# Optimal Strategy for the Two-way Cooperation in International Logistics Based on Variational Method

**Wen Song**[1, 2]

[1]*Economics and Management School, Wuhan University, Wuhan, Hubei, 430072, China*
[2]*Guangxi Vocational and Technical College of Communications, Nanning, Guangxi, 530023, China*

Abstract
This paper focuses on the sustainable development of international trade and international logistics, and considers the construction of the two-way cooperation system in the international logistics based on the factors of the variational method. On the basis of the logistics perspective, the paper takes into account the customer value exploration, and abstracts its essence into the procedure of "logistics cooperation". For this international logistics dynamic coordination issue, on the basis of the long-term cooperative relationship between member companies in the international logistics, this paper adopts the variational method in the continuous time background to establish the differential game model for all the international logistics parties on the two-way cooperation system of the international trade and international logistics, by solving the model, it finds that when the quasi constraint equations have solutions, Nash equilibrium exists, and thus the corresponding optimal differential game can be obtained. These theoretical findings have clarified the significance of the long-term and stable relations of cooperation for the realization of the international logistics coordination, which has certain inspiration and reference value for the implementation of the dynamic coordination of the international logistics in China. Finally, the paper makes numerical analysis on the relevant conclusion.
Key words: INTERNATIONAL LOGISTICS, DYNAMIC GAME, VARIATIONAL METHOD, COOPERATIVE SYSTEM.

## 1. Introduction

International logistics management and coordination is an important management idea and method for modern enterprises to achieve the development in an increasingly competitive living environment. The characteristics of the independence and mutual interdependency in the interests and decision-making among the member companies in the logistics result in the gap of the expected efficacy between the centralized and the decentralized decision-making model in the international logistics. To solve this imbalance of individual and collective rationality, currently the countermeasures are mainly from the perspective of incentive and coordination [1]. On the other hand, due to the features of personalization and diversification in customer demand, the product life cycle is dramatically shortened, more and more products are showing the features of strong seasonality and short sales cycle. Therefore, the study based on Newsvendor model for the coordination issue of short sales cycle of the international logistics has become an important field of research in the current international logistics management [2]. Due to the features of the products, it is natural for people to focus on considering the one-time transaction among members of the international logistics --- or game - relationship, and obtain a lot of relevant research results, including the design for the international logistics coordination strategy [3-5], information asymmetry measures [6-7], the horizontal competition model of supplier or retailer [8-9], the influence of the policymaker's risk preference [10-11], and so on.

However, in fact, the cooperation relationship between international logistics companies in the short sales cycle often shows the long-term and dynamic characteristics. Time is undoubtedly an important factor in the cooperation and game relations between the international logistics members, and it even can be said that the entire international logistics cooperation process is full of game in the continuous time background [12]. However, due to the fact that it is very complicated and difficult to describe the influence of the continuous time on the decision-making of the international logistics member businesses, it is hard to build the decision-making model. Therefore, currently there is lack of relatively in-depth research on the international logistics coordination issue based on the continuous time and dynamic game. However, in the practice of international logistics, the same or even more important is the content of the long-term game throughout the entire cooperation period, such as joint forecast, joint investment, advertising, as well as cooperation and other aspects.

As an initial attempt of research on this important field of study, this paper investigates an important issue in the international logistics dynamic coordination and game based on the analysis of dynamic coordination of international logistics and its analysis methods and tools: The differential game of two-way cooperation system of the international trade and international logistics based on continuous time.

**2. Dynamic, Variational and Differential Game: New Elements of International Logistics Coordination**

In the common international logistics decision optimization issues, the general expression of the decision-making issue of the decision-makers is as follows:

$$\max_{x\in R} y = f(x), \; s.t. x \in D \tag{1}$$

The solution to this decision problem is the optimal value $x = x^*$ of the decision variable x, where $x^*$ is a real number.

While for the dynamic optimization decision-making issues with the time context, the decision making function of the decision-makers will be changed into:

$$\max_{x(t)} y = f(x(t)), \; x(t) = x^*(t) \tag{2}$$

Its optimal solution is $x(t) = x^*(t)$, where $x^*$ is a function

Therefore, the issue of decision-making is essentially the optimization issue for functional y, when the solutions and algorithms for the original optimization issues are not applicable, this paper will introduce the variational method in the discussion of the differential game for the international trade and international logistics two-way cooperation system in the following sections.

In particular, for the game issue of the dynamic coordination of the international logistics in continuous time, as the decision-makers (game participants) cannot guarantee that the optimal decision at one moment will still be the optimal in the time that follows up, therefore, the international logistics game participant strategy (decision variables) should also be a function of the time, in other words, at this time the game between the members of the international logistics become a differential game, or the differential countermeasure.

Differential game stems from the research on the military confrontation strategy of two sides, which is also applied in the management area currently, including the field of investment, duopoly competition issues, etc. [22~25]. There are a few researches carried out by domestic scholars to introduce the differential game into the international logistics advertising strategy [26~28], However, due to the complexity of differential game issue itself, it can only obtain the analytic constant solution for the decision variables (functions) independent of the time, which in turn does not reflect the association with the time, and loses the essential value to analyze the differential game issue, hence with certain deficiencies.

**3. The Two-way Cooperation System of the International Trade and International Logistics: Rethinking the International Logistics Coordination**

In the one-off international logistics trading relations, to explore how to realize the international logistics maximum expected revenue (benchmarking) in the centralized decision-making model in the actual decentralized decision-making model, that is, based on the improvement of Pareto, the international logistics expected revenue in the maximum decentralized decision-making model. Zhang Jianjun et al [21] has proposed the long-term cooperation relationship between the international logistics member companies, which also reveals the limitation of the one-time international logistics partnerships. At the same time, it is necessary to further emphasize that not only the connection between the members of the international logistics businesses are long-term. The connection between the international logistics and its market and customers (or partnership) is also long term. This means that it is very necessary to include the management that customer requires into the research field of international logistics management, to combine the exploration of customer demand and customer value and the optimization of the international logistics member businesses decision-making, i.e. greater of (or even maximization) the original maximum international logistics expected revenue.

