Image Reconstruction for Denoising Based on Compressive Sensing

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Abstract

Due to the disadvantage of large amounts of data computation and image quality degradation of classical reconstruction algorithm, a novel adaptive method of image reconstruction denoising based on compressive sensing is proposed. Firstly, the wavelet approximate coefficients and detail coefficients from the image noise are Gaussian distribution, and have different variances in different levels. Secondly, the noise image is divided into image blocks of a certain size, a new compression sensing block reconstruction method has been used to recover small block coefficients. Finally, the reconstructed denoising images are obtained based on recovered detail coefficients and approximate coefficients by the separation of small block wavelet inversed transform. Experimental results show that this method is feasible and available, compared with pure wavelet denoising and block image, signal-to-noise ratio has been improved highly, the image noise has been removed effectively and the reconstructed image quality has been improved highly.

Key words: COMPRESSIVE SENSING, IMAGE DENOISING, WAVELET TRANSFORM, SPARSE REPRESENTATION

1. Introduction

As a new field derived from computer and internet science, the digital image processing is widely in people's daily life. In processing of acquisition, encoding, transmitting image signal, various noises contaminates the image inevitably, affecting the quality of image and its subsequent processing. In recent years image denoising has received more attention as an important part of image pre-processing. Its objective is to recover the best estimate of the original image from its nosy version. Now there are more and more in-depth studies on image denoising, and it will be the development of this field with breaking the traditional pattern and bringing in new technology.

In document [1], the author analyses the inter-scale distribution characteristics of the coefficients at finer scale of the noise image, and propose a statistical model of noise coefficient correlation of intra-scale and inter-scale, called the zero tree like structure of noise coefficient distribution inter finer-scale, and the method of block-wise Bayes shrinkage threshold. In document [2], the author divides the whole image into blocks and deals with them separately. The denoising effect could be improved to a certain degree, but some discontinuities can be found in the areas of adjacent blocks. In document [3], image is decomposed by multi-wavelet packet into a series of approximate piecewise constant sub-images, and sub-images are processed separately by TV model. The above methods are all not considered that sparse representations and actual circumstances flexibly reduce the sampling rate and the costs of data processing.

Compressive sensing is an emerging science in recent years, it blazed a new path for signal processing field, the core idea of which is using few measurement data with much smaller than the Nyquist sampling rate to realize signal reconstruction, and can save the cost of signal processing [4]. Compared with traditional denoising methods, denoising method based on compressed sensing theory can bring the advantages of sparse representations into full play, adapt to actual circumstances flexibly, reduce the sampling rate and the costs of data processing, and get better denoising results.

This paper mainly research on the applications of compressive sensing in image denoising field, constructing the mathematical model of image denosing based on compressive sensing. The main work of this paper as follows: firstly, the framework of compressive sensing is introduced, including sparse representation, the construction of measurement matrix and the reconstruction algorithm; Second, a new compression sensing block reconstruction method has been used to recover small block coefficients. Thirdly, the reconstructed denoising images are obtained based on recovered detail coefficients and approximate coefficients. Finally, several contrast tests was done for comparing with classis denoising algorithms. The experiments show that this method is feasible and effective.

2. Compressive sensing

Compressed sensing image processing mainly contains the sparse representation of the signal, and the three steps of the encoding measurement and denoising algorithm.

The first step, if the signal $X \in R^N$ is compressible in an orthogonal basis or tight frames, it is possible of transforming coefficients $\Theta = \Psi^T X$, Θ is equivalent to Ψ in sparse representation [4];

The second step, the observation matrix Φ with $M \times N$ dimension is designed, which is a stable and is not related to the transformation base Ψ, the observation set $Y = \Phi\Theta = \Phi\Psi^T X$ was obtained through Θ measurement, the process can also be expressed as the signal X non adaptive observation $Y = A^{CS}$ ($A^{CS} = \Phi\Psi^T$) through the matrix A^{CS}, it is called as CS information operator[5-6];

Third step, the exact or approximate approximation $\hat{X}$ is solved by use of the optimization problem of 0-norm [7], the main operators of the algorithm include positive selection, memory cells producing, hyper-mutation and similar antibodies suppression.

