Dynamics analysis and simulation of steering screw on automobile recirculating ball type steering gear

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Abstract
A steering screw is an important transmission part of automobile recirculating ball type steering gear. Its structure and processing quality have a great influence on steering accuracy and controlling sensitivity of the steering wheel. Based on spiral groove model and Hertz elastic contact theory, it analyzed contact deformation of balls. This paper also establishes the stiffness models of the

steering screw in coupling the torque and axial force, and derives the vibration modal function under the torque equation. According to the simulation of screw geometry model, the paper verifies the correctness of the theoretical model and points out the dangerous section of the steering screw. The paper provides a theoretical basis for designing rational structure of steering screw.
Key words: STEERING SCREW, DEFORMATION MODEL, STIFFNESS MODEL, MODAL FUNCTION, SIMULATION

1. Introduction

A steering screw is an important spiral transmission part for automobile hydraulic assistance steering gear which is recirculating ball type, it changes steering wheel's rotary motion into linear motion of the rack piston[1]. The structure and manufacturing precision of the steering screw have a great influence on steering accuracy and controlling sensitivity of the steering wheel[2]. The steering screw is a hollow shaft. It's inner of big cylinder and input shaft form the valve controlling fluid direction. The pats of big cylinder end transmit torque of steering wheel and the surface of small cylinder is processed spiral groove. Balls move circularly in closed spiral groove, which push the rack piston to move. While the sector gear of rocker shaft rotates following the rack piston moving, the vehicle turns. Therefore, it is necessary to research the steering screw on dynamic analysis and simulation[3,4].

2. Mathematical model of spiral groove

A fixed Cartesian Coordinates $o-xyz$ is established, which z-axis is coincident with the axis of steering screw[5]. Moving $o-xyz$ from a point o to a point o' along x-axis, the distance between the points is r_i, i.e. $\overline{oo'}=r_i$. The coordinates $o-xyz$ is rotated rounding x-axis, which angle between of y-axis and y'-axis is β. So the moving coordinates $o'-x'y'z'$ is get, in which the normal section of spiral groove is a semi-circle as shown in Figure1.

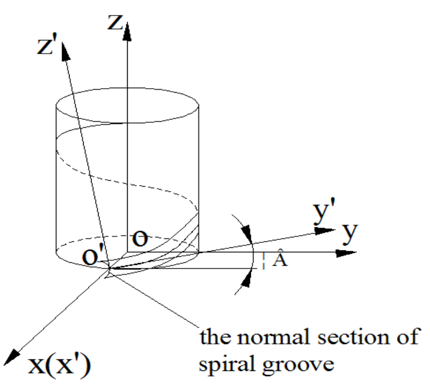

Figure 1. Coordinates of spiral groove on the steering screw

Take spiral groove formula for normal section of the steering screw in the coordinates $o'-x'y'z'$

$$\begin{cases} X' = R\cos\gamma \\ Y' = 0 \\ Z' = R\sin\lambda & \left(-\dfrac{\pi}{2} < \lambda < \dfrac{\pi}{2}\right) \end{cases} \tag{1}$$

R—Radius of spiral groove;
γ—Angle of spiral groove arc.

The transformation of coordinate points between moving coordinates $o'-x'y'z'$ and fixed coordinates $o-xyz$

$$\begin{cases} X = X' + r_i \\ Y = Y'\cos\beta - Z'\sin\beta \\ Z = Y'\sin\beta + Z'\cos\beta \end{cases} \tag{2}$$

β—Helical angle of spiral groove.

The general equation for spiral surface can be followed as

$$\begin{cases} x = X\cos\alpha - Y\sin\alpha \\ y = X\sin\alpha + Y\cos\alpha \\ z = Z + \dfrac{\alpha}{r_i}\tan\beta \end{cases} \tag{3}$$

Substituting equation (1) and equation (2) for equation (3), the mathematical model of spiral groove could be get as follows.

$$\begin{cases} x = (r_i - R\cos\gamma)\cos\alpha + R\sin\gamma\sin\beta\sin\alpha \\ y = (r_i - R\cos\gamma)\sin\alpha - R\sin\gamma\sin\beta\cos\alpha \\ z = R\sin\gamma\cos\beta + \dfrac{\alpha}{r_i}\tan\beta \end{cases} \tag{4}$$

3. Deformation of steering screw

Because the pitch of steering screw is very short, the balls could be taken as uniform arrangement on the circumference. So total radial loads are approximately zero on the contact surface between balls and spiral groove, and axial loads push the rack piston.