However, it is worth noting that: In the current study and model of thinking, the demand of final customers is considered to be the objectively established background --- though it may be a function of the sales price, it can also be a function of the amount of stock, it can even be unknown --- in the field of international logistics management, international logistics coordination, from the perspective of international logistics to discuss the exploration of customer value, there are relatively few relevant research. While in the field of business management, especially in the field of marketing, the development of customer value is one of the key issues that people attach importance to. Similarly, the focus on customer value is also a core issue in the demand management, and research on corporate revenue management has achieved fruitful results. In so many studies, the positive impact of good cooperation of enterprises on consumer demand has been revealed. The British famous international logistics and international logistics management specialist John Christopher pointed out that: The competition among enterprises has been replaced by the competition among international logistics. In this context, the cooperation in international logistics is an important factor in the international logistics competition.

However, in the environment of international logistics, similarly, as the members of the international logistics businesses have independent interests, the customer value (demand) management (including cooperation maintaining issue) also encounters a big problem: Every business can perform its own demand management, but how the demand management of enterprises have interaction with each other (game) so as to improve the Pareto and thus achieve the optimal benefit?

In terms of international logistics cooperation issue, it is very important to cooperate with the most downstream businesses which are in direct contact with customers and have a great impact on the final consumers --- however, customers' cooperation judgment and evaluation results on their direct contact enterprises (usually the most downstream international logistics enterprises) are not determined independently and completely by the own action of the business itself, it is also influenced by the efforts of all enterprises in the entire international logistics, such as on-time delivery, product (raw materials, spare parts) quality, advertising, etc. and all kinds of cooperation relations of the member enterprises in the international logistics, which means: From the perspective of the entire system of the international logistics rather than an individual business, to include the cooperation and its relationship with customer needs into the research of the international logistics coordination and game field as a direction with relatively important value. For this reason, this paper will attempt to analyze the game of international logistics parties to maintain and enhance the cooperation based on the concept of "Two-way cooperation system for international trade and international logistics". The so-called "two-way cooperation system of the international trade and international logistics" is clearly defined in this paper as: The degree of customer's trust for the entire international logistics.

It is noted that the two-way cooperation system of the international trade and international logistics is dependent on the effort and the cumulative time variable of the individual member of the international logistics enterprises. Therefore, the game between the member companies of international logistics has become a dynamic game with uninterrupted time, or differential game.

**4. Assumptions of the Model and General Description**

Investigate the two-stage international logistics constituted by single manufacturer and single retailer, with long-term cooperation relationships:

First, explain the meaning of the related symbols:

(1). t: Time variable;

(2). $C_M(t)/C_R(t)$: Manufacturer / retailer's level of efforts in maintaining the two-way cooperation system of international trade and international logistics;

(3). $C_M(t)/C_R(t)$: Manufacturer / retailer's maintenance costs for the two-way cooperation system of international trade and international logistics;

(4). $\pi_{SC}(t)$ The output of international logistics at time t;

(5). $\prod_M(t)/\prod_R(t)$: Manufacturer / retailer's earnings at time t;

(6). $T\prod_M(t)/T\prod_R(t)$: Manufacturer / retailer's benefits from the entire cooperation time;

(7). $\Theta = \Theta(t)$: The two-way cooperation system of international trade and international logistics.

Assuming:

(1). According to the research in the marketing field, as the cooperation and maintenance costs are an increasing function of the level of effort, which shows the characteristics of strictly convex, assuming both can meet

$C_M(t)=\frac{\mu_M(t)}{2}e_M^2(t)$, $C_R(t)=\frac{\mu_R(t)}{2}e_M^2(t)$, where

$\mu_R(t)$, $\mu_R(t)$ influence coefficient.

Similarly, the higher the maintenance level of both international logistics cooperation parties is, the higher the two-way cooperation system of the international trade and international logistics will be, and showing strict characteristics of convex, it can be assumed that the two-way cooperation system of international trade and international logistics differential equation that meets the state change (according to Nerlove-Arrow model [29]) is as follows:

$$\begin{cases}\Theta'(t)=\rho_M(t)e_M(t)+\rho_R(t)e_R(t)-\xi(t)\Theta(t)\\ \Theta(t)\big|_{t-0}=\theta_o\geq 0\end{cases} \tag{3}$$

In which, $\theta_o$ is the initial value of the two-way cooperation system of international trade and international logistics: and $\rho_M(t)>0$、$\rho_R(t)>0$ are the influential factors of the maintenance level of cooperation of manufacturers and retailers to the two-way cooperation system of international trade and international logistics respectively; $\xi(t)>0$ represents the rate of decay of the two-way cooperation system of international trade and international logistics due to external factors.

(2). Within an infinite length of time of cooperation, the two parties of the international logistics have the same discount factor at any moment, denoted by $\delta$;

(3). Assuming the distribution ratio of income determined by the strength of the two parties between manufacturers and retailers is $\omega(t)$[6], namely:

$$\Pi_M(t)=\omega(t)\big(\pi_{SC}(t)-C_R(t)\big),\quad \Pi_R(t)=\big(1-\omega(t)\big)\big(\pi_{SC}(t)-C_M(t)-C_R(t)\big) \tag{4}$$

.