2.1. Signal sparse representation

The signal transformation coefficients vector is expressed as $\Theta = \Psi^T X$ in the orthogonal basis, assumed $0 < p < 2$ and $R > 0$ predictable, these signal coefficients can be formed as

$$\|\Theta\|_P \equiv \left(\sum_i |\theta_i|^P \right)^{1/P} \le R \tag{1}$$

where the coefficient vector Θ is sparse in some sense. It can be guaranteed that the signal is sparse only by choosing the suitable base for the signal, so that the signal can be recovered and the sparse representation of the transformation matrix can be measured by the transform coefficients [8]. To satisfy the signal that has a power decay rate, the recovery can be obtained by using the theory of compressed sensing, and the reconstruction error can be formed as

$$E = \left\| \hat{X} - X \right\|_2 \le C_r \cdot (K / \log N)^{-6r}$$

(r =1/p – 1/2,0<p<1) (2)

The condition of the transformation group is the orthogonal basis of the orthogonal basis. In an orthogonal basis, the optimal orthogonal basis, which is adaptive to the characteristics of a signal can be approximated by an orthogonal basis, the signal is transformed to the most sparse signal.

The hotspot of the study of sparse representation is sparse decomposition of the signal in a redundant dictionary. This is a new kind of signal representation theory: the use of a complete redundancy function library to replace the basis function, called redundant dictionary, the dictionary elements are known as atoms. The choice of a dictionary should be as good as possible in the structure of the signal that is approximated, and its structure can be without any restrictions [9]. The best linear combination of K atom is found in the redundant dictionary to represent a signal, which is called sparse approximation or highly nonlinear approximation. From the nonlinear approximation perspective, sparse signal approximation contains two aspects: one is according to the objective function from a given the Kikuyu choose good or best basis; the second is to select the best K combination from the good base [10-12].

2.2. Selection of measurement matrix

The observation matrix Φ with $M \times N$ dimensions is designed, which is stable and unrelated to the transformation basis Ψ, and the important information is not destroyed when the sparse vector Θ is reduced from N to M dimension, the low speed sampling problem is solved [13-16].

Observation process actually is the use of observation matrix Φ with $M \times N$ dimension and M row vector $\{\varphi_j\}_{j=1}^{M}$ of sparse coefficient vectors are projected, namely, the inner product is calculated by Θ and observation vector $\{\varphi_j\}_{j=1}^{M}$, got M observations $y_j = <\Theta, \varphi_j> (j = 1,2,.. \text{ M})$, called as observation vector $Y = (y_1, y_2, \ldots, y_M)$, it can be formed as

$$Y = \Phi\Theta = \Phi\Psi^T X = A^{CS} X \tag{3}$$

The sampling process is non-adaptive, and Φ may not been changed according to the X signal, the observation is no longer a signal of the point sampling but a more general K linear functional. If the observation matrix Φ and sparse matrix Ψ are not coherent, the RIP property is satisfied in a large probability for A^{CS}. The three conditions of observation matrix are necessary, and most of the three conditions are obtained, which can be used as the observation matrix, such as partial Fourier set, partial Hadamard set, uniform distribution of random projection (Random Projection uniform). However, after observing the above various kinds of observation matrix, it can only guarantee a very high probability to restore the signal, and can't guarantee the accurate reconstruction of the signal one hundred percent [17-19]. For any stable reconstruction algorithm, whether there is a real deterministic observation matrix is still a problem to be studied.

2.3. Signal reconstruction

Because the number M of observations is much smaller than the signal length N, it has to be faced with the solution of the underdetermined equations $Y = A^{CS} X$ [20-21], the $p-$norm of the vector $X = \{x_1, x_2, \ldots, x_n\}$ is defined as

$$\|X\|_p = \left(\sum_{i=1}^{N} |x_i|^p\right)^{1/p} \tag{4}$$

Where it is $0-$ norm just at $p = 0$, which actually represents the number of nonzero entries [22-24]. Thus, the problem of solving the underdetermined equations: $Y = A^{CS} X$ is transformed into a minimum $0-$norm problem when the signal X is sparse or compressible, it is defined as

$$\min \|\Psi^T X\|_0 \quad \text{S.T.} \quad A^{CS} X = \Phi\Psi^T X = Y \tag{5}$$

However, it needs a linear C_N^K combination of all non-zero position in M of the list, which may get the optimal solution [25-28]. Solution (5) numerical calculation is very unstable and it is difficult to solve the problem of NP, which will be solved in a more simple l_1 optimization problems will produce the same solution (requirements not related between Φ and Ψ), which is defined as

$$\min \|\Psi^T X\|_1 \quad \text{S.T.} \quad A^{CS} X = \Phi\Psi^T X = Y \tag{6}$$

where the slight difference makes the problem into a convex optimization problem, so it can be easily simplified to linear programming problem.