3.1. Torsional deformation and torsional rigidity of steering screw

The big cylinder surface of steering screw is supported by a bearing, and small cylinder is a free. When steering screw turned, its total torque M consists of steering wheel torque M_1 and resistance torque M_2 coming from wheel tyres[6]. As the structure of oil holes and oil grooves is relatively small, it can be ignored while analyzing structure deformation. The simplified structure of the steering screw can be shown in Figure 2.

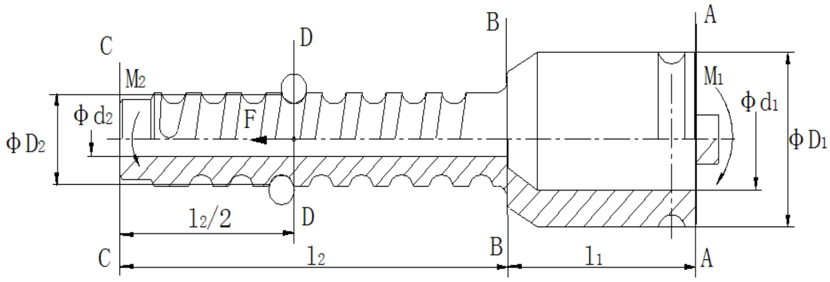

Figure 2. Simplified structure of steering screw

Torsional deformation of section A is

$$\varphi_A = \frac{Tl_1}{GI_{p1}} \quad (5)$$

l_1—Distance between cross section A and cross section B;

G—Modulus of elasticity in shearing;

I_{p1}—Polar moment of inertia section A.

In the formula (5), M and G can be expressed as

$$M = M_1 - M_2 \quad (6)$$

$$G = \frac{E_1}{2(1+\nu_1)} \quad I_{p1} = \frac{W_p D_1}{2} \quad (7)$$

E_1—Equivalent modulus of elasticity;

ν_1—Poisson's ratio;

W_p—Section modulus of torsion;

D_1—Big cylinder diameter of steering screw.

The steering screw is a hollow shaft, therefore

$$W_p = \frac{\pi D_1^3 (1-\alpha_1^4)}{16} \quad (8)$$

α_1—The ratio between internal diameter d_1 and external diameter D_1 of hollow shaft.

Substituting equation (6), equation (7) and equation (8) into equation (5), so φ_A can be get as follow.

$$\varphi_A = \frac{64M(1+\nu_1)l_1}{\pi D_1^4 E_1 \left[1-\left(\frac{d_1}{D_1}\right)^4\right]} \quad (9)$$

In the same way, the torsional deformation of section B and section C can be expressed as follows.

$$\varphi_B = \frac{64M(1+\nu_1)l_2}{\pi D_2^4 E_1 \left[1-\left(\frac{d_2}{D_2}\right)^4\right]} \quad (10)$$

$$\varphi_C = 0 \quad (11)$$

l_2—Distance between cross section Band cross section C;

d_2 and D_2——Internal diameter and external diameter of small cylinder of steering screw .

So the maximum torsional deformation of steering screw is

$$\varphi_Z = \varphi_A + \varphi_B + \varphi_C \quad (12)$$

i.e.

$$\varphi_Z = \frac{64M(1+\nu_1)}{\pi D_1^4 E_1}\left[\frac{l_1}{1-\left(\frac{d_1}{D_1}\right)^4} + \frac{l_2}{1-\left(\frac{d_2}{D_2}\right)^4}\right] \quad (13)$$

3.2. Axial deformation of steering screw

The axial force of the steering screw comes mainly from the axial loads of extrusion pressure of balls. According to Hooke's law, the axial deformation of the steering screw can be as follow.

