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Influence of operational mode loading on fatigue life with cyclic unstable inelastic materials

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Abstract
The analytical method for determination of material fatigue lifetime under the cyclic loading with random stress amplitude operational process using the model of ultimate plasticity exhaustion is presented. Feature of the given method is that random process of loading is not schematized by block one using known schemes, but real loading is used. An attempt to consider damage under very high fatigue random loading is undertaken.

Introduction.

For many machines and structures the main type of operational loading is random one. Depending on the type and purpose of the product loading can be a heavy, medium or light force impact mode on its elements, and can be described by one of the known functions of loading amplitudes probability accumulation. Accumulated experimental data on the loading of certain types of products allow to define both the form of such function and the value of its parameters. The subsequent steps in the evaluation of the elements fatigue life working in such loading conditions, are to determine the stress-strain state in the element, the choice of an appropriate hypothesis of fatigue damage summation, fatigue characterizations of the material used. Despite the recommendations on the summation hypotheses choice, this step in the sequence of the elements fatigue life evaluation is one of the most uncertain [1-3].

In this connection, the basic provisions of the limit plasticity exhaustion model proposed earlier by the authors [4-6], is applied to determine the durability at random loading. It does not require acceptance of the hypothesis of fatigue damages summation, as the degradation (damage) of the material is considered as a continuous hardening / softening before the exhaustion of a material plasticity resource, causing the limit state - destruction.

Modeling of random cyclic process

To simulate the random process (description of the changes the stress amplitude in the cycle) will use the normal law of distribution modernized using the Heaviside with the condition that the random amplitude loading is not beyond the range defined by scope of the proposed model reflecting the conditions that the stress amplitude value is within the predetermined area:

$$f\left(\sigma_a, M_\sigma, D_\sigma\right) = \frac{C}{\sqrt{2\pi \cdot D_\sigma}} \cdot e^{-\frac{(\sigma_a - M_\sigma)^2}{2 \cdot D_\sigma}} \cdot H_1\left(\sigma_a - \sigma_{a,d}\right) \cdot H_1\left(\sigma_{a,u} - \sigma_a\right) \tag{1}$$

$$\sigma_{a,d} \le \sigma_a \le \sigma_{a,u} \text{ or } P(\sigma_a \le \sigma_{a,d} \cup \sigma_{a,u} \le \sigma_a) = 0,$$

where $f(\sigma)$ – the probability density of normal law of distribution; M_σ and D_σ – the expectation and dispersion of a discrete random value σ; $P(\sigma_1 \le \sigma \le \sigma_2)$ – the probability that an independent value σ is in the range from σ_1 to σ_2; C – coefficient of the probability distribution interval that depends on the selected range (determined by the condition $P(\sigma_{a,d} \le \sigma_a \le \sigma_{a,u})=1$); $\sigma_{a,u}$, $\sigma_{a,d}$ – upper and lower values of stress amplitude, respectively; $H_r(\sigma)$ – Heaviside unit function expressed as:

$$H_r\left(x\right) = \left\{0, x < 0; \quad r, x = 0; \quad 1, x > 0\right\}. \tag{2}$$

where r – parameter specifying the function value at the discontinuity point of the first kind.

Using equations (1) - (2) in Fig. 1 the probability densities for the upgraded normal law of distribution are built and the modeling of random process of stress amplitude values change (Fig. 2a) and sinusoidal multicycle loading signal restored by it (2b) are demonstrated.

Determination of the material durability at random loading

Before finding the durability at random loading we define optional versions of random loading variation ranges combinations (load region) and of areas in which the material fatigue damage accumulation (damaged area) may be considered by the model. The value of stress in modeled random process can vary from 0 до σ_K. Fatigue damages will not be accumulated at values of the random stress amplitude below σ_{nd} (σ_{nd} – gigacycle fatigue limit). The scope of model (damage area) at multicycle fatigue is a stress range from σ_{-1} to σ_K, and at gigacycle one - from σ_{nd} to σ_{-1}.