$T\Pi_M\big(e_M(t),\Theta(t),t\big),T\Pi_R\big(e_M(t),\Theta(t),t\big)\in C^2(0,+\infty)$

(5). Taking into account the monotonic increase of the income on the cooperation, assuming at any time t, the international logistics total revenue $\pi_{SC}(t)$ with cooperation $\Theta(t)$ meets: $\pi_{SC}(t)=\varphi(t)\Theta^2(t)$, in which $\varphi(t)$ is the influential factor to the international logistics total revenue of the two-way cooperation system for the international trade and international logistics.

**5. The Differential Game Model for the Two-way Cooperation System of the International Trade and International Logistics**

**5.1 Decisions of the Two Parties in the International Logistics**

From the above assumptions, it can be seen that:

All the expected revenue of manufacturers is:

$$T\Pi_M\big(e_M(t)\big)=\int_0^{+\infty}\omega(t)\big(\pi_{SC}(t)-C_M(t)-C_R(t)\big)e^{-\delta}dt$$

Thus the manufacturer's decision issue is:

$$\max_{eM(t)} T\Pi_M\big(e_M(t)\big)=\int_0^{+\infty}\omega(t)\big(\pi_{SC}(t)-C_M(t)-C_R(t)\big)e^{-\delta}dt \tag{5}$$

Prove:

Firstly, consider the manufacturer's decision:

By the principle of optimality:

If $e_M^*(s)(t\leq s)$ is the two-way cooperation system of the international trade and international logistics for manufacturer from time t

Noted that:

For $\Delta t\to 0$ ,there is:

And: $\Theta(t+\Delta t)\approx\Theta(t)+\Theta'(t)\Delta t$

$$\Rightarrow T\Pi_M^*\big(\Theta(T+\Delta t),T+\Delta t\big)\approx T\Pi_M^*\big(\Theta(t)+\Theta'(t)\Delta t,\mathrm{T}+\Delta t\big)$$

Similarly, the decision issue of retailers is:

$$\max_{R^{\omega}} T\Pi_R\big(e_R(t)\big)=\int_0^{+\infty}\big(1-\omega(t)\big)\big(\pi_{SC}(t)-C_M(t)-C_R(t)\big)e^{-\delta}dt \tag{7}$$

Taking into account the difficulty of solving the analytical solutions to the above-mentioned differential game when the environment variables $\mu_M(t),\mu_R(t),\omega(t),\varphi(t)$ etc. change with the time, in order to simplify the problem, assuming that each environment variable is constant as follows:

$$.\mu_M(t)=\mu_M,\mu_R(t)=\mu_R,\omega(t)=\omega,\varphi(t)=\varphi,\rho_M(t)=\rho_M,\rho_R(t)=\rho_R$$

**5.2 Differential Game Model and Game Equilibrium Analysis**

Noted: $e_M^*(t)$、$e_R^*(t)$ are the solutions for the decision issue of manufacturers and retailers respectively:

$$L_M\left(e_M(t),\Theta(t),t\right)=\omega\left(\pi_{SC}(t)-C_M-C_R\right)$$

$$L_M\left(e_R(t),\Theta(t),t\right)=(1-\omega)\left(\pi_{SC}(t)-C_M-C_R\right)$$

The optimal revenue value function for manufacturer after the cooperation state $t\Theta(t)$ is:

$$T\Pi_M^*\left(\Theta(t),t\right)=\max_{e_M(t)}\int_t^{+\infty}L_M\left(e_M(s),\Theta(s),s\right)e^{-\delta}ds \quad (6)$$

The optimal revenue value function for manufacturer after the cooperation state $t\Theta(t)$ is:

$$T\Pi_R^*\left(\Theta(t),t\right)=\max_{e_R(t)}\int_t^{+\infty}L_R\left(e_R(s),\Theta(s),s\right)e^{-\delta}ds \quad (7)$$

Then the propositions are:

Proposition 1 Differential game issue of the two-way cooperation system of international trade and international logistics, with Hamilton-Jacobi-Bellman equation:

$$\begin{cases}\dfrac{\partial T\Pi_M^*}{\partial t}=-\max\limits_{e_M\omega}\left(L_M\left(e_M,\Theta,t\right)\right)e^{-\delta}+\dfrac{\partial T\Pi_M^*}{\partial\Theta}\left(\rho_M e_M+\rho_R e_R-\xi\Theta\right)\\ \dfrac{\partial T\Pi_R^*}{\partial t}=-\max\limits_{e_R\omega}\left(L_R\left(e_R,\Theta,t\right)\right)e^{-\delta}+\dfrac{\partial T\Pi_R^*}{\partial\Theta}\left(\rho_M e_M+\rho_R e_R-\xi\Theta\right)\end{cases} \quad (8)$$

$\Theta(t)$ The initial optimal cooperation maintenance effort level, then $e_M^*(s)(t\le s)$ must be from time $\Theta(t+\Delta t)$, the two-way cooperation system of the international trade and international logistics $\Theta(t+\Delta t)$ initial optimal international trade and international logistics two-way cooperation system maintenance effort level.