$$\Delta l = \varepsilon_1 l_1 + \varepsilon_2 l_2 \quad (14)$$

ε_1 and ε_2—Normal strain of different diameters, $\varepsilon_1 = \frac{F}{E_1 A_1}$, and $\varepsilon_2 = \frac{F}{E_1 A_2}$. In the equations, F is axial force, A_1 and A_2 are equivalent area in the section of different diameters. Substituting these variables into equation (14), thus

$$\Delta l = \frac{F_1}{E_1 A_1} + \frac{F_2}{E_1 A_2} \quad (15)$$

The axial stiffness of steering screw is expressed as follow.

$$K_g = \frac{F}{\Delta l} = \frac{E_1 A_1 A_2}{l_1 A_2 + l_2 A_1} \quad (16)$$

3.3. Contact deformation of screw groove surface

Moving along its axis, the rack piston pushes the balls cycling movement, which transfers extrusion pressure from the rack piston to steering screw. So the screw groove surface deforms on touching point. It is a elastic deformation. If we ignore the influence of lubricating oil and manufacturing errors is ignored, the surface compress stress is approximate ellipsoid on the contact point between the ball and the spiral groove based on Hertz elastic contact theory.

Therefore, the principal curvatures of the ball are get as follow[7].

$$\rho_1 = \rho_2 = \frac{1}{r_z} \tag{17}$$

r_z—Radius of ball.

In the same way, the principal curvatures of the spiral groove are expressed as follows.

$$\rho_{11} = -\frac{1}{R}, \quad \rho_{12} = \frac{\cos\beta\cos\gamma}{r_i + r_z\cos\gamma} \tag{18}$$

Contact deformation coefficient is

$$\tau = \sqrt{\frac{(\rho_{11}-\rho_{12})^2+2(\rho_{11}-\rho_{12})(\rho_{21}-\rho_{22})\cos2\psi+(\rho_{21}-\rho_{22})^2}{\rho_{11}+\rho_{12}+\rho_{21}+\rho_{22}}} \tag{19}$$

Substituting equation (17) and equation (18) into equation (19), contact deformation coefficient is shown as follow.

$$\tau = \frac{|\rho_{11}-\rho_{12}|}{\sqrt{\rho_{11}+\rho_{12}+\rho_{21}+\rho_{22}}} \tag{20}$$

Thus, normal contact deformation is the formula as follow.

$$\delta_n = \frac{3P}{2\pi a}\left(\frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2}\right)\Gamma \tag{21}$$

In the formula, E_2 is elastic modulus, v_2 is Poisson's ratio of the ball, P is normal pressure of steering screw, and Γ is determined by τ elliptic integral. a and b represent respectively half-long axis and half-short axis of the ellipsoid on the contact point.

a and b can be expressed as the formula (22).

$$a = a^*\sqrt[3]{\frac{3P}{E'\sum_{i=1,j=1}^{2}\rho_{ij}}}, \quad b = b^*\sqrt[3]{\frac{3P}{E'\sum_{i=1,j=1}^{2}\rho_{ij}}} \tag{22}$$

E'—Equivalent elastic modulus.

$$\frac{2}{E'} = \frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2} \tag{23}$$

$$a^* = \sqrt[3]{\frac{2k^2\Omega}{\pi}} \quad b^* = \sqrt[3]{\frac{2\Omega}{\pi k}} \tag{24}$$

In the formula'(24),

$$k = 1.0339\left(\frac{\sum_{i=1}^{2}\rho_{2i}}{\sum_{i=1}^{2}\rho_{1i}}\right)^{0.636}, \quad \Omega = 1.0003+0.5968\left(\frac{\sum_{i=1}^{2}\rho_{1i}}{\sum_{i=1}^{2}\rho_{2i}}\right)$$

4. Modal function of steering screw under torsional vibration

A steering screw is two stepped shaft. Spiral groove is milled on the small cylinder surface of steering screw, and its axial section is a nonlinear periodic change. When researching the natural frequency of the steering screw, the small cylinder could be simplified as an equivalent shaft. According to Euler-Bernoulli beam theory, a vibration equation of the steering screw could be established[8].

Studying an infinitesimal section on the steering screw, a differential equation can be established as follow.

$$\frac{\partial}{\partial x}\left(GI_p(x)\frac{\partial\theta}{\partial t^2}\right) - J(x)\frac{\partial^2\theta}{\partial t^2} = 0 \tag{25}$$

$J(x)$——Moment of inertia;

$\theta(x,t)$——Torsion angle.