3. Image discrete wavelet transform in two dimensional

Wavelet a function or signal $\psi(x)$ when a function of space $L^2(R)$ accord with (7) and (8)

$$R^* = R - \{0\} \tag{7}$$

$$C_\psi = \int_{R^*} \frac{|\psi(x)|^2}{|\omega|} d\omega < \infty \tag{8}$$

for any function or signal $f(x)$, the wavelet transform is defined as

$$W_f(a,b) = \int_R f(x)\overline{\psi_{(a,b)}(x)}dx = \frac{1}{\sqrt{|a|}}\int_R f(x)\overline{\psi(\frac{x-b}{a})}dx \tag{9}$$

where a sequence of two dimensions is obtained by the two one of $\{p_{l,j}\}$ and $\{q^i_{l,j}\}$, i=1,2,3, that is to say, $p_{l,j} = p^1_l p^2_j, p^1_{l,j} = p^1_l q^2_j, q^2_{l,j} = q^1_l p^2_j$, $p^3_{l,j} = q^1_l q^2_j$. Then a reconstruction algorithm is defined as

$$c_{k+1;n,m} = \sum_{l,j}\left(p_{n-2l,m=2j}c_{k;l,j} + \sum_{j=1}^{3} q^i_{n-2l,m-2j} d^i_{k;l,j}\right) \tag{10}$$

wavelet reconstruction of the data transfer diagram is showed in Figure 1 the coefficients distribution of the two dimensional discrete wavelet transform is defined as

$$S_j f(n,m)\{W_j^1 f(n,m), W_j^2 f(n,m), W_j^3 f(n,m)\}_{j=-j,\ldots,-1}\}_{(n,m)} \in Z\times Z \quad (11)$$

two dimensional signal orthogonal wavelet decomposition coefficient is composed, which is showed in Figure 2.