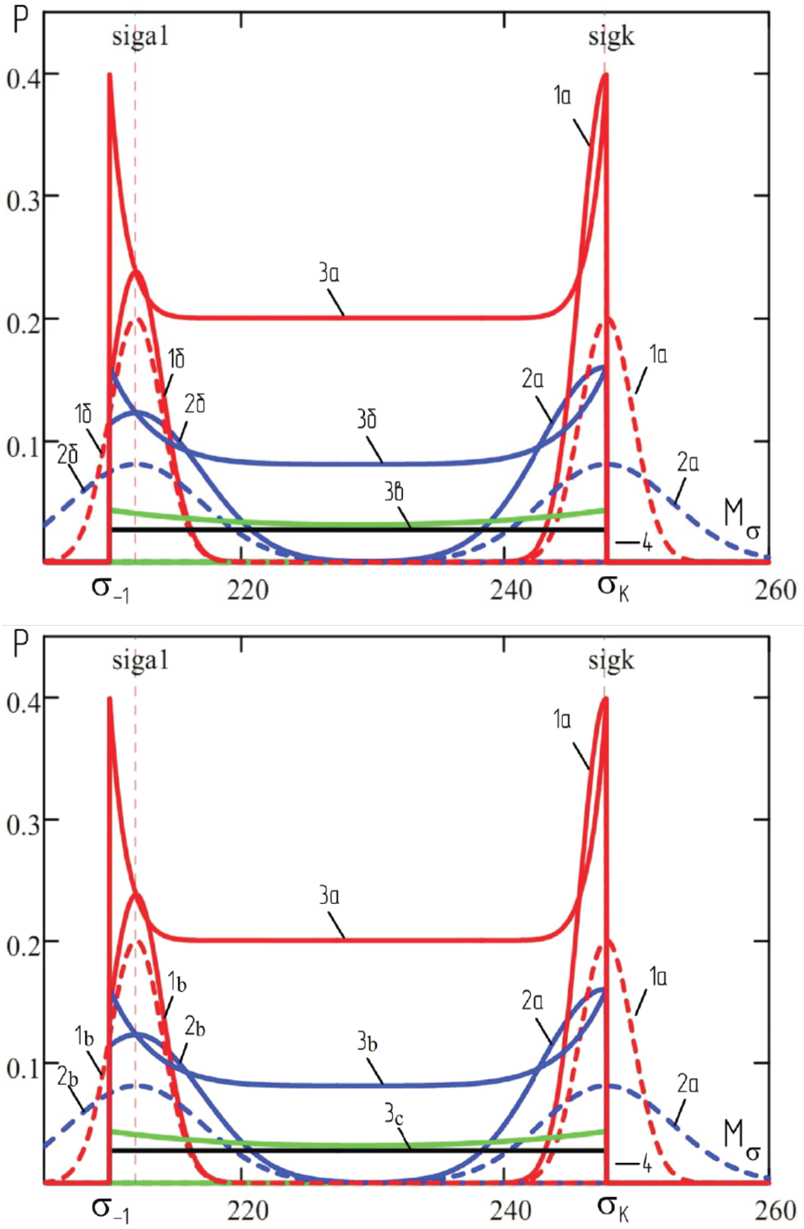

Figure 1 - The image of probability densities depending on σ_a at a random process modeling by the normal law of distribution with parameters $\sigma_{a,u} = \sigma_K$, $\sigma_{a,d} = \sigma_{-1}$ (area is marked by vertical lines). 1a, 2a and 2b, 2b - the probability densities of the normal law of distribution for M_σ = 250 and 212 MPa, at D_σ equal respectively to 2 and 5. Dashed lines - canonical normal law of distribution, solid lines - modernized one (1). 3a, 3b, 3c - changing of coefficient C (1) at D_σ equal respectively to 2, 5, 20 (not to scale). 4 - probability density at a uniform law of distribution.

Table 1 - Possible options of stresses $\sigma_{a,nd}$ and $\sigma_{a,d}$.