As can be known from equation (6) that:

$$\begin{aligned}&T\Pi_M^*\left(\Theta(t+\Delta t),t+\Delta t\right)=\max_{e_R(t)}\int_{t+\Delta t}^{+\infty}L_M\left(e_M(s),\Theta(s),s\right)e^{-\delta}ds\\ &\Rightarrow T\Pi_M^*\left(\Theta(t),t\right)=\max_{e_M(t)}\left(\int_t^{t+\Delta t}L_M\left(e_M(s),\Theta(s),s\right)e^{-\delta}ds+\int_{t+\Delta t}^{+\infty}L_M\left(e_M(s),\Theta(s),s\right)e^{-\delta}ds\right)\\ &=\max_{e_M(t)}\left(\int_t^{1+\Delta t}L_M\left(e_M(s),\Theta(s),s\right)e^{-\delta}ds+\Pi_M^*\left(\Theta(t+\Delta t),t+\Delta t\right)\right)\end{aligned} \quad (9)$$

Perform Taylor expansion on $T\Pi_M^*\left(\Theta(t)+\Theta'(t)\Delta t,t+\Delta t\right)$:

$$\int_t^{1+\Delta t}L_M\left(e_M(s),\Theta(s),s\right)e^{-\delta}ds\approx L_M\left(e_M(t),\Theta(t),t\right)e^{-\delta}\cdot\Delta t \quad (10)$$

$$T\Pi_M^*\left(\Theta(t)+\Delta t,t+\Delta t\right)\approx T\Pi_M^*\left(\Theta(t),t\right)+\frac{\partial T\Pi_M^*}{\partial\Theta}\Theta'\Delta t+\frac{\partial T\Pi_M^*}{\partial t}\Delta t \quad (11)$$

Substitute equation(10)and equation(11)into equation(9)to obtain the approximate value of $T\Pi_M^*\left(\Theta(t),(t)\right)$:

$$T\Pi_M^*\left(\Theta,t\right)=\max_{e_M(t)}$$

$$\begin{aligned}&\left(L_M\left(e_M,\Theta,t\right)e^{-\delta}\cdot\Delta t+T\Pi_M^*\left(\Theta,t\right)+\frac{\partial T\Pi_M^*}{\partial\Theta}\Theta'\Delta t+\frac{\partial T\Pi_M^*}{\partial t}\Delta t\right)\Rightarrow 0=\max_{e_M(t)}\left(L_M\left(e_M,\Theta,t\right)e^{-\delta}+\frac{\partial T\Pi_M^*}{\partial\Theta}\Theta'+\frac{\partial T\Pi_M^*}{\partial t}\right)\\ &\Rightarrow 0=\max_{e_M(t)}\left(L_M\left(e_M,\Theta,t\right)e^{-\delta}+\frac{\partial T\Pi_M^*}{\partial\Theta}\Theta'+\frac{\partial T\Pi_M^*}{\partial t}\right)\\ &\Rightarrow\frac{\partial T\Pi_M^*}{\partial t}=-\max_{e_M(t)}\left(L_M\left(e_M,\Theta,t\right)e^{-\delta}+\frac{\partial T\Pi_M^*}{\partial\Theta}\Theta'\right)\end{aligned}$$

Thus obtain the Hamilton-Jacobi-Bellman equation for decision problem (6) and (4):

$$\Rightarrow\frac{\partial T\Pi_M^*}{\partial t}=-\max_{e_M(t)}\left(L_M\left(e_M,\Theta,t\right)e^{-\delta}+\frac{\partial T\Pi_M^*}{\partial\Theta}\left(\rho_M e_M+\rho_R e_R-\xi\Theta\right)\right) \quad (12)$$

Retailer decision issue (7)'s Hamilton-Jacobi-Bellman equation can be obtained by complete parallel analysis.

In particular, if

$T_M(\Theta(t)) = e^{\delta}T\prod_M^*(\Theta(t),t)$、$T_R(\Theta(t)) = e^{\delta}T\prod_R^*(\Theta(t),t)$

then:

Proposition 2 International logistics differential game issue (4) (5), the differential game with the optimum two-way cooperation system of international trade and international logistics

$\Theta^*(t) = e^{\delta}\left(\theta_0 + \frac{q}{p}\right) - \frac{q}{p}$ (where $p = \frac{2\rho_M^2 a_1^*}{\omega\mu_M} + \frac{2\rho_R^2 a_2^*}{(1-\omega)\mu_R} - \xi, q = \frac{\rho_M^2 a_2^*}{\omega\mu_M} + \frac{\rho_R^2 b_2^*}{(1-\omega)\mu_R}$)

and Nash equilibrium solution is

$$\left(e_M^*(t) = \frac{\rho_M\left(2a_1^*\Theta^*(t)+b_1^*\right)}{\omega\mu_M}, e_R^*(t) = \frac{\rho_R\left(2a_2^*\Theta^*(t)+b_2^*\right)}{(1-\omega)\mu_R}\right),$$

if the unknown parameters $a_1, a_2, b_1, b_2, c_1$ and $c_2$ equation group(which is called quasi constraint equation group).

Hence Proposition 1 is proved.

$$\begin{cases} \omega\varphi + \frac{2\rho_M^2 a_1^2}{\omega\mu_M} - \frac{2\omega\rho_R^2 a_2^2}{(1-\omega)^2\mu_R} + \frac{4\rho_R^2 a_1 a_2}{(1-\omega)\mu_R} - (2\xi+\delta)a_1 = 0 \\ \frac{2\rho_M^2 a_1 b_1}{\omega\mu_M} - \frac{2\omega\rho_R^2 a_2 b_2}{(1-\omega)^2\mu_R} + \frac{2\omega\rho_R^2(a_1b_1 + a_2b_1)}{(1-\omega)\mu_R} - (\xi+\delta)b_1 = 0 \\ \frac{\rho_M^2 b_1^2}{2\omega\mu_M} - \frac{\omega\rho_R^2 b_2^2}{2(1-\omega)^2\mu_R} + \frac{\rho_R^2 b_1 b_2}{(1-\omega)\mu_R} - \delta c_1 = 0 \\ (1-\omega)\varphi + \frac{\rho_R^2 a_2^2}{(1-\omega)\mu_R} - \frac{2(1-\omega)\rho_M^2 a_1^2}{\omega^2\mu_R} + \frac{4\rho_M^2 a_1 a_2}{\omega\mu_M} - (2\xi+\delta)a_2 = 0 \\ \frac{2\rho_R^2 a_2 b_2}{(1-\omega)\mu_R} - \frac{2(1-\omega)\rho_M^2 a_1 b_1}{\omega^2\mu_R} + \frac{2\rho_M^2(a_1b_2 + a_2b_1)}{\omega\mu_M} - (\xi+\delta)b_2 = 0 \\ \frac{\rho_R^2 b_2^2}{2(1-\omega)\mu_R} - \frac{(1-\omega)\rho_M^2 b_1^2}{2\omega^2\mu_M} + \frac{\rho_M^2 b_1 b_2}{\omega\mu_M} - \delta c_2 = 0 \end{cases}$$