Order

$$\theta(x,t) = n\sum_{j=1}\Phi_j(x)q_j(t) \tag{26}$$

$\Phi(x)$——Modal function of the steering screw;

$q(t)$——Vibration form.

Substituting equation (26) for equation (25), it makes respectively the variable x and t partial derivative, and separates the variables. The equations can be get as follows.

$$\frac{d}{dx}\left(GI_p(x)\Phi_j'(x)\right) + \omega_n^2 J(x)\Phi_j(x) = 0 \tag{27}$$

$$q''(t) + \omega_n^2 q(t) = 0 \tag{28}$$

ω_n—Natural frequency.

Polar moment of inertia $I_p(x)$ can be expressed by the step function $H(x)$ as follow[9].

$$I_p(x) = I_{p1}[H(x-x_0)+H(x-x_1)] + I_{p2}[H(x-x_1)+H(x-x_2)] \tag{29}$$

In the formula, x_i is the distance between two sections. Substituting equation (29) into equation (27), the equation could be get as follow by means of the nature of the function δ [10

$$GI_p(x)\Phi_j''(x)+G\left[-I_{p1}(x)\Phi_j'(x_1)\delta(x-x_1)+I_{p2}(x)\Phi_j'(x_2)\delta(x-x_2)\right]+\omega_n^2 J(x)\Phi_j(x)=0 \tag{30}$$

The steering screw is a uniform mass shaft, so $I_{p1}(x)=I_{p2}(x)$.The equation (30) can be simplified as follow.

$$\Phi_j''(x)+\omega_n^2\frac{J(x)}{GI_p(x)}\Phi_j(x)=0 \tag{31}$$

Order $a_i^2=\frac{GI_{pi}(x)}{J_i(x)}$ the function can be expressed by the step function.

Therefore,

$$\frac{J(x)}{GI_p(x)}=\sum_{i=1}^{2}\frac{1}{a_i^2}\left[H(x-x_{i-1})-H(x-x_i)\right] \tag{32}$$

If the equation (32) substitutes the equation (31), thus,

$$\Phi_j''(x)+\omega_n^2\Phi_j(x)\sum_{i=1}^{2}\frac{1}{a_i^2}\left[H(x-x_{i-1})-H(x-x_i)\right]=0 \tag{33}$$

Then, the initial value of the equation (33) can be get by means of Laplasse transform of the function as follow.

$$\Phi_j(x)=\Phi_j(0)\sum_{i=1}^{2}\left[H(x-x_{i-1})-H(x-x_i)\right]W^{(i-1)}\left\{\psi_1\left[\frac{\omega_n}{a_i}(x-x_i)\right]\right\}+\Phi_j'(0)\sum_{i=1}^{2}\left[H(x-x_{i-1})-H(x-x_i)\right]W^{(i-1)}\left\{\psi_2\left[\frac{\omega_n}{a_i}(x-x_i)\right]\right\} \tag{34}$$

In the equation,

$$\begin{cases}\psi_1\left[\frac{\omega_n}{a_i}(x-x_i)\right]=\cos\frac{\omega_n}{a_i}(x-x_i)\\ \psi_2\left[\frac{\omega_n}{a_i}(x-x_i)\right]=\frac{a_i}{\omega_n}\sin\frac{\omega_n}{a_i}(x-x_i)\end{cases}$$

$$\begin{cases}W\left\{\psi_1\left[\frac{\omega_n}{a_i}(x-x_i)\right]\right\}=\left[\psi_1\left(\frac{\omega_n}{a_i}x_i\right)\frac{I_{pi}}{I_{pi+1}}\psi_1'\left(\frac{\omega_n}{a_i}x_i\right)\right]\left\{\psi_1\left[\frac{\omega_n}{a_{i+1}}(x-x_i)\right]\psi_1\left[\frac{\omega_n}{a_{i+1}}(x-x_i)\right]^T\right\}\\ W\left\{\psi_2\left[\frac{\omega_n}{a_i}(x-x_i)\right]\right\}=\left[\psi_2\left(\frac{\omega_n}{a_i}x_i\right)\frac{I_{pi}}{I_{pi+1}}\psi_2'\left(\frac{\omega_n}{a_i}x_i\right)\right]\left\{\psi_2\left[\frac{\omega_n}{a_{i+1}}(x-x_i)\right]\psi_2\left[\frac{\omega_n}{a_{i+1}}(x-x_i)\right]^T\right\}\end{cases}$$