Option №	I	II	III	IV
Damage ranges, $\sigma_{a,nd}$	σ_{-1}	σ_{-1}	$\sigma_{-1}/2$	$\sigma_{-1}/2$
Loading ranges, $\sigma_{a,d}$	σ_{-1}	$\sigma_{-1}/2$	$\sigma_{-1}/2$	0

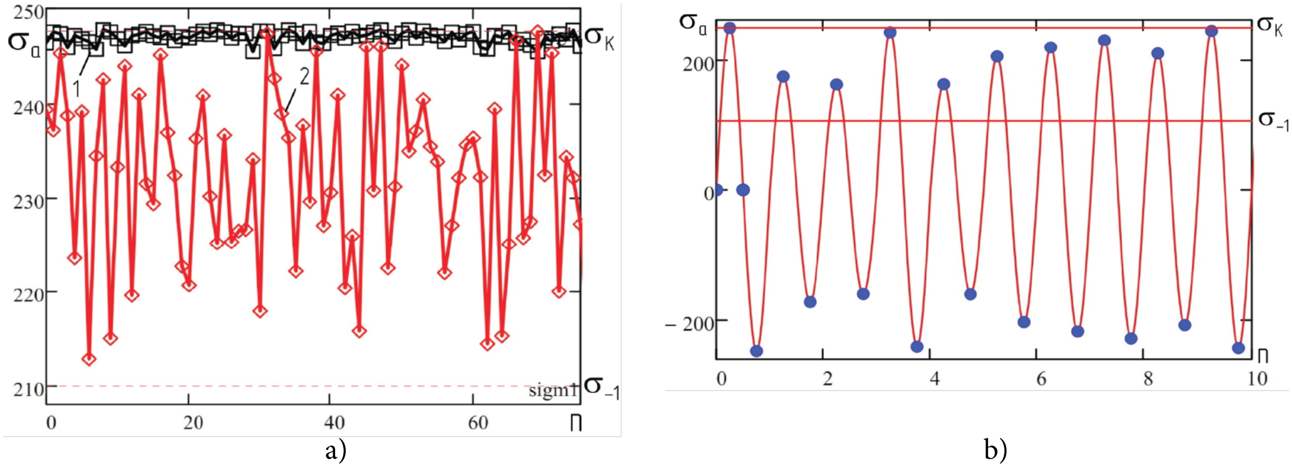

Figure 2 - Modeling of process of a random stresses amplitude change (a) and a random sinusoidal multicycle loading (b), from the number of loading cycles, at M_σ = 250MPa with parameters $\sigma_{a,u} = \sigma_K$, $\sigma_{a,d} = \sigma_{-1}$ for a and $\sigma_{a,u} = \sigma_K$, $\sigma_{a,d} = \sigma_{nd}$ fo b. Curves 1 and 2 in Fig. 2a, and the curve in Fig. 2b are obtained at D_σ equal to 1, 25 and 50 respectively. Determination areas are marked by horizontal lines.

Changing the damaged area by change of lower range of model definition $\sigma_{a,nd}$ (can take values σ_{nd} or σ_{-1}) and varying the lower range of loading area $\sigma_{a,d}$ (can take values 0, σ_{nd} or σ_{-1}), the following options of loading and damage areas combinations are possible, which lead to different kinetics of damages accumulation (Table. 1).

For values of the critical $\sigma_{a,cr}$ and upper $\sigma_{a,d}$ stress amplitudes for all cases of Table 1 take the critical stress according V.S. Ivanova σ_K [5].

Using the basic equation of limit plasticity exhaustion model (15), as well as some equations given above, the fatigue curves for the cases presented in Table 1 (Fig. 3 shows the case III) can be constructed. At a random loading mode fatigue curves are constructed taking into account the condition $\sigma_a = M_\sigma$. For calculation, the following parameters variation limits of the normal law of distribution are used: $M_\sigma = \sigma_{a,nd} \ldots \sigma_K$ in increments of 1 and D_σ = 1 ... 200 in increments of 5.

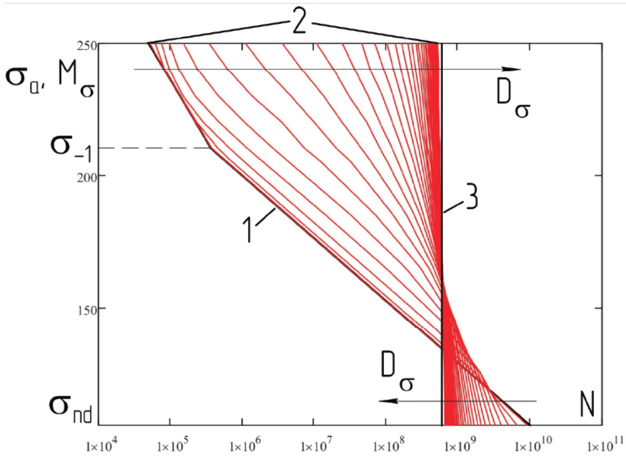

Figure 3 - Comparison of fatigue curves (multicycle experimental, gigacycle hypothetical) with the calculated curves obtained at a random loading mode: 1 - experimental fatigue curve; 2 - a family of fatigue curves for the random loading mode, described by equation (1) (the D_σ increase is shown by arrow); 3 - fatigue curve at a random loading with uniform distribution.