With solution($a_1^*, a_2^*, b_1^*, b_2^*, c_1^*, c_2^*$).

Prove:

From $T_M(\Theta(t)) = e^{\&}T\prod_M^*(\Theta(t),t), T_R(\Theta(t)) = e^{\&}T\prod_R^*(\Theta(t),t)$ get:

$$\frac{\partial T\prod_M^*(\Theta(t),t)}{\partial t} = -\delta e^{-\&}T_M(\Theta(t)), \frac{\partial T\prod_R^*(\Theta(t),t)}{\partial t} = -\delta e^{-\&}T_R(\Theta(t)) \tag{13}$$

$$\frac{\partial T\prod_M^*(\Theta(t),t)}{\partial t} = -\delta e^{-\&}T_M(\Theta(t)), \frac{\partial T\prod_R^*(\Theta(t),t)}{\partial t} = -\delta e^{-\&}T_R(\Theta(t))$$

Substitute the above expression into the Hamilton-Jacobi-Bellman equation of the differential game issue, Hamilton-Jacobi-Bellman equation's equivalent form is obtained:

$$\begin{cases} \delta T_M(\Theta(t)) = \max\limits_{e_M\omega}\left(L_M(e_M,\Theta,t) + T_M'(\Theta(t))(\rho_M e_M + \rho_R\rho_R - \xi\Theta)\right) \\ \delta T_R(\Theta(t)) = \max\limits_{e_R\omega}\left(L_R(e_R,\Theta,t) + T_R'(\Theta(t))(\rho_M e_M + \rho_R\rho_R - \xi\Theta)\right) \end{cases} \tag{14}$$

Solving:

By the first-order conditions:

$$\begin{cases} \frac{\partial\left(L_M(e_M,\Theta,t) + T_M'(\rho_M e_M + \rho_R\rho_R - \xi\theta)\right)}{\partial e_M} = 0 \\ \frac{\partial\left(L_R(e_R,\Theta,t) + T_R'(\rho_M e_M + \rho_R\rho_R - \xi\theta)\right)}{\partial e_R} = 0 \end{cases}$$
$$\Leftrightarrow \begin{cases} -\omega\mu_M e_M + \rho_M T_M' = 0 \\ -(1-\omega)\mu_R e_R + \rho_R T_R' = 0 \end{cases} \tag{15}$$
$$\Leftrightarrow \begin{cases} e_M = \frac{\rho_M T_M'}{\omega\mu_M} \\ e_M = \frac{\rho_R T_M'}{(1-\omega)\mu_R} \end{cases}$$

Substitute(15)into(14)to obtain:

$$\begin{cases} \delta\Gamma_M(\Theta)=\max\limits_{e_M(t)}\left(\omega\varphi\Theta^2+\dfrac{\rho_M^2\Gamma_M'}{2\omega\mu_M}-\dfrac{\omega\rho_R^2\Gamma_R'^2}{2(1-\omega)^2\mu_R}+\dfrac{\rho_R^2\Gamma_M'\Gamma_R'}{(1-\omega)\mu_R}-\xi\Theta\Gamma_M'\right) \\ \delta\Gamma_R(\Theta)=\max\limits_{e_R(t)}\left((1-\omega)\varphi\Theta^2+\dfrac{\omega\rho_R^2\Gamma_R'^2}{2(1-\omega)\mu_R}-\dfrac{(1-\omega)\rho_M^2\Gamma_M'\Gamma_R'}{2\omega^2\mu_M}+\dfrac{\rho_M^2\Gamma_M'\Gamma_M'}{\omega\mu_M}-\xi\Theta\Gamma_R'\right) \end{cases} \quad (16)$$

Noted the order characteristic of the above differential equation group, assuming that it has the solution on $\Theta$ for the m-order polynomial form, then due to the left of the equation is m times, the highest on the right is $2(m-1)$ times, therefore, from $\Gamma_M(\Theta)$、$\Gamma_R(\Theta)$, it can be known that $m=2$.

