to \infty}(\vec{v}_0))\ldots) \right\| = 0, \qquad (6)$$

which proves the Theorem statement.

Thus, a sufficient condition for the stability of equation (1) is the satisfaction of the condition (5).

Theorem 2. For $k_u \to \infty$ condition (5) is a necessary and sufficient condition for asymptotic stability in the region D of the equation (1).

Proof. Implementation of the sufficiency follows from Theorem 1. Let's prove the necessity. Let the equation (1) is stable, i.e., for it (2) is fulfilled, from this expression there follows the relation $\lim_{k_u \to \infty} \|\varphi_{k_u}(\vec{v}_i)\| = 0$ Comparing the last relation with the condition (5) its implementation for $k_u \to \infty$ and any $x_i \in D$, $\vec{v}_i \neq \vec{0}$ can be concluded. Further, according to Theorem 1, the necessary and sufficient conditions of stability for the equation (4) coincide with a necessary and sufficient condition for the stability of equation (1), which proves the theorem.

Theorem 3. If the system (1) is stable it is always possible to choose such large k_u (the case of $k_u \to \infty$ is not excluded), at which a sufficient condition for stability (5) will be executed.

Proof. The stability of the equation (1) is equivalent to the ratio $\lim_{k \to \infty} \|\varphi_{k_u}(\vec{v}_i)\| = 0$. According to the definition of the function limit, it can be written [4]: for an arbitrary $\vec{v}_i \in D$, $\vec{v}_i \neq \vec{0}$ there exists such a $N > 0$, that for $k_u \geq N$ the inequality $\|\vec{v}_i\| > \|\varphi_{k_u}(\vec{v}_i)\|$ is satisfied, which coincides with the stability condition (5). which proves the Theorem statement.

From the above theorems the technique for studying the stability of the system described by equation (1) is follows. Equation of the multiplicity system (4), which investigated on stability by the method of contracting maps (verification of condition (5)) is constructed. The larger the multiplicity parameter of k_u, the closer the assessment to the actual stability region. Checking the condition (5) in an analytical form can be carried out only in rather trivial cases, in general, it is necessary to use numerical procedures.

Definition 1. The above-mentioned technique of investigating the stability of the closed control systems will be called as multiple equations method (MEM). Let's compare the method for multiple equations and simulation modeling by the required computational costs. For example, it is necessary to determine the asymptotic stability of the equation (1) in the region *D*. Since it is impossible to check the stability at all points in region of *D*, therein N_{check} points are selected (for example, located at the nodes of a uniform rectangular grid). Using these points as the initial, the numerical solution of difference equation (1) is performed. If in all checked points the system is asymptotically stable, then the conclusion about the stability in the region of D is made. For example, to determine the stability it is necessary to find a solution of equation (1) with duration of N_{sol} counts, then the total number of function $f(\cdot)$ evaluations, which required for the stability checking by simulation method will be as follows

$$N_{sim} = N_{check} \cdot N_{sol}. \qquad (7)$$

In determining the stability in the region *D* by the multiple equations method we will use the same N_{check} initial points in which the condition (5) is checked. The total number of $f(\cdot)$ function evaluations, which required for the stability checking, in this case will be the following

$$N_{MEM} = N_{check} \cdot k_u , \qquad (8)$$

where k_u, as before, the multiplicity of the equation (4).

Comparing the formulas (7) and (8) it can be concluded that the amount of calculations required in the application of the multiple equations method compared to simulation modeling is less in N_{sol} / k_u times. Here is an explanatory example.

An example of the multiple equations method

The present section provides the examples of the stability analysis of the local automatic control system over a channel "mixture ignition temperature - the temperature in the vacuum chamber 24" using multiple equations method.

The conducted statistical studies of the control object over a channel "mixture ignition temperature - the temperature in the vacuum chamber 24" [2] made it possible to develop a functional block diagram of the fuzzy ACS of surface temperature of the mixture under the ignition furnace [2] and its block diagram (Fig. 1).

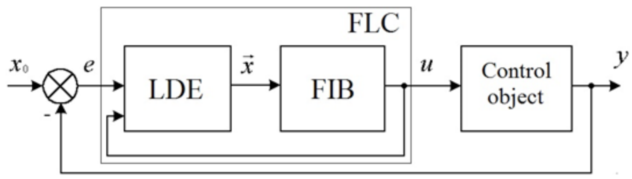

Figure 1. The block diagram of the fuzzy ACS over a channel "mixture ignition temperature - the temperature in the vacuum chamber 24"