As can be seen from Fig. 3, at high dispersion (D_σ> 100) the longevities value of with a a random normal law of distribution described by equation (1) are almost indistinguishable and aspire to durability, defined by the uniform law of distribution.

For the analysis of fatigue curves families obtained on the options of Table 1, using the limit plasticity exhaustion model, we introduce the coefficient of fatigue damages summation:

$$A\left(M_\sigma, D_\sigma\right)=\sum_{i=1}^{N\left(M_\sigma, D_\sigma\right)} \frac{1}{N\left(M_{\sigma,i}\right)}, \quad (3)$$

where $N(M_\sigma, D_\sigma)$ - durability at a random loading with parameters M_σ and D_σ; $N(M_{\sigma,i})$ - the durability, determined by the fatigue curve.

Compare the coefficients $A(M_\sigma,D_\sigma)$, obtained by (3), with the coefficients obtained by the linear hypothesis of damages summation - $A_L(M_\sigma,D_\sigma)$. From Figure 1, and according to (1) the summation coefficient by linear hypothesis exists as:

$$A_L\left(M_\sigma, D_\sigma\right)=\left(\int_{\sigma_{a,nd}}^{\sigma_u} f\left(\sigma_a, M_\sigma, D_\sigma\right) d\sigma_a\right)^{-1}. \quad (4)$$

Using equations (3) and (4) present the results of fatigue curves calculation for the 45 steel, in the form convenient for comparison (Fig. 4).

For options I and III, the linear hypothesis of damages summation gives a result equal to 1. This is due to the fact that the areas of loading and damage are the same, which means that each cycle is counted at damages summation. For variants I and III the summation coefficient defined by the proposed model ranges from 0.8 to 1, which indicates the impossibility of applying the linear summation hypothesis. For options II and IV the summation coefficients, calculated according to (3) and (4) change from 0.8 to 2.9 according to the proposed model, and from 1 to 3.6 by the linear hypothesis. As can be seen, the damages summation linear hypothesis gives higher upper and lower values of summation coefficients, compared with the modal one. In calculating from formulas (3) and (4) the cycles without causing damages are accounted, and this is due to the value of summation coefficients greater than 1 (without accounting of which the result would be less than 1). It is evident that the proposed summation model non-linearly takes into account the fatigue damages and gives a more conservative estimation compared to the linear hypothesis.

From Fig. 4 it follows that during the change of stress amplitude dispersion from 0 to 50 the summation coefficient varies over a wide range, but at the dispersion approach to 200, the summation coefficient A and AL , for each case of Table 1 tends to its asymptotic value. The extremes of summation coefficient are observed in the area of loading amplitude dispersions change from 25 to 50. Starting from the Fig. 4 analysis the summation coefficients extremes A and AL were analyzed for the characteristic points of the fatigue curve, for the model and linear damages summation hypothesis of (datum are presented in Table 2).