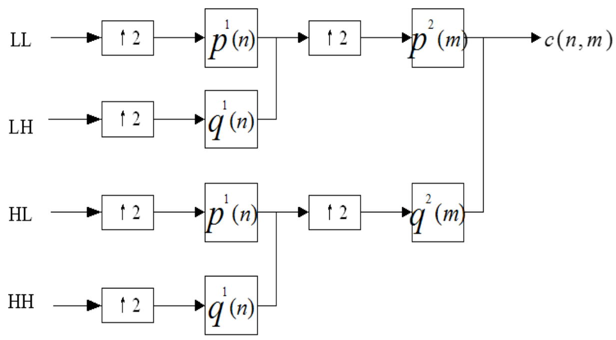

Figure 1. Schematic diagram of wavelet reconstruction data flow

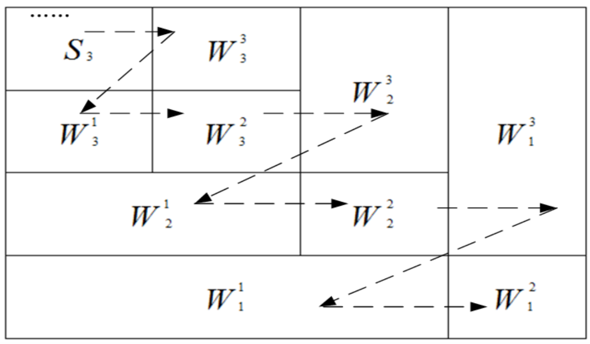

Figure 2. Two dimensional orthogonal wavelet decomposition coefficients

where each of them can be considered as an image, the $f(x,y)$ wavelet coefficients of the high frequency components of the vertical direction are given by $W_j^1 f(n,m)$, the wavelet coefficients of the high frequency components in the horizontal direction are given by $W_j^3 f(n,m)$, the wavelet coefficients of the high frequency components of the diagonal direction are given by $W_j^2 f(n,m)$, and the wavelet coefficients of the low frequency components are given by $S_J f$. Thus, if expressing respectively $S_j f(n,m)$, $W_j^1 f(n,m)$, $W_j^2 f(n,m)$, $W_j^3 f(n,m)$ by use of S_J, W_j^1, W_j^2, W_j^3, it can obtained the transform coefficient after 2: 1 sub-sample for images of (referred to as sub images), then either image can be decomposed into 3J+1 discrete sub image among j=-J, ... , -1, the expression of S_J, W_j^1, W_j^2, W_j^3 is an approximation for the S_J W_j^i (i=1, 2, 3; j=-J,…,-1) is the details in different directions and different resolutions , if the original image has a N2 pixel, the sub images S_J, W_j^1, W_j^2, W_j^3, respectively have $2^i N$ pixel, so the total number of N_T pixels after the decomposition is defined as

$$N_T = 4^{-J} N^2 + 3[\sum_{j=-1}^{-j} 4^i] N^2 = N^2 \quad (12)$$

where it can be seen, the total number of pixels after the decomposition unchanged.

Digital image, which is based on the digital image of the image, is the digital image. The result of the image is a huge number matrix, image processing is completed on this matrix [29]. So, a two-dimensional digital signal d_{mn} can be seen as $S_0 f(n,m)$, which is defined as

$$d_{mn} = S_0 f(n,m) = f(x,y)^* \varphi_0(-x,-y)(n,m) = \iint_{R^2} f(x,y)\varphi(x-n, y-m)dxdy \quad (13)$$

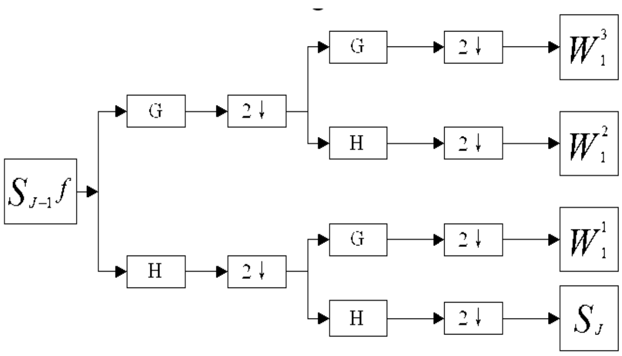

Figure 3. Block diagram of two-dimensional wavelet transform

Mallat algorithm is similar to the one dimensional case. Since the two one dimensional wavelet transform to achieve a two-dimensional wavelet transform, so the line of the matrix of wavelet transform, then the column of wavelet transform. The block diagram of the two-dimensional wavelet transform is shown in Figure 3.

4. Image denoising model based on compressed sensing

The image noise with mean 0, variance sigma σ^2 additive white noise, remember to V, which contain image observation model for Y=X+V[30]. The y for noisy image, X is a clean image, V is additive noise. If in containing noise y ($Y \in R^{d \times N}$), in form of overlapping pixel extraction tick $\sqrt{N} \times \sqrt{N}$ image block, and in a column in the form of a vector ($y_i \in R^n$) are arranged, namely: $y_i = R_i Y$ (R_i is extraction block matrix, which extract the image block y_i from Y [31-33].

According to the theory of compressed sensing, $y = x + w = \emptyset a$, where a is sparse impression based on the transformation $\emptyset$, if the image is clean image with not containing noise, we can solve $a = \arg min \|a\|_0 \quad s.t.\ \emptyset a = x$, Visible, clean images exist sparse representation, however, after adding noise, the noise has destroyed its sparse representation, but, it can restore the original image, so as to achieve the purpose of removing noise, ARFMIN is defined as

$$\arg min \left\{ \mu \|Y - X\|_2^2 + \sum_i \partial_i \|a\|_0 + \sum_i \|\emptyset a - y\|_2^2 \right\} \tag{14}$$

where the sparse representation of the clean image is estimated, and then the reconstructed image is restored, so the noise is removed.