The function meets the conditions: $W^{(i-1)}=W^{(i-2)}$ $W^{(1)}=W$, $W^{(0)}=1$.The boundary conditions as $\Phi_j(0)=0$ and $\Phi_j'(L)=0$ substituted for the equation (34), the modal function of steering screw could be get as follow.

$$\tan\frac{\omega_n}{a_1}l_1=\frac{a_1J_1}{a_2J_2}\cot\frac{\omega_n}{a_2}l_2$$

5. Analysis of motion simulation

There is a recirculating ball steering gear. Its total torque M is 108 $\mathrm{N\cdot m}$. Axial force F is 857N, which comes from extrusion pressure of balls to the steering screw, and acts on the cross section D as shown in Figure 2. The material of the steering screw is 20CrMnTi, its modulus of elasticity is 205GPa, and Poisson's ratio is 0.27. The big cylinder diameter $D_1=\phi 44$ mm, the hole diameter $d_1=\phi 28$ mm, the shaft length l_1=55 mm, the small cylinder equivalent diameter $D_2=\phi 22$ mm, the hole diameter $d_2=\phi 10$ mm, the shaft length $l_2=100$ mm, the distance CD = 50 mm.

The 3D steering screw model is established by UG NX 8.0, and the 3D model is exported from UG NX into ABAQUS 6.11. The deformation contour of the steering screw is shown in Figure 3, which is changed by the coupling of torque and axial force, and the stress contour is shown in Figure 4. Table 1 gives the value of deformation and stress in some sections.

According to Table 1, it could be known that the further to the fixed end the severer torsion deformation, the smaller diameter of steering screw the smaller torsion deformation in the same cross section. The maximum torsional deformation of steering screw is $\varphi_Z=6.327\times10^3$, which calculation based on theory analysis, the simulation result is $\varphi_Z=6.516\times10^3$, and the error is 2.98%. Simplifying steering screw structure equivalently made a difference between theory analysis and simulation in the value. Although the model of theory analysis exists errors, the deformation value and the regularity that come from these formulas are correct.

From Figure.4, the maximum stresses of principal stress and yield stress on the cross section D arise on the interface of balls and steering screw. Thus, the section is a dangerous section of the steering screw, which should be checked emphatically in design.