In Fig. 1 the following designations are accepted: M_1 and M_2 – Pulse-amplitude modulators with zero-order hold, which working synchronously with the sampling period of T_0. Dynamic element is described by the transfer function

$$W(p) = -\frac{k_0}{1 + p \cdot T} .$$

Since the flow rate of natural and blast gases mixture with calorific properties of Q = 2800÷ 3000 Kcal/Nm³ under the excess air coefficient of 1,2÷1,4Qg=700 m³/hour provides the ignition temperature of the mixture of 1100°C, which for accordance of production process conducting conditions to the Technological instruction TI 228-AP-56-2003 allows to complete the process of sintering over the 24-th vacuum chamber [2] (the indirect indicator is the maximum temperature of 220°C in 24-th vacuum chamber) as a reference signal use the value of 220°C. The fuzzy inference block (FIB) is described by a set of fuzzy rules:

Π_1: if x^1 is *P* and x^2 –arbitrary, then $u = 1$,
Π_2: if x^1 is *N* and x^2 – arbitrary, then $u = -1$,
Π_3: if x^1 – arbitrary and x^2 is *P*, then $u = 1$,
Π_4: if x^1 – arbitrary and x^2 is *N*, then $u = -1$,
Π_5: if x^1 is *Z* and x^2 is *Z*, then $u = 0$,

where x^1 and x^2 – 1-st and 2-nd components of vector $\vec{x}$ respectively.

The membership functions of fuzzy sets Z, P and N are shown in Fig. 2. Sugeno fuzzy inference algorithm of 0-th order with the aggregation of input variables *min* is used [4].

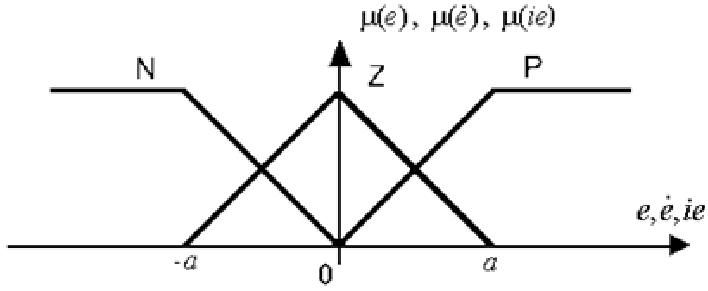

Figure 2. The functions of fuzzy variables N, Z, and P

It is necessary to determine a limit (maximum) parameter k_0 value at which the system remains stable.

Equation (1) in this case takes the form:

$$\vec{v}_{i+1} = \begin{bmatrix} e^{-\frac{T_0}{T}} & 0 \\ -k_1 & 0 \end{bmatrix} \cdot \vec{x}_n + \begin{bmatrix} (1 - e^{-\frac{T_0}{T}}) \cdot k_0 \\ 0 \end{bmatrix} \cdot y([-1, 0] \cdot \vec{v}_i, [0, 1] \cdot \vec{v}_i) ,$$

where $$y(x^1, x^2) = \frac{-\mu_N(x^1) + \mu_P(x^1) - \mu_N(x^2) + \mu_P(x^2)}{\mu_N(x^1) + \mu_P(x^1) + \mu_N(x^2) + \mu_P(x^2) + \min(\mu_Z(x^1), \mu_Z(x^2))}$$

is the function realized by fuzzy logic controller (FLC).

Let's choose the region $v_1 \in [-1, 1]$, $v_2 \in [-1, 1]$ as the region *D*. Let's arrange the check points in the nodes of a uniform rectangular grid 100×100 ($N_{check} = 10000$). The number of $f(\cdot)$ function evaluations, which required for solution obtaining, choose of $N_{sol} = 1000$. Results of computational experiment are given in Table 1.

Table 1. Results of computational experiment

	Simulation modeling	The multiple equations method		
		$k_u = 1$	$k_u = 2$	$k_u = 4$
The number of calculations $f(\cdot)$	10^7	10^4	$2 \cdot 10^4$	$4 \cdot 10^4$
The obtained limit value k_0	20.5	7.5	15.5	18.5
Relation to the true value	100 %	36.6 %	75.6 %	90.2 %

Table 1 shows that with a multiplicity of $k_u = 4$ it is possible to determine more than 90% of the true stability region, while the amount of calculations compared with the use of simulation modeling for this is 250 times smaller.

Statistical variant of the stability analysis method.

The choice of checkpoints in the regular grid nodes is above considered. Often, to reduce the amount of check points N_{check} is possible by placing them in the region D randomly. To determine the required number of check points is possible using the technique described in the monograph [5].

Let's put forward a suggestion that at an arbitrary point of region *D* the inequality (5) is satisfied which, according to proven theorems is equivalent to the asymptotic stability of the system in the region *D*. For example, experience is conducted, which consists in checking of inequality(5) in a random point of region *D* selected by a uniform law. Let's consider the event *A*, which consists in the fact that (5) is not satisfied as a result of the above described experiment. Let's formulate the null hypothesis H_0: the probability of event *A* is equal to 0, i.e. $P(A)=0$.