Algorithm description:

Algorithm 1 image sparse representation denoising

Input: y, A, d, $\in$

Initialization: $k \leftarrow 0, r_0 \leftarrow y, C \leftarrow \emptyset, \Delta \leftarrow \infty, \hat{x} \leftarrow 0$;

Repeat until $\Delta < \in$

p=arg $min \|a\|_0$

$x \leftarrow \emptyset a$

$k \leftarrow k + 1$

$C \leftarrow C \cap \{p\}$

$\hat{x}_k \leftarrow \arg min \{\mu \|Y - X\|_2^2\}$

$\hat{x}_k \leftarrow \hat{x}_k + \sum_i \partial_i \|a\|_0 + \sum_i \|\emptyset a - y\|_2^2$

$r_k \leftarrow y - \sum_{i \in C} A^H [i] \hat{x}_k[i]$

$\Delta \leftarrow \|r_k\|_2$

Output: $\hat{x}_k$

5. Experiments and analysis

In order to test this section algorithm is effective and compared existing typical to denoising algorithm (K-MAP denoising algorithm B-PDE denoising algorithm), in clean image with zero mean, variance sigma square of the additive white Gaussian noise, with the three different to denoising algorithm with a picture of a noisy image denoising and multiple operation and obtain the optimal value are analyzed and compared.

Using the improved OMP algorithm of measured data for image denoising recovery algorithm in this paper using wavelet multi-scale transform to noise image processing, using the standard Gaussian random matrix as the measurement matrix phi, random measurement of the sparseness of the data.

Standard Lena images, House images and Cameraman images are selected to compare with the algorithm of the images, and the Gauss white noise with zero mean value is added in the three images, and the results are shown in Figure 4.

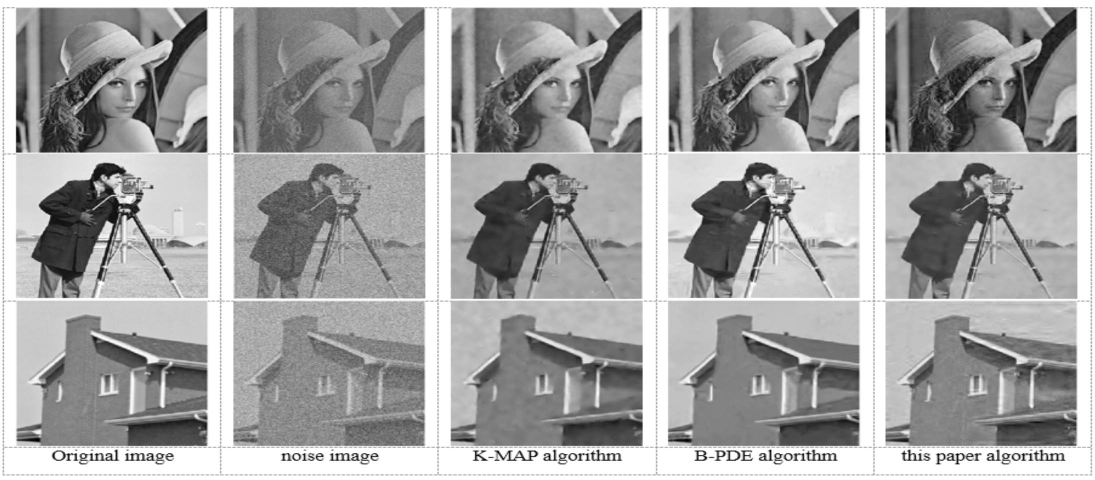

Figure 4. Comparison of three types of denoising algorithm

In order to verify the algorithm in the environment with high noise denoising effect, this chapter selects the Gaussian white noise standard difference of sigma = 40, from the experimental simulation effect diagram can be evident. In this chapter, the algorithm can preserve the image details, where the table 1 for the different images in different noise algorithm under the condition of recovery image, with a PSNR value.

Table 1. The comparison of PSNR values of different denoising algorithms

	Lena	Camera	House
Noise image	16.38	16.41	16.09
K-MAP algorithm	28.95	26.17	29.40
B-PDE algorithm	29.87	26.49	30.67
This paper algorithm	29.95	27.28	30.95

The experimental results show that under the same condition, the PSNR value of the reconstructed image is higher than that of K-MAP and B-PDE, From table 2, we can see that this algorithm has some validity.

6. Conclusions

On the basis of the traditional compressed sensing reconstruction algorithm, a new image reconstruction algorithm is proposed. Using the technique of block compression and wavelet threshold denoising algorithm, the noisy image is reconstructed. The experimental results show that the fast reconstruction algorithm can significantly reduce the amount of computation, greatly shorten the running time of the program, and highly improve the quality of image reconstruction.

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