Thus assuming function $\Gamma_M(\Theta)$、$\Gamma_R(\Theta)$ has the following expression:

$$\begin{cases} \Gamma_M(\Theta)=a_1\Theta^2+b_1\Theta+c_1 \\ \Gamma_M(\Theta)=a_2\Theta^2+b_2\Theta+c_2 \end{cases} \quad (17)$$

Where $\Gamma_M(\Theta)$、$\Gamma_R(\Theta)$ and $c_2$ are all unknown constants

Substitute (17) into equation (16) to obtain:

$$\begin{cases} \delta a_1\Theta^2+\delta b_1\Theta+\delta c_1=\omega\varphi\Theta^2+\dfrac{\rho_M^2(2a_1\Theta+b_1)^2}{2\omega\mu_M}-\dfrac{\omega\rho_R^2(2a_2\Theta+b_2)^2}{2(1-\omega)^2\mu_R}+\dfrac{\rho_R^2(2a_1\Theta+b_1)(2a_2\Theta+b_2)}{(1-\omega)\mu_R}-\xi(2a_1\Theta+b_1)\Theta \\ \delta a_2\Theta^2+\delta b_2\Theta+\delta c_2=\omega\varphi\Theta^2+\dfrac{\rho_R^2(2a_1\Theta+b_1)^2}{2\omega\mu_R}-\dfrac{\omega\rho_M^2(2a_2\Theta+b_2)^2}{2(1-\omega)^2\mu_M}+\dfrac{\rho_M^2(2a_1\Theta+b_1)(2a_2\Theta+b_2)}{(1-\omega)\mu_M}-\xi(2a_2\Theta+b_2)\Theta \end{cases}$$

$$\Rightarrow\begin{cases} \delta a_1=\omega\varphi+\dfrac{2\rho_M^2a_1^2}{\omega\mu_M}-\dfrac{2\rho_M^2a_2^2}{(1-\omega)^2\mu_R}+\dfrac{4\rho_R^2a_1a_2}{(1-\omega)\mu_R}-2\xi a_1 \\ \delta b_1=\dfrac{2\rho_M^2a_1b_1}{\omega\mu_M}-\dfrac{2\omega\rho_R^2a_2b_2}{(1-\omega)^2\mu_R}+\dfrac{2\rho_R^2(a_1b_2+a_2b_1)}{(1-\omega)\mu_R}-\xi b_1 \\ \delta c_1=\dfrac{2\rho_M^2b_1^2}{2\omega\mu_M}-\dfrac{\omega\rho_R^2b_2^2}{2(1-\omega)^2\mu_R}+\dfrac{\rho_R^2b_1b_2}{(1-\omega)\mu_R} \\ \delta a_2=(1-\omega)\varphi+\dfrac{2\rho_R^2a_2^2}{(1-\omega)\mu_R}-\dfrac{2(1-\omega)\rho_M^2a_1^2}{\omega^2\mu_M}+\dfrac{4\rho_M^2a_1a_2}{\omega\mu_M}-2\xi a_2 \\ \delta b_2=\dfrac{2\rho_R^2a_2b_2}{(1-\omega)\mu_R}-\dfrac{2(1-\omega)\rho_M^2a_1b_1}{\omega^2\mu_M}+\dfrac{2\rho_M^2(a_1b_2+a_2b_1)}{\omega\rho_M}-\xi b_2 \\ \delta c_2=\dfrac{\rho_R^2b_2^2}{(1-\omega)\mu_R}-\dfrac{(1-\omega)\rho_M^2b_1^2}{2\omega^2\mu_M}+\dfrac{\rho_M^2b_1b_2}{\omega\mu_M} \end{cases} \quad (18)$$

Then from the known conditions of the proposition, it can be known that the solution to the equation group is also the solution to the quasi constrained equation $a_1^*,a_2^*,b_1^*,b_2^*,c_1^*,c_2^*$.

Thus function $\Gamma_M(\Theta)$、$\Gamma_R(\Theta)$ has the expressio n:

$$\begin{cases} \Gamma_M^*(\Theta)=a_1^*\Theta^2+b_1^*\Theta+c_1^* \\ \Gamma_M^*(\Theta)=a_2^*\Theta^2+b_2^*\Theta+c_2^* \end{cases} \quad (19)$$
$$\begin{cases} \Gamma_M^{*\prime}(\Theta)=2a_1^*\Theta+b_1^* \\ \Gamma_M^{*\prime}(\Theta)=2a_2^*\Theta+b_2^* \end{cases}$$

Substitute equation(19)into equation(15)to obtain:

$$\begin{cases} e_M^*(\Theta)=\dfrac{\rho_M(2a_1^*\Theta(t)+b_1^*)}{\omega\mu_M} \\ e_R^*(\Theta)=\dfrac{\rho_M(2a_1^*\Theta(t)+b_1^*)}{\omega\mu_M} \end{cases} \quad (20)$$

Substitute equation (20) into state change differential equation (3) to obtain:

$$\Theta'(t)=\left(\frac{2\rho_M^2a_1^*}{\omega\mu_M}+\frac{2\rho_R^2a_2^*}{(1-\omega)\mu_R}-\xi\right)\Theta(t)+\left(\frac{\rho_M^2b_1^*}{\omega\mu_M}+\frac{\rho_R^2b_2^*}{(1-\omega)\mu_R}\right) \quad (21)$$

The following is the solution of differential equations (21):

Note

$$p=\frac{2\rho_M^2a_1^*}{\omega\mu_M}+\frac{2\rho_R^2a_2^*}{(1-\omega)\mu_R}-\xi,\ q=\frac{\rho_M^2b_1^*}{\omega\mu_M}+\frac{\rho_R^2b_2^*}{(1-\omega)\mu_R}$$

then the general solution to the differential equation (21) is: $\Theta(t)=e^{\int pdt}\left(\int_{qe}^{-\int pdt}dt+c\right)$

Where c is any constant

Thus, according to the state change differential equation boundary conditions $\Theta(t)|_{t=0} = \theta_0 \geq 0$ ,special solution of $\Theta(t)$ that meets the boundary conditions can be obtained: $\Theta^*(t) = e^{pt}\left(\theta_0 + \int_0^t qe^{-pt}ds\right)$

i.e.:

$$\Theta^*(t) = e^{pt}\left(\theta_0 + \frac{q}{p}\right) - \frac{q}{p}$$

Thus Proposition 2 is proven.