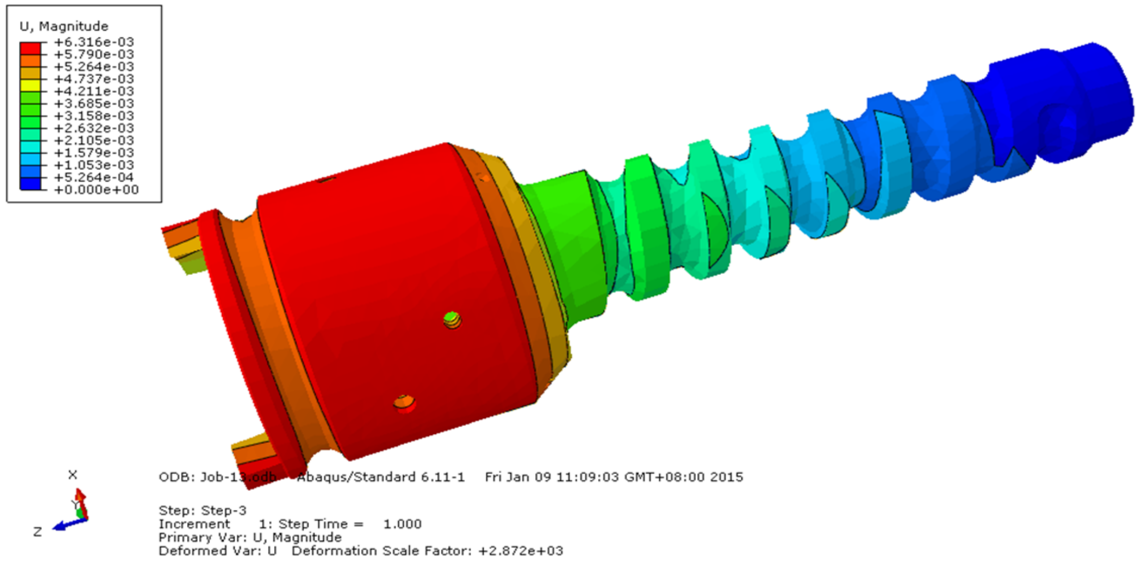

Figure 3. Deformation contour of the steering screw by coupling tension and torque

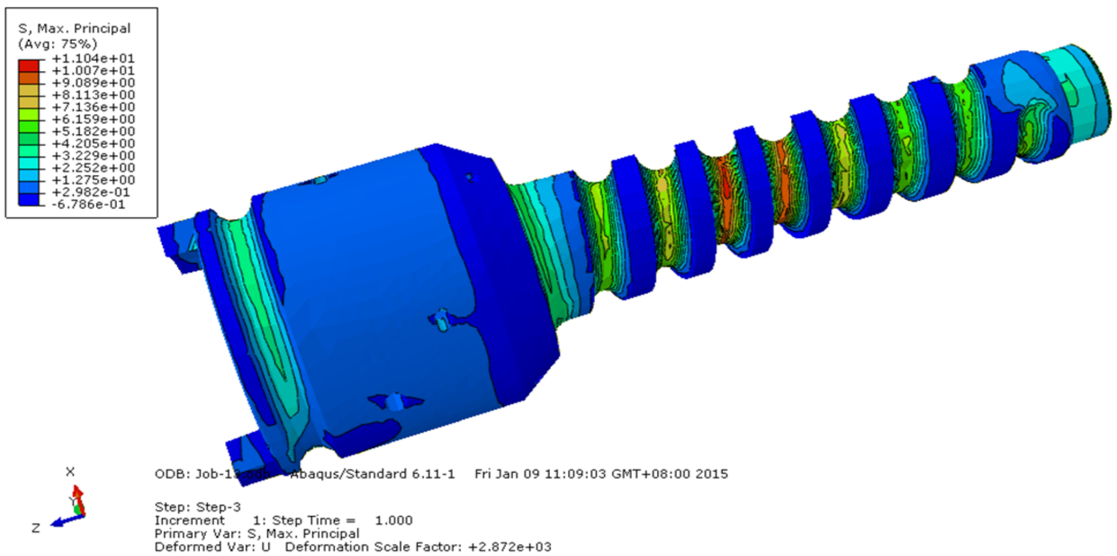

Figure 4. Stresscontour of the steering screw by coupling tension and torque

Table 1. Value of deformation and stress in some sections

Sections	Section C	Section D	Section B	Section A
Axial deformation of analysis /mm	0	6.974×10^{-4}	1.375×10^{-3}	1.643×10^{-3}
Axial deformation of simulation /mm	0	9.176×10^{-4}	1.426×10^{-3}	1.972×10^{-3}
Torsion deformation of analysis /mm	0	3.041×10^{-4}	6.083×10^{-3}	6.327×10^{-3}
Torsion deformation of simulation /mm	0	3.105×10^{-4}	6.217×10^{-3}	6.516×10^{-3}
Max-principal stress /MPa	0	11.04	5.821	8.436

6. Conclusions

The curved surface equation of spiral groove which is based on the working principle of steering screw is established, and the deformation of steering screw coupling torque and axial force is analyzed in this paper. According to Hertz elastic contact theory, the contact deformation model is derived. The function of torsional vibration could be get by means of step function $H(x)$ and function δ. Analyzing the geometry model's simulation contour, it is known that the deformation degree is different in directions and the stress differ in sections. The maximum stress exists in the contact surface of balls and the steering screw, but its cross-sectional area is the smallest. So the section is dangerous. In addition to checking its structural strength, improving the processing quality of the spiral groove surface which can avoid the stress concentration in order to weaken strength is also an important part that a designer should notice.

Acknowledgements

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