Let, as a result of the experiments the event *B* was observed, which consists in the fact that verifiable assumption was satisfied i.e. the event *A* was never observed.

Let's suppose further that the probability of event *A* is equal to *p*, it is required to find the maximum value of probability of an event *A p1*, which, together with the results of experiments with a given confidence level of β. The probability of an event *B* occurrence in N_{check} experiments is defined by the formula [5]:

$$P_N(B) = (1-p)^{N_{check}}, \qquad (9)$$

by equating $P_N(B) = 1 - \beta$ and $p = p1$ we get

$$p1 = 1 - (1-\beta)^{1/N_{check}}. \qquad (10)$$

From whence, in order that the system was stable in the region *D* with confidence level of β with error probability less than $p1$, it is sufficient

$$N_{check} = \frac{\ln(1-\beta)}{\ln(1-p1)}, \qquad (11)$$

Once to ensure in the inequality (5) fulfillment in random points of region D.

Example 2: Let's assume that the confidence level $\beta = 0.9$, error probability less than pl =0.01, Then, according to formula (11) it is necessary to conduct $N_{check} = 459$ checks of inequality (5) in random points of region D.

Accuracy estimation of the multiple equations method

In applying the multiple equations method it is important to choose the right value of k_u. For a small k_u it is possible to define only a small part of the stability region in the space of its parameters. However, with increasing the value of k_u the amount of required computations also increases.

Let's consider the empirical method of the parameter k_u choosing in the multiple equations method, proceeding from the specified accuracy.

Let's consider the parameter θ, which characterizes the region of the system stability. As such parameter the hypervolume of the stability region in the parameter space can be selected, for example (in the one-dimensional case θ – is the limit value of the investigated system, in which it remains stable).

Let's suppose that some positive integer constants k and m are selected, and for $k_u = k$ the value of θ_k is obtained, for $k_u = m \cdot k - \theta_{m\cdot k}$, and for $k_u = m^2 \cdot k - \theta_{m^2\cdot k}$.

According to Theorem 2 the ratio $\lim_{k\to\infty} \theta_k = \theta$ is carried out.

The absolute error is determined by the formula: $\Delta\theta_k = \theta - \theta_k$.

Let's suppose (empirically) that the absolute error is associated with a parameter of k_u by following ratio:

$$\hat{\Delta\theta}_{k_u} = A k_u^{\,p}, \qquad (12)$$

where A, p – are the constant parameters.

In this case, the parameter of p can be estimated by the Aitken method [7]:

$$p = \ln\left[\frac{\theta_{mk} - \theta_{m^2k}}{\theta_k - \theta_{mk}}\right] / \ln(m). \qquad (13)$$

Using the first formula of Runge the absolute error of θ_{m^2k} parameter can be estimated [6, 7]:

$$\hat{\Delta\theta}_{m^2k} = \frac{\theta_{m^2k} - \theta_{mk}}{m^{-p} - 1}. \qquad (14)$$

Solving (13) and (14) simultaneously we obtain:

$$\hat{\Delta\theta}_{m^2\cdot k} = \frac{-(\theta_{m^2\cdot k} - \theta_{m\cdot k})^2}{\theta_{m^2\cdot k} - 2\theta_{m\cdot k} + \theta_k}. \qquad (15)$$

Assuming that $p < -1$, it is possible to get a simple, but more rough estimate of the absolute error:

$$\hat{\Delta\theta}_{m^2k} = \theta_{m^2k} - \theta_{mk}. \qquad (16)$$

Example 3. Let's consider the results of the multiple equations method application presented in Example 1 (Table 1).

Let's choose $m = 2$, $k = 1$. The calculation by (13), (14) gives $p = -1{,}415$, $\hat{\Delta\theta}_{m^2k} = 1{,}8$.

Estimation of relative error of the stability region determination:

$$\hat{\delta}_{m^2k} = \frac{\Delta\theta_{m^2k}}{\theta_{m^2k}} 100\% = 9{,}730\%.$$

The actual relative error is:

$$\delta_{m^2k} = \frac{\theta - \theta_{m^2k}}{\theta} 100\% = 9{,}756\%.$$

Coincidence of error estimation and its true value, appear to can be recognized satisfactorily. More rough estimate of parameter θ by (16) gives

$$\Delta\theta_{m^2k} = 3.$$

Estimation of the relative error in this case:

$$\hat{\delta}_{m^2k} = \frac{\Delta\theta_{m^2k}}{\theta_{m^2k}} 100\% = 16{,}216\%$$

and can also be found to be satisfactory.

Conclusion

The multiple equations method proposed in the article can be widely used in the stability computer analysis of the fuzzy-logic control systems of complex technological processes.

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