**6. Case Analysis on the Two-way Cooperation System of the International Trade and International Logistics**

The following is the calculation analysis on the examples for the above differential game:

The specific values for the given parameters are as follows:

$$\mu_M = 0.5, \rho_M = 0.5, \mu_R = 0.3, \rho_R = 0.8$$
$$\omega = 0.6, \delta = 0.9, \xi = -5, \varphi = 0.3, \theta_0 = 0.5$$

Substitute the above parameters into the quasi constraint equation to obtain:

$$a_1^* = -0.016730924, b_1^* = 0.089190424$$
$$a_2^* = -0.013526144, b_2^* = 0.054732274$$

Thus

$$p = \frac{2\rho_M^2 a_1^*}{\omega\mu_M} + \frac{2\rho_R^2 a_2^*}{(1-\omega)\mu_R} - \xi = -0.472163741$$
$$q = \frac{\rho_M^2 b_1^*}{\omega\mu_M} + \frac{\rho_R^2 b_1^*}{(1-\omega)\mu_R} = 0.366230815$$

Thereby to obtain the optimal differential game for the two-way cooperation system of the international trade and international logistics is:

$$\Theta^*(t) = e^{pt}\left(\theta_0 + \frac{q}{p}\right) - \frac{q}{p} = 0.28e^{-0.47t} - 0.78$$

The Nash input equilibrium solution to the two-way cooperation system of the international trade and international logistics is:

Manufacturer's optimal differential game on the two-way cooperation system maintenance and effort level of the international trade and international logistics:

$$e_M^* = \frac{\rho_M\left(2a_1^*\Theta^*(t) + b_1^*\right)}{\omega\mu_M} = 0.015e^{-0.47t} - 0.096$$

(2)Retailer's optimal differential game on the two-way cooperation system maintenance and effort level of the international trade and international logistics:

$$e_R^* = \frac{\rho_R\left(2a_2^*\Theta^*(t) + b_2^*\right)}{(1-\omega)\mu_R} = -0.0007e^{-0.47t} + 0.225$$

The corresponding function changes are illustrated as follows:

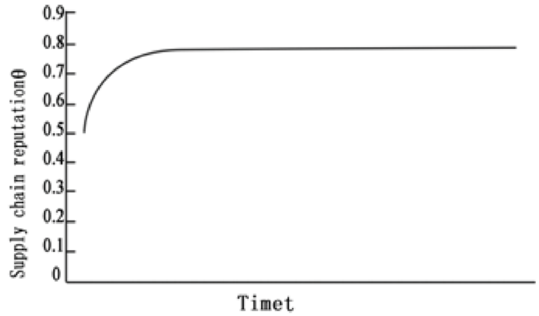


**Figure 1**. The Change of the Two-way Cooperation System of International Trade and International Logistics Over Time

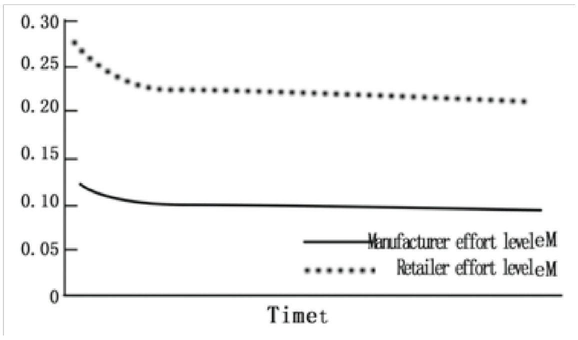


**Figure 2**. The Change of the Optimal Level of Cooperation Efforts in All Aspects of International Logistics Over Time

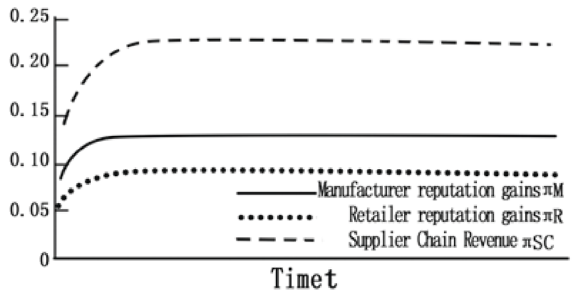


**Figure 3**. The Change of the Optimal Cooperation Benefit Time of All Parties in the International Logistics

As can be seen from Figure 1, 2 and 3: In the range of the above parameters,

(1). The optimal game on the input of the maintenance of the two-way cooperation system of the international trade and international logistics changes over time

This reflects the time correlation of the game of both parties in the international logistics: Since game is closely related to time, the optimal game at some point may no longer be able to bring the maximum benefit to the decision-makers in the next time point may, therefore, dynamic game that changes with time is the game with practical value. This fully shows that the differential game method has great value in the dynamic game of the international logistics.

(2). The two-way cooperation system of international trade and international logistics, and the cooperation input of the two parties of the international logistics have the trend of time stability.

A notable feature that Figures 1 and 2 show is that the change of the two-way cooperation system of international trade and international logistics, the cooperation input of manufacturers and retailers over time, after the early stages of relatively severe adjustment, tend to be stabilized in the latter stage. Figure 3 on the changes of the revenue of both parties of the international logistics and the entire international logistics also verifies the similar conclusion.

This means that, in the long-term relationship of game, the international logistics system has the characteristics of autonomy, which tends to be balanced, in other words, the dynamic changes in the international logistics system will be effectively controlled.

Thus, the differential game analysis on the above two-way cooperation system of the international trade and international logistics reflects from the other side that the long-term stable cooperative relations between the partners of international logistics is conducive to the stability of the international logistics.