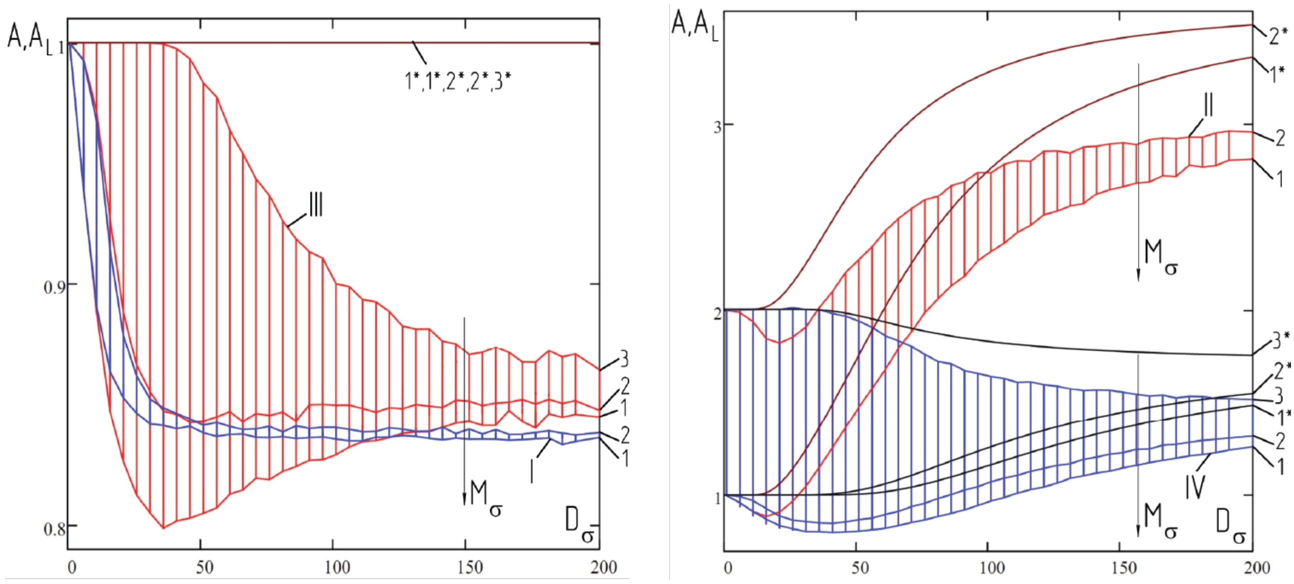

Figure 4 - Changing the summation coefficients A and AL on the stress amplitude dispersion: a - cases I, III, b - cases II, IV frpm Table 1. Numbering of I, II, III, IV areas with vertical hatching corresponds to the range of summation coefficient variation A for cases of Table 3; 1, 2, 3 - curves constructed for Mσ, equal respectively to σ_K, σ_{-1}, σ_{nd}. The figures marked with an asterisk correspond to the calculated data, counted by the linear hypothesis of damages summation. The arrow indicates the increase of the expectation Mσ (amplitude loading).

Table 2 - Comparison of the values of summation coefficient extrema A and AL for the characteristic points of fatigue curve.

σ	σ_{-1}				σ_K			
$A(D_\sigma)$	I	II	III	IV	I	II	III	IV
$A(0)$	$\frac{1}{1}$	$\frac{2}{2}$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$

$A(10)$	0.84 / 1	2.95 / 3.52	0.86 / 1	1.32 / 1.54	0.82 / 1	2.79 / 3.35	0.85 / 1	1.26 / 1.48
$A_E(D_\sigma)$	0.86(25) / 1	1.8(21) / 2(15)	0.84(40) / 1	0.85(40) / 1(40)	0.83(25) / 1	0.85(21) / 1(15)	0.79(40) / 1	0.8(40) / 1(40)

Figures correspond to the numbering of Table 1. Figures in brackets - dispersion value for the summation coefficient. Above the line - calculation on model, below the line - by linear hypothesis of damages summation.

As can be seen from Table 2, the durability evaluation by linear damage summation hypothesis is more inflated (by 20%) compared with the estimation by model of ductility exhaustion.

Conclusions

An attempt was made to solve the problem of durability calculation and limit state under random symmetric sinusoidal loading by the previously developed model of limit plasticity exhaustion for the multicycle fatigue area. The model is extended to gigacycle fatigue and applied to find the durability, fatigue damage and construction of fatigue curves families at a random loading according to the normal law for various M_σ and D_σ. A feature of the proposed method is that the loading is not schematized by block by the known patterns, and the real kinetics of loading is used. An important aspect of the model is that the data to determine its parameters are known mechanical characteristics and fatigue curves for smooth samples.

The article examined in detail the analytical method of random loading modeling, corresponding to the specified parameters, and 4 most possible from the point of view of authors of the damage and real loading account areas combinations are analyzed. The durability evaluation at a random loading of the proposed model ismore conservative compared to the linear hypothesis of damages summation, as it accounts the non-linear summation. Using the model shows a significant effect of the dispersion of the normal random loading process on the total accumulated fatigue damage and thus fatigue life.

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