In addition, in order to simplify the problem, this paper takes the relevant parameters as constant. However, in the above differential game on the two-way cooperation system of the international trade and international logistics analysis process, as whether these variational problems are solvable on the functional, whether the solution can be expressed, with analytic form strictly dependents on the specific structure of the differential equation of the functional, while in practice, the descriptive forms of parameters typically are vastly different, therefore, in the non-degenerate case, the analytical solution is difficult to be obtained, thus, it is necessary to make the research on numerical solution of such differential equation to be the important research object in this direction.

**7. Conclusion and Outlook**

The only source of income for international logistics is the final consumer; therefore, the effort of all members of the entire international logistics to develop the value for customers has a very important role for the revenue of the international logistics. Through the analysis on the nature of this issue, this paper abstracts the issue into the issue of constructing the two-way cooperation system of the international trade and international logistics. At the same time, considering that the relationship of cooperation and game among the members of the international logistics enterprises are usually long-standing in practice, this paper takes the cooperation time into account, and build the differential game model for the two-way cooperation system of the international trade and international logistics in the scenario of continuous time and dynamic game, and looks for the conditions for Nash equilibrium, so as to draw certain valuable conclusions. These conclusions have illustrated the significance of cooperation, and will provide certain inspiration and reference for decision-makers in practice.

**Acknowledgements**

This work is supported by Guangxi university research projects in 2014, Project Name: Empirical Study of Guangxi's Foreign Trade Logistics Development impact, Project NO.: YB2014517.

**References**

1. Chen, H. K., Chou, H. W., & Chiu, Y. C. (2007) On the modeling and solution algorithm for the reverse logistics recycling flow equilibrium problem. *Transportation Research Part C: Emerging Technologies*, 15(4), p.p. 218-234.
2. Zhou, X., Xie, R., Zhang, X., Wang, C., & Wu, X. (2009) Pansystems traffics, logistics and knowledge rediscovery. *Kybernetes*, 38(1/2), p.p. 217-228.
3. Chanintrakul, P., Coronado Mondragon, A. E., Lalwani, C., & Wong, C. Y. (2009) Reverse logistics network design: a state-of-the-art literature review.*International Journal of Business Performance and Supply Chain Modelling*, 1(1), p.p. 61-81.
4. Hoekman, B., & Mattoo, A. (2011) Services trade liberalization and regulatory reform: re-invigorating international cooperation. *World Bank Policy Research Working Paper Series*, p.p.1-25
5. Junginger, M., Van Dam, J., Zarrilli, S., Mohamed, F. A., Marchal, D., & Faaij, A. (2011) Opportunities and barriers for international bioenergy trade. *Energy Policy*, 39(4), p.p. 2028-2042.
6. Marlow, P., & Nair, R. (2008) Service contracts-An instrument of international logistics supply chain: Under United States and European Union regulatory frameworks. *Marine Policy*, 32(3), p.p. 489-496.
7. Hummels, D. (2007) Transportation costs and international trade in the second era of globalization. *The Journal of Economic Perspectives*, p.p. 131-154.
8. 5. Gali J. (2005) Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies*, 72 (3), p.p. 707-734.
9. Hill, C. (2008) International business: Competing in the global market place. *Strategic Direction*, 24(9).
10. Vaidyanathan, G. (2005) A framework for evaluating third-party logistics. *Communications of the ACM*, 48(1), p.p. 89-94.

11. 9. Markus Reisinger, Emanuele Tarantino. (2015) Vertical Integration, Foreclosure, and Productive Efficiency. *Rand Journal of Economics*, 46 (3), p.p. 461-479.

12. Li, Y., Shu, C., & Xiong, L. (2011) Information services platform of international trade based on E-commerce. *Advances in Information Sciences and Service Sciences*, 3(1), p.p. 78.

# Supply Chain Management Model of Garment Company With R&D Perspective

**Xin Zhang**

*Jiaxing University, Jiaxing 314200, Zhejian, China*

Abstract
The supply chain has not been given the anticipated recognition it needs in garment industry. This is due to the overdependence on the old management techniques that have less priority for the supply. The article proposed a SCM(supply chain management) model with R&D perspective in order to solve the R&D efficiency and moral risk among of garment companies. It seeks to construct the cooperative model in the integrated supply chain through studying the investment strategy among the members. The model indicated that the garment company would increase the R&D investment when the average interest will be distributed in the supply chain cooperative R&D process. There are different results of the importance of incorporating the supply chain into the activities of enterprises in the garment industry. The study shows that the garment companies interest should be distributed in average in their supply chain alliance. Especially, they must strengthen cooperation if they want to achieve the success when the market risk increased.
Key words: GARMENT COMPANY, R&D FORM, INTEREST DISTRIBUTION, SUPPLY CHAIN MANAGEMENT.

## 1. Introduction

Many firms have ventured into the garment industry in China without the basics of management of the production and distribution procedures.[1]This information shall be deeply analyzed in this document as a way to provided firsthand information to the firms on the essence of the supply chain. Various garment firms shall be analyzed to identify the techniques they have employed to ascertain if they are fit for the success of the business.[2]The great importance is the significance of supply chain management toward the improvement of the product quality. Besides this, how does the supply chain contribute to the improvement of the innovation though such things as fashion. Various businesses have come up with methods that are not fit for the success of the businesses in terms of the supply chain, as a result called for the need to improve the supply chain methods that are already in use within most of the Chinese firms [3].

Some firms have already adopted proper supply chain management methods through they still lack proper management procedures. For a firm to fit properly to the ever changing needs in the supply chain, innovation should always be given a priority. Once the interests of the market shift, the supply chain should equally shift since the markets will be interested in latest designs and low priced products. An example for changes to the supply chain is through the removal of the supply chain intermediaries who mostly make additional expenditure